Master's thesis proposal : Fundamental limits of passive cloaking on a frequency bandwidth : numerical investigation.

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1 Scientific context and proposed work

While cloaking at a single frequency has been established (even experimentally [12]) for passive cloaks (i.e., cloaks without active energy sources), it is much less certain that one can cloak over an interval of frequencies as metamaterial are very dispersive media (see [3, 14] and [4]), i.e., highly frequency dependent. Yet, passive cloaks have only been explored for single frequency waves which is unrealistic. Indeed by nature, a time-dependent electromagnetic wave has a whole characteristic frequency bandwidth. Hence, one can achieve cloaking at one frequency, but at the same time increases the signature of the object at another frequency making the cloak device useless for broadband signals.

Is it possible to construct a cloak made of a passive electromagnetic material that cloaks an object over a broad frequency band? If not, what are the fundamental limitations to cloaking and performance bounds on cloaking devices over a finite frequency range? Finally in that case, can one synthesize broadband cloaks which achieve the optimal bounds which significantly decrease the signature of an object on a given frequency band and thus makes the object hardly detectable in this frequency range?

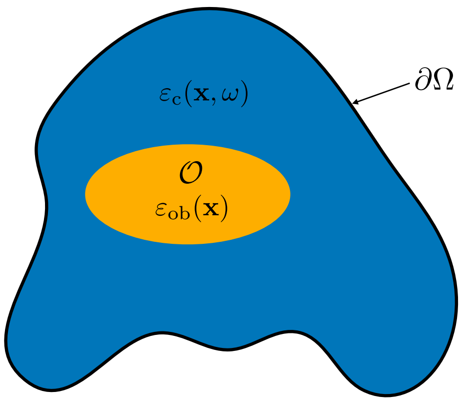

Figure 1: Near-field cloaking refers to the case where the observer is assumed to have access to the Dirichlet-to-Neumann operator associated with a dispersive invisibility cloak (in blue) whose permittivity is given by $\varepsilon_c(\mathbf{x},\omega)$, and which surrounds an obstacle $\mathcal{O}$ (in yellow) with permittivity $\varepsilon_{ob}(\mathbf{x})$.

To address these questions, M. Cassier, G. W. Milton and A. Welters proved that cloaking was impossible over a finite frequency band by establishing quantitative bounds and physical limits to this phenomenon in quasistatics in the far field regime [4] and in the near field regime [2] where the observer has respectively access to the poloarizability tensor or to the Dirichlet-to-Neumann (DtN) map of the cloaking device to probe the system. More precisely, in the near filed regime, the observer can probe the system by imposing a complex voltage at the boundary of the cloaking device and measures on this boundary the resulting normal displacement of the electric field. In other words, he probes the medium by knowing the DtN operator associated to the system over the considered frequency interval.

To obtain these fundamental limits on broadband cloacking, the key mathematical tools come from the theory of Herglotz functions [6] (i.e., analytic functions of the upper-half plane with non-negative imaginary part). The properties (integral representations [3, 8, 13], sum rules [1] and [4], continued fraction expansions of these functions [11] and [5]) are associated with powerful tools in complex analysis that are used to derive optimal bounds to constrain the dispersion of a passive linear systems [10, 13, 9, 14] and [4] (where the existence of such functions is guaranteed by the causality and passivity of the system in the time-domain [7, 14, 3, 4]).

The internship will be divided into two parts: a theoretical component and a numerical component. In the theoretical part, the master's student will become familiar with the theoretical framework of passive electromagnetic materials and the results obtained in [2]. In the numerical part, the student will use the open-source softwares Gmsh [15] and GetDP [16] to evidence whether the bounds established in [2] in the context of near-field cloaking are numerically sharp for two-or-three-dimensional systems. Specifically, the goal will be to design cloaks that approach the fundamental limits of cloaking established in [2].

2 Desired experience

We are looking for a candidate with a strong background in electromagnetism and with good programming skills. A good knowledge in complex analysis is desirable but not mandatory at all.

3 Practical information

The internship will be localize at the Institute Fresnel in Marseille.

3.1 International collaboration

This internship is also in collaboration with **Graeme Milton** (University of Utah) and **Aaron Welters** (Florida Institute of Technology). It may be possible to discuss a potential PhD topic with all relevant partners to pursue after the internship.

References

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