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Strength of tensor force and s-d-shell effective interactions

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The s-d-shell effective interaction is derived from the Bonn NN potential, using a G-matrix folded-diagram method. It is found that due to the relatively weak-tensor-force characteristic for the Bonn potential, the effective interaction matrix elements, particularly those with isospin $T=0$, come out generally more attractive than in previous derivations which were based on conventional local strong-tensor-force NN potentials. This renders the results obtained with the Bonn potential in considerably better agreement with the recent empirical s-d-shell matrix elements of Wildenthal.

Recently, it has been pointed out by the Bonn group[1,2] that the quantitative description of the low-energy nucleon-nucleon (NN) data is consistent with a nuclear tensor force component which is remarkably weaker than commonly assumed. The Bonn group also finds that the NN interaction derived from relativistic meson theory typically contains nonlocalities which lead to a weak tensor force at low energies.

Commonly employed potentials[3-5] are parametrized in terms of local functions which can be obtained in the nonrelativistic, local approximation to one-meson-exchange Feynman amplitudes. With such local expressions the empirical two-nucleon parameters sensitive to the nuclear tensor force (e.g., the deuteron quadrupole moment and asymptotic D/S state ratio, as well as the ϵ_1 mixing parameter up to about 300-MeV lab energy resulting from a phase-shift analysis of elastic NN scattering data) can be fitted only if a relatively strong tensor force is assumed. A practical measure for the strength of the tensor-force component contained in a nuclear potential is the predicted D-state probability of the deuteron, P_D. While from nonrelativistic, local potentials[6] $P_D \approx 6\%$ is obtained (the precise values are the following: Reid,[3] 6.5%; Paris,[4] 5.8%; Argonne V_{14},[5] 6.1%), the Bonn potential[1,2] predicts 4.4%. The strength of the tensor force presently is the most uncertain part of the nuclear force at low energy.

Microscopic nuclear structure predictions—which are based on the bare NN interaction as input—can depend rather sensitively on the strength of the nuclear-tensor potential. Therefore, since the tensor force cannot be uniquely pinned down by two-nucleon data, one may try to obtain complementary information on this issue from the nuclear many-body system.

Recently, the relevance of the tensor force for the bound three-nucleon system has been investigated systematically and demonstrated clearly.[2,7] Applying the Bonn potential, a triton binding energy of 8.35 MeV is obtained,[7] while other potentials[3-5] predict about 7.5 MeV (the experimental value is 8.48 MeV). Also, the other empirically known quantities of the three-nucleon system (e.g., the charge radius[8] and the ^{3}H-^{3}He binding energy difference[9] fall into place for the case of the more attractive Bonn prediction.

The binding energy of nuclear matter is also known to be very sensitive to the strength of the tensor force.[2,10] In an energy *versus* density plot, the saturation points as predicted by conventional nuclear-matter calculations using a variety of two-nucleon potentials are located along a band which has become known as the "Coester line." An essential parameter of this Coester band is the strength of the tensor force at low energy (as measured by P_D) with low-P_D potentials predicting more binding energy than high-P_D potentials. The former, however, usually gives too high a saturation density as compared with the empirical value. As is well known, one can lower the saturation density by including the relativistic effects, which are repulsive and strongly density dependent. Thus, applying an attractive (low P_D) potential and including relativistic effects, nuclear-matter saturation can be rather satisfactorily explained.[2,11]

Nuclear-structure problems discussed so far are dealing only with nuclear ground states. Another important and interesting area are the excited states of nuclei. In a microscopic approach, one starts from a bare NN potential and derives the Brueckner G matrix which, in turn, is used to calculate certain classes of diagrams defining the effective interaction in an open shell, $V_{\rm eff}$. The matrix elements of $V_{\rm eff}$ can then be used to calculate, for example, the excited states of an open-shell nucleus.

The first work along this line was done by Kuo and Brown,[12] who derived matrix elements of the effective interaction between two nucleons outside an ^{16}O core. As is well known, these s-d-shell matrix elements have been remarkably successful, at least qualitatively, in nuclear shell-model calculations. They were, however, derived some twenty years ago and since that time more realistic NN potentials as well as more systematic many-body methods for calculating these matrix elements have been developed. It should be worthwhile, then, to incorporate these improvements into the calculation. An attempt in this direction has been carried out by Shurpin, Kuo, and Strottman.[13] They performed a folded-diagram calculation of these matrix elements using both Reid and Paris NN interactions. The resulting matrix elements seem to

have a general deficiency when compared with the recent and highly successful empirical matrix elements of Wildenthal.[14] As to be illustrated shortly, a main problem appears to be that, in general, there is not enough attraction provided by the calculated matrix elements. This fact was recently stressed also by Daehnick[15] who found, particularly for the $T=0$ matrix elements, a large discrepancy between theory and experiment due to a general lack of attraction on the theoretical side.

Theoretical work done so far has used conventional, local, and strong-tensor-force NN potentials, only. In view of the problems mentioned, it is natural to raise the question as to how sensitive the effective interaction is with regard to the bare two-nucleon potential used as input in these calculations. As discussed, in particular, there is latitude in the strength of the tensor force. It is the purpose of this paper to examine the influence of the strength of the bare two-nucleon tensor force on the theoretically derived effective interaction.

To proceed, let us first briefly describe our procedures[13,16] for deriving the shell-model effective interactions. A first step is to calculate the model-space G matrix defined by the integral equation

$$G(\omega)=V_{NN}+V_{NN}Q_2\frac{1}{\omega-Q_2TQ_2}Q_2G(\omega)\,, \qquad (1)$$

where we use a shell-model Pauli exclusion operator specified by $(n_1,n_2,n_3)=(3,10,21)$.[13] V_{NN} stands for the NN potential, such as the Bonn A potential. Using a matrix inversion method we have solved the above equation in an essentially exact way. In fact, G is calculated as the sum of two terms, G_F, the free-space G matrix and ΔG in which the effect of Q_2 is entirely contained.[13,16]

The effective interaction is then expressed as a folded-diagram series, grouped according to the number of folds. Namely,

$$V_{\rm eff}=F_0+F_1+F_2+F_3+F_4+\cdots\,, \qquad (2)$$

where F_n denotes a $(N+1)$ Q-box term connected with n sets of folded lines. For example, the three-time folded term has the form

$$F_3=-Q\int Q\int Q\int Q\,. \qquad (3)$$

We include in the Q box all the two-body and one-body valence irreducible diagrams up to second order in the model-space G matrix of Eq. (1). (Namely, diagrams D1 to D7 and U of Ref. 13.) We note that only the two-body terms are retained in the folded-diagram series of $V_{\rm eff}$. This is because the one-body terms are presumably already contained in the experimental single-particle (sp) energies which are to be employed in the present framework. In our calculation we shall use harmonic-oscillator wave functions, to be specified by its frequency parameter $\hbar\omega$. In fact, here we use $\hbar\omega=14$ MeV. The folded-diagram series is found to converge satisfactorily; the terms with more than three folds appear to be negligibly small as has been observed and discussed in Ref. 13.

Using the above formalism we have calculated s-d-shell effective interaction matrix elements using the Bonn A potential.[2] This is a relativistic meson-exchange potential defined in the framework of the Blanckenbecler-Sugar reduction of the Bethe-Salpeter equation, and it has a weak tensor force with a deuteron D-state prediction of 4.4%.

It turns out that the effective interaction matrix elements derived from the weak-tensor-force Bonn A potential are in substantially better agreement with the empirical Wildenthal matrix elements than in former derivations in which the stronger-tensor-force potentials of Reid or Paris were used. The better agreement, in general, is due to an increase of attraction. The overall improvement can be seen in the total χ^2 (Ref. 17) calculated for the 63 matrix elements which is 20.1, 24.9, and 25.3 for Bonn A, Reid, and Paris, respectively. For the $T=0$ matrix elements, only, it is 15.0, 17.5, and 18.5, and for $T=1$, one obtains 5.1, 7.4, and 6.8, again for Bonn A, Reid, and Paris, respectively.

In Figs. 1–3, we give an overview of our results for all 63 matrix elements calculated. Some representative results of our calculation are presented in Table I. Consider first the $(abcd,JT)=(4444,01)$ matrix element. The result given by the Paris potential is -2.22 MeV, which is significantly weaker than the Wildenthal result of -2.82 MeV. The result given by the Bonn A potential is -2.77 MeV, which is considerably closer to the empirical value. Recall that the Bonn A potential has a considerably weaker tensor force than that of the Paris potential, the deuteron D-state properties for them being 4.4% and 5.8%, respectively. A similar improvement for the $(abcd,JT)=(4444,10)$ matrix element is also observed. Here the Paris result is -1.01 MeV and the Bonn A one is -1.49 MeV which is rather close to the empirical value -1.63 MeV of Wildenthal. It appears to be a general feature that the effective interaction derived from the weak-tensor-force Bonn A potential is, overall, more attractive

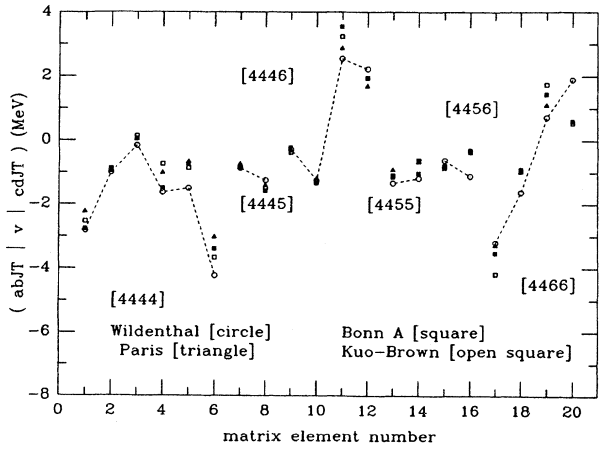

FIG. 1. Comparison of s-d-shell matrix elements. As explained in the caption of Table I the sp orbitals are denoted by numerals 4, 5, and 6. Matrix elements of common $[abcd]$ are grouped together and within each group the ordering for various (J,T) values is $(T=1$, increasing $J)$ to the left of $(T=0$, increasing $J)$. For example, the eight matrix elements of the [4646] multiplet have, counting from left to right, $(JT)=(1,1)$, $(2,1)$, $(3,1),\ldots,(3,0)$, and $(4,0)$.

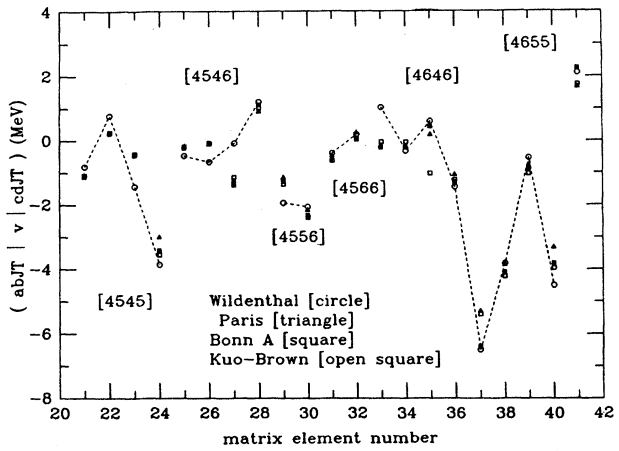

FIG. 2. See caption of Fig. 1.

and consequently yields a better agreement with the empirical matrix elements.

It may be instructive to also compare those matrix elements not listed in Table I to see if there is some general trend. For this purpose let us now look at Figs. 1–3, where we have plotted the entire set of the 63 *sd*-shell matrix elements calculated from the Bonn *A* and Paris potentials together with those of Wilthenthal and of Kuo and Brown. It is seen that the general trend of the Wildenthal matrix elements is rather well reproduced by all the calculated matrix elements. For instance, the general trend of the empirical [4646] multiplet is well reproduced by the calculated ones as indicated in Fig. 2. This by itself is an encouraging result, since, in our opinion, both the *NN* potentials and the many-body methods for the calculation of these matrix elements are still "rather far from being perfect" and a number of improvements can still be made for them. For example, the calculations contain only the two-body part of the effective interaction while actually there are many-body forces and the empirical matrix elements are supposed to contain, in an effective way, some

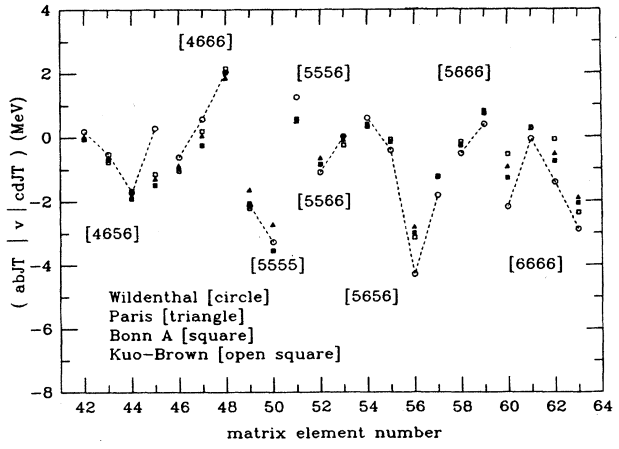

FIG. 3. See caption of Fig. 1.

TABLE I. Shell-model matrix elements $\langle abJT \mid V_{\text{eff}} \mid cdJT \rangle$ (in units of MeV) calculated from the Bonn *A* (Ref. 2) and the Paris potential (Ref. 4), and from the work of Kuo and Brown (Ref. 12), as denoted. The empirical matrix elements are taken from the analysis by Wildenthal (Ref. 14). The orbital notation is $4 = 0d_{5/2}$, $5 = 1s_{1/2}$, and $6 = 0d_{3/2}$.

TJ	*abcd*	Bonn	Kuo-Brown	Empirical	Paris
10	4444	−2.77	−2.53	−2.82	−2.22
	4455	−1.13	−1.09	−1.32	−0.89
	4466	−3.51	−4.11	−3.19	−3.09
	5555	−2.05	−2.21	−2.12	−1.61
	5566	−0.83	−0.84	−1.08	−0.69
	6666	−1.28	−0.54	−2.18	−0.95
01	4444	−1.49	−0.74	−1.63	−1.01
	4455	−1.02	−0.61	−1.18	−0.67
	4466	+1.45	+1.75	+0.72	+1.12
	5555	−3.53	−3.54	−3.26	−2.73
	5566	+0.03	−0.24	+0.03	−0.06
	6666	−0.76	−0.08	−1.42	−0.51

contribution from these many-body forces.

As shown in the figures, a number of calculated matrix elements are not attractive enough, as already mentioned earlier. An example is the [6666] multiplet of Fig. 3. Here the Bonn *A* matrix elements are more attractive but are still not adequate compared to the empirical values. It appears that large differences happen mostly for the matrix elments which involve the $d_{3/2}$ orbit. Some possible reasons for this are the following. First, this orbit is nearly unbound and it is probably not well represented by a harmonic oscillator wave function as is done in the present work. A better way is to employ a more realistic wave function, such as that given by a Woods-Saxon or Hartree-Fock calculation, for this orbit.

A more pertinent reason may be the following. The sp energies are treated as adjustable parameters in Wildenthal's calculation, while in our folded-diagram formalism we are supposed to use the $A = 17$ experimental sp energies whose relative spectrum is (0, 0.87, 5.08) MeV for the orbits 4, 5, 6, respectively. In contrast, the corresponding "best fit" sp energies used by Wildenthal are (0, 0.784, 5.595) MeV. In fact the total two-body effective Hamiltonian H_{eff}, i.e., the sum of the sp energies and the effective interaction matrix elements, for the [6666] multiplet given by our calculation is actually quite close to that of Wildenthal. The above comparison may have brought with it an important message, concerning the many-body forces which have not been considered in our derivation. The above multiplet involves only two valence nucleons and we have reproduced Wildenthal's H_{eff} rather well. Many-body effective forces are not involved in this case. But when there are more than two $d_{3/2}$ nucleons, for example, such forces will have to be calculated in order to reproduce Wildenthal's H_{eff}. We feel that Wildenthal's choice of the sp energies contains, to some extent, an effective way to compensate for the many-body forces which have not been explicitly considered in his empirical matrix elements.

In summary, we have derived the matrix elements for the s-d-shell effective interaction from the weak-tensor-force Bonn potential, using a G-matrix folded-diagram method. The essential result is that these matrix elements turn out generally more attractive than in former derivations in which more conventional local strong-tensor-force potentials were applied. This increase of attraction leads to a substantially better agreement with the empirical s-d-shell matrix elements. The present results derived from the Bonn potential together with those obtained earlier, like the successful prediction of the triton binding energy[7] and the quantitative explanation of nuclear-matter saturation,[2,11] may indicate that modern genuine meson-theoretic potentials allow for a more consistent description of nuclear structure than traditional nuclear-force models.

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