Photonic theory : Green's functions, Scattering and Density of states

1. Electron wavelength Use the relativistic formula for a point particle to find the de Broglie wavelength of an electron whose kinetic energy is $K = 2$ eV. Give a numerical value (in metres and in nanometres).

Why the energy $E = h\nu$ in non-relativistic quantum mechanics refers to kinetic energy

In non-relativistic quantum mechanics, the relation $E = h\nu$ connects the frequency ν of a particle's matter wave to its *mechanical* energy, that is, its kinetic (and possibly potential) energy rather than the total relativistic energy including the rest mass term mc^2.

Intuitive explanation. The rest energy mc^2 is an enormous constant that does not affect the particle's dynamics. Adding a constant to all energies merely multiplies the wavefunction by an overall phase factor,

$$\psi(t) = e^{-iE_{\text{tot}}t/\hbar} = e^{-i(mc^2t/\hbar)}\, e^{-i(E_{\text{kin}}t/\hbar)}.$$

The first exponential, $e^{-imc^2t/\hbar}$, corresponds to a uniform and unobservable phase rotation. Only the second term, containing E_{kin}, governs the observable time evolution of the wavefunction.

From relativity to Schrödinger. Starting from the relativistic relation

$$E^2 = p^2c^2 + m^2c^4,$$

one finds, for $p \ll mc$,

$$E = mc^2 + \frac{p^2}{2m} - \frac{p^4}{8m^3c^2} + \cdots .$$

When we insert this into a plane wave

$$\psi(\mathbf{r}, t) = e^{i(\mathbf{p}\cdot\mathbf{r}-Et)/\hbar} = e^{-imc^2t/\hbar}\, e^{i(\mathbf{p}\cdot\mathbf{r}-\frac{p^2}{2m}t)/\hbar},$$

the first factor again produces only a global, unobservable phase. Discarding it leads directly to the Schrödinger equation, where $E = h\nu$ refers to the kinetic (and potential) energy.

In summary: in quantum mechanics, only *energy differences* affect physical observables. The rest energy mc^2 simply shifts all levels by a constant and can be ignored. Hence, in the non-relativistic limit, the E in $E = h\nu$ represents the kinetic energy of the particle.

Compton versus de Broglie wavelength

Two characteristic wavelengths can be associated with a massive particle, depending on which form of its energy is used in the relation $E = h\nu$.

1. Compton wavelength (rest-energy based):

$$\lambda_C = \frac{h}{mc}.$$

It corresponds to using the *total* relativistic energy $E = mc^2$ in $E = h\nu$. The associated frequency $\nu_C = mc^2/h$ describes the extremely rapid internal phase rotation of the particle's wavefunction:

$$\psi(t) \sim e^{-imc^2 t/\hbar}.$$

For an electron,

$$\lambda_C = 2.426 \times 10^{-12} \text{ m}.$$

The Compton wavelength marks the scale where relativistic and quantum effects (e.g. pair creation) become important; a particle cannot be localized to better than about λ_C without creating new particles.

2. de Broglie wavelength (momentum based):

$$\lambda_{\text{dB}} = \frac{h}{p}.$$

It follows from the *kinetic* energy or momentum of motion, and governs interference and diffraction phenomena. For a 2 eV electron,

$$\lambda_{\text{dB}} \approx 8.7 \times 10^{-10} \text{ m}.$$

In summary:

$$\boxed{\text{Compton: } E = mc^2 \Rightarrow \lambda_C = h/(mc)} \quad \text{vs.} \quad \boxed{\text{de Broglie: } p = h/\lambda_{\text{dB}}.}$$

The Compton wavelength is a fixed relativistic constant for a given particle, whereas the de Broglie wavelength depends on its velocity and describes its wave-like behavior in motion.

2. Calulate the wavelength of a 2eV photon. Compare this to the deBroglie wavelength of the electron.

3. Suppose we have a material medium that absorbs a single frequency so strongly that the imaginary of its complex susceptibility, $\chi(\omega) = \chi'(\omega) + i\chi''(\omega)$ can be approximated as a delta function :

$$\chi''(\omega) = K\delta(\omega - \omega_r) \tag{1}$$

where K is a real constant. This idealized model corresponds to an infinitely narrow absorption line and is therefore unphysical, but it illustrates clearly how dispersive features arise from absorption.

Use the Kramer's Krönig relations to determine the real part of the susceptibility, $\chi'(\omega)$

4. We consider small spherical scatterers composed of a noble metal (or another plasmonic material) with diameters below a few tens of nanometers, embedded in a homogeneous

background medium with real constitutive parameters ε_b and μ_b. Assuming that the particles are much smaller than the in-medium wavelength λ of visible light, their electromagnetic response can be modeled by an induced electric dipole moment $\boldsymbol{p}(\omega)$.

If the particle response is linear and isotropic, the induced dipole moment is related to the incident electric field $\boldsymbol{E}_{\text{inc}}(\omega)$ through the polarizability $\alpha(\omega)$:

$$\boldsymbol{p}(\omega) = \epsilon_0 \varepsilon_b \alpha(\omega) \boldsymbol{E}_{\text{inc}}(\omega) \ .$$

From considerations involving the Poynting vector and radiation flux, the extinction and scattering cross sections of an electric dipole scatterer can be expressed in terms of polarizability, $\alpha(\omega)$, as:

$$\sigma_{\text{ext}} = \kappa \Im m \left\{ \alpha(\omega) \right\} \qquad \sigma_{\text{scat}} = \frac{\kappa^4}{6\pi} \left| \alpha(\omega) \right|^2 \ . \tag{2}$$

where $\kappa = \frac{\omega}{c}\sqrt{\varepsilon_b \mu_b} \equiv \frac{2\pi}{\lambda}$ with λ the wavelength in the host (background) medium

Energy conservation in the scattering process imposes the inequality:

$$\frac{\kappa^3}{6\pi} \left| \alpha(\omega) \right|^2 \leq \Im m \left\{ \alpha(\omega) \right\} \ . \tag{3}$$

(a) What are the physical dimensions of σ and α? Briefly justify why these dimensions are physically reasonable.

(b) Express the inequality of eq.(5) in terms of the cross-sections given in eq.(4). The inequality becomes an equality in the case where scattering occurs in the absence of loss into material degrees of freedom. What does this imply for the relationship between the extinction, scattering, and absorption cross sections?

(c) Write $\alpha(\omega) = \alpha'(\omega) + i\alpha''(\omega)$, with α' and α'' real.

i. Rewrite Eq. (5) in terms of α' and α''.

ii. Deduce the maximum allowed value of α''. *Hint:* The bound is saturated when $\alpha' = 0$.

iii. Hence obtain an upper bound on σ_{ext}.

(d) Recalling that the wavenumber is related to the wavelength in the external medium by, $\kappa = 2\pi/\lambda$, use the results of the previous question, and the approximation $\pi \simeq 3$ to find the often stated (but seldom derived) *unitary limit* for a dipole interaction cross section: $\sigma \leq \frac{\lambda^2}{2}$.

5. The *quasi-static* expression for the electric dipolar polarizability of a sphere of volume, V, and dielectric permittivity ε_s, immersed in a background medium of dielectric permittivity, ε_b, can be written:

$$\alpha_{\text{qs}} \equiv \lim_{\omega \to 0} \alpha(\omega) = 3V \frac{\varepsilon_s - \varepsilon_b}{\varepsilon_s + 2\varepsilon_b} \ . \tag{4}$$

This expression is valid in the electrostatic (Rayleigh) limit, ($\kappa R \ll 1$), for non-magnetic, isotropic materials.

(a) If a peak in the cross sections is observed near $\lambda = 365\,\text{nm}$, what condition must be approximately satisfied by the real part of the metal permittivity ε_s relative to ε_b at this wavelength?

(b) A commonly employed model for including 'radiative damping' to the quasi-static polarizability is given by:

$$\alpha(\omega) \simeq \frac{\alpha_{\rm qs}}{1 - i\frac{\kappa^3}{6\pi}\alpha_{\rm qs}} \,. \tag{5}$$

What is the limiting value of $\alpha(\omega)$ when $\alpha_{\rm qs} \to \infty$? Comment on this result based on your results from question 4. Why is this model superior to simply using the quasi-static value for polarizability ?

(c) Even though the refinement proposed in Eq.(10) for including the radiative damping of the polarizability is crucial in many situations, it is often ignored by people modeling plasmonic particles with diameters less than 20nm. Explain why (hint: consider Eqs. (8) and (10)), and the real and imaginary parts of a typical Drude permittivity in the visible range as shown in Fig.(1).

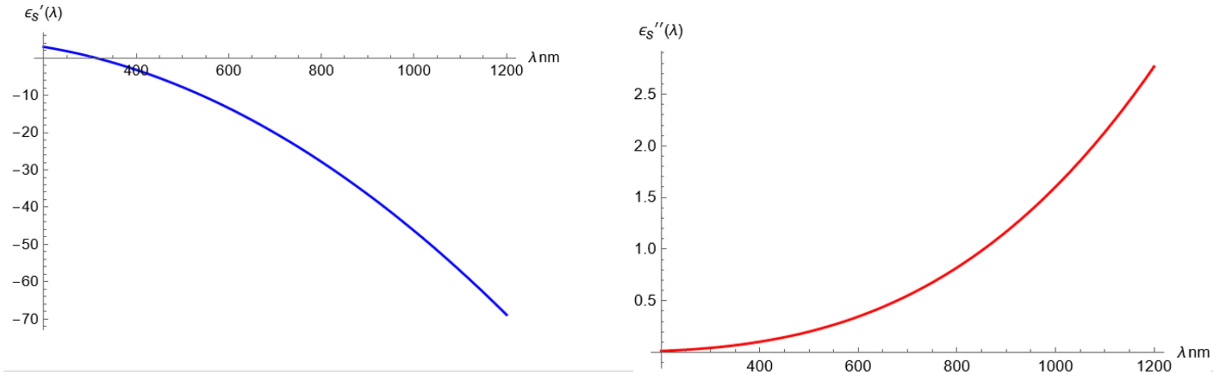

Figure 1: Real part of the permittivity (left: Re$[\varepsilon_s]$) and its imaginary part (right: Im$[\varepsilon_s]$) of an 'idealized' Drude permittity for a plasmonic conductor in the visible range.

6. **Free-space electromagnetic Green's function :**
The electric-field free-space dyadic Green's function relates an oscillating electric dipole source to the the resulting electric field in the frequency domain :

$$\overleftrightarrow{g}(\boldsymbol{r}) = \frac{e^{i\kappa r}}{4\pi\kappa^2 r^3}\mathcal{P.V.}\left\{(1-i\kappa r)\left(3\boldsymbol{u}_r\boldsymbol{u}_r - \overleftrightarrow{\mathbb{I}}\right) + \kappa^2 r^2\left(\overleftrightarrow{\mathbb{I}} - \boldsymbol{u}_r\boldsymbol{u}_r\right)\right\} - \frac{\overleftrightarrow{\mathbb{I}}}{3\kappa^2}\delta^3(\boldsymbol{r}) \ , \tag{6}$$

where $\boldsymbol{u}_r \equiv \frac{\boldsymbol{r}}{r}$ is the unit vector in the radial direction, $\kappa \equiv \frac{\omega\sqrt{\varepsilon_b\mu_b}}{c} \equiv \frac{2\pi}{\lambda_b}$, is the wavenumber in the external(background) medium, and finally $\mathcal{P.V.}$ stands for principal value (associated with a 3D exclusion volume at the origin on account of the field being undefined when $r \to 0$. The Green's function $\overleftrightarrow{g}$, allows one to directly obtain the time harmonic electric field of an oscillating dipole, $\boldsymbol{p}$, positioned at the coordinate system origin and oriented along a unit vector $\boldsymbol{n_p}$ (*i.e.* $\boldsymbol{p} = |\boldsymbol{p}|\boldsymbol{n}_p$). Since this field is produced by a source, (like a coaxial feed cable or induced by an "incident" field), we henceforth refer to it as the "scattered" field :

$$\begin{aligned}\boldsymbol{E}^{(\mathrm{s})}(\boldsymbol{r}) &= \frac{\omega^2}{\epsilon_0 c^2}\overleftrightarrow{g}(\boldsymbol{r})\cdot\boldsymbol{p} \\ &= \mathcal{P.V.}\left\{\frac{e^{i\kappa r}}{4\pi\epsilon_0\varepsilon_b r^3}\left[\kappa^2 r^2\left[\boldsymbol{p} - \boldsymbol{u}_r\left(\boldsymbol{u}_r\cdot\boldsymbol{p}\right)\right] + (1-i\kappa r)(3(\boldsymbol{u}_r\cdot\boldsymbol{p})\boldsymbol{u}_r - \boldsymbol{p})\right]\right\} \\ &\qquad - \frac{\boldsymbol{p}}{3\epsilon_0\varepsilon_b}\delta^3(\boldsymbol{r}) \ .\end{aligned} \tag{7}$$

a) Identify the **near-field** part of the scattered field, $\boldsymbol{E}^{(\mathrm{s})}$ and explain the physical relevance of the $-\frac{\boldsymbol{p}}{3\epsilon_0\varepsilon_b}\delta^3(\boldsymbol{r})$ term in eq.(13), including a discussion of why this term is directed in the direction opposite to the dipole, $\boldsymbol{p}$.

b) It is convenient to describe the dipole field mathematically by aligning the dipole moment along the z-axis so that the dipole moment is written $\boldsymbol{p} = |\boldsymbol{p}|\boldsymbol{n_p} \longrightarrow |\boldsymbol{p}|\boldsymbol{u}_z$, which can be induced by an electromagnetic plane wave propagating along the x-axis, *i.e.* $\boldsymbol{k}/|\boldsymbol{k}| = \boldsymbol{u}_x$. The scattered fields are in spherical coordinates are then:

$$\boldsymbol{E}^{(\mathrm{s})}(\boldsymbol{r}) = E_r^{(\mathrm{s})}(r,\theta)\boldsymbol{u}_r + E_\theta^{(\mathrm{s})}(r,\theta)\boldsymbol{u}_\theta + E_\phi^{(\mathrm{s})}(r,\theta)\boldsymbol{u}_\phi \tag{8}$$

We can obtain expressions for these components by using the relation,

$$\boldsymbol{u}_z = \cos\theta\boldsymbol{u}_r - \sin\theta\boldsymbol{u}_\theta \tag{9}$$

in Eq.(13). Determine the expressions for $E_r^{(\mathrm{s})}$, $E_\theta^{(\mathrm{s})}$, and $E_\phi^{(\mathrm{s})}$ that ignore the delta function contribution.

c) Since we know the scattered electric field, the scattered magnetic field is obtained from the Maxwell equation:

$$i\omega\boldsymbol{B} = \boldsymbol{\nabla}\times\boldsymbol{E} \ , \tag{10}$$

the constitutive relation in the background medium, $\boldsymbol{H}_{\mathrm{S.I.}} = \frac{\boldsymbol{B}}{\mu_b\mu_0}$, and the curl in spherical coordinates:

$$\begin{aligned}\boldsymbol{\nabla}\times\boldsymbol{E}(r,\theta,\phi) = \frac{\boldsymbol{u}_r}{r\sin\theta}\left[\frac{\partial\left(E_\phi\sin\theta\right)}{\partial\theta} - \frac{\partial E_\theta}{\partial\phi}\right] + \frac{\boldsymbol{u}_\theta}{r}\left[\frac{1}{\sin\theta}\frac{\partial E_r}{\partial\phi} - \frac{\partial\left(rE_\phi\right)}{\partial r}\right] + \\ + \frac{\boldsymbol{u}_\phi}{r}\left[\frac{\partial\left(rE_\theta\right)}{\partial r} - \frac{\partial\left(E_r\right)}{\partial\theta}\right] \ .\end{aligned} \tag{11}$$

Find the expression for $\boldsymbol{H}^{(s)}(r,\theta,\phi)$. What do you remark about, $\boldsymbol{H}^{(s)}$, in the near and far fields respectively ?

d) A question of significant practical importance is to determine the total power radiated by the electric dipole field in eq.(13). At a fundamental level, this is done by determining the associated $\boldsymbol{H}^{(s)}$ field and then using the Poynting vector to calculate radiation flux leaving any closed volume surrounding the dipole. Such calculations can most easily be done in the far-field limit where the expressions for fields are the simplest. One can show that that in the far-field limit, the scattered, $\boldsymbol{H}^{(s)}$ field is related to the scattered $\boldsymbol{E}^{(s)}$ field via the relation :

$$\lim_{r\to\infty} \boldsymbol{H}^{(s)} \to \frac{\kappa}{\omega\mu_b\mu_0}\boldsymbol{u}_r \times \boldsymbol{E}^{(s)} = \sqrt{\frac{\varepsilon_b}{\mu_b}}\eta_0 \boldsymbol{u}_r \times \boldsymbol{E}^{(s)} \ . \tag{12}$$

Show that this relation is indeed consistent with the full expression for $\boldsymbol{H}^{(s)}$ found in the previous question.

e) Now we derive the radiated power of a dipole: The formula for the time averaged Poynting vector, in a time-harmonic formulation is:

$$\langle \boldsymbol{\Pi} \rangle_T = \frac{1}{2}\Re\left\{\boldsymbol{E}^{(s),*} \times \boldsymbol{H}^{(s)}\right\} \tag{13}$$

Use this formula to calculate the time averaged Poynting vector in the far-field, and then integrate this over the surface of a sphere at infinity to determine the average power, $\langle P_{\text{rad}}\rangle_T$, radiated by the oscillating electric dipole.

f) From here on we connect Green's function to LDOS and radiated power. The component of the dipole's electric field parallel to $\boldsymbol{p}$ ($E_z(r,\theta) \equiv \boldsymbol{n_p}\cdot\boldsymbol{E}^{(s)}$) plays an important role in the quantum theory of spontaneous emission and in antenna theory. Use eq.(13) to demonstrate that:

$$\begin{aligned} E_z^{(s)}(r,\theta) &= \frac{|\boldsymbol{p}|}{4\pi\epsilon_0\varepsilon_b}\frac{\exp(i\kappa r)}{r}\mathcal{P}.\mathcal{V}.\left[\kappa^2\sin^2\theta - i\kappa\frac{3\cos^2\theta-1}{r} + \frac{3\cos^2\theta-1}{r^2}\right] \\ &= \kappa^3\frac{|\boldsymbol{p}|}{4\pi\epsilon_0\varepsilon_b}\frac{\exp(i\chi)}{\chi}\mathcal{P}.\mathcal{V}.\left[\sin^2\theta - i\frac{3\cos^2\theta-1}{\chi} + \frac{3\cos^2\theta-1}{\chi^2}\right] \ . \end{aligned} \tag{14}$$

where we defined $\chi \equiv \kappa r$.

g) Green function theory establishes that the local density of states (or local density of modes) is related to the *imaginary part* of $E_z^{(s)}(r,\theta)$ in eq.(27). This connection is workable since even though the real part of $E_z^{(s)}$ diverges as $r \to 0$, its imaginary component, $\Im[E_z^{(s)}(r)]$ remains finite in this limit. To verify this behavior for small values of r, use the Taylor expansion of the exponential function for small arguments,

$$\exp(i\chi) = 1 + i\chi - \frac{\chi^2}{2} - i\frac{\chi^3}{6} + \dots \tag{15}$$

Approximating $\exp(i\chi)$ up to third order in χ is shown in eq.(28), find the imaginary part of $\lim_{r\to 0} E_z^{(s)}(r,\theta)$.

h) The time averaged emitted power, $\langle P_e\rangle_T$ delivered by a time-harmonic *emitter*, located within a volume $\mathcal{V}$, is equal to the negative of the power carried out by the electric field, $\boldsymbol{E}$, acting on the time harmonic current $\boldsymbol{j}$, that produced the field:

$$\langle P_e\rangle_T = -\frac{1}{2}\iiint \Re\left\{\boldsymbol{E}^*\cdot\boldsymbol{j}\right\} d^3r \ . \tag{16}$$

The current created by an electric dipole placed at the origin is:

$$\boldsymbol{j} = -i\omega\boldsymbol{p}\delta^3(\boldsymbol{r}) \ . \tag{17}$$

Use these expression to calculate the power emitted by the dipole source of the field, and discuss its relationship to the radiated power, $\langle P_{\rm rad}\rangle$ calculated in part e).

i) One can compare the imaginary part of the field at the origin, found in part g) with the expressions in eqs.(12) and (13) to demonstrate the important result for local density of states calculations that :

$$\Im\left\{\boldsymbol{n_p}\cdot\overleftrightarrow{\boldsymbol{g}}(\boldsymbol{0})\cdot\boldsymbol{n_p}\right\} = \frac{\kappa}{6\pi} \ . \tag{18}$$

Expressing the Green's function in Fourier space using the same techniques as we saw in class for the scalar Helmholtz equation, one finds the formula for the Density Of States (DOS) in terms of the dyadic Green's function :

$$\rho_{\rm DOS}(\omega) = \frac{6\varepsilon_b\mu_b\omega}{\pi c^2}\Im\left\{\boldsymbol{n_p}\cdot\overleftrightarrow{\boldsymbol{g}}(\boldsymbol{r},\boldsymbol{r})\cdot\boldsymbol{n_p}\right\} \ . \tag{19}$$

Question: Use the result of eq.(34) in eq.(35) to express the homogeneous medium density of states in terms of the speed of light in the background medium, $c_b \equiv \frac{c}{n_b} = \frac{c}{\sqrt{\varepsilon_b\mu_b}}$. How does this correspond to the expression arrived at by counting arguments in the calculation of black-body radiation ?

j) Express the emitted power, $\langle P_e\rangle_T$, determined in (i), in terms of the homogeneous medium density of states of the background medium, $\rho_{\rm DOS}(\omega)$, $|\boldsymbol{p}|^2$ and ω.

7. **Purcell factor and Local density of states:** Edward Mills Purcell was the first person to realize, in 1947, that an emitter's decay rate could be modified by its external environment.

Purcell initially formulated this concept in the context of a resonant cavities with a resonant frequency near a quantum mechanical transition frequency. He considered a high-Q resonance scenario where the decay rate enhancement would be largely independent of the emitter's position within the cavity. In this setting, he derived an expression, F_P for the enhancement of spontaneous decay as:

$$F_P = \frac{3}{4\pi^2}\left(\frac{\lambda}{n}\right)^3\frac{Q}{V} \tag{20}$$

where Q was the Quality factor of the resonance, and V the *mode volume* which was taken to be the physical volume of the cavity.

However, experiments later revealed that spontaneous decay rates could even be modified by open structures, like as a mirror, near the emitter. This discovery sparked decades-long debates about how the Purcell factor should be adapted to account for decay rate modifications in open systems. It is now generally accepted that as long as F_P is allowed to depend on the particle's position, $\boldsymbol{r}$, with respect to its local environment then it can be defined as:

$$F_P(\boldsymbol{r}) \equiv \frac{\tau_0}{\tau(\boldsymbol{r})} = \frac{\Gamma(\boldsymbol{r})}{\Gamma_0} \tag{21}$$

where Γ_0 is the spontaneous decay rate of the emitter measured in the vacuum, and $\Gamma(\boldsymbol{r})$ is its modified decay rate at a position $\boldsymbol{r}$, with the local environment typically structured at the wavelength scale in order produce significant deviations of F_P from 1.

Calculations of $F_P(\boldsymbol{r})$ tacitly assume that the emitter is sufficiently small with respect to the wavelength and the scale of the local environment, that the decay rates of an emitter at a given position is determined by the radiation reaction deduced from system's *total* Green function, $\boldsymbol{n}_p \cdot \overleftrightarrow{\boldsymbol{G}}(\boldsymbol{r},\boldsymbol{r}) \cdot \boldsymbol{n_p}$, a quantity conventionally called the Local Density Of States (LDOS). For an emitter in a homogeneous medium, but which has nearby "scattering" materials, one can write:

$$\overleftrightarrow{\boldsymbol{G}} \equiv \overleftrightarrow{\boldsymbol{g}} + \overleftrightarrow{\boldsymbol{G}}_s \tag{22}$$

where $\overleftrightarrow{\boldsymbol{g}}$ is the homogeneous medium Green's function, and $\overleftrightarrow{\boldsymbol{G}}_s$ is the "scattering" Green's function that takes into account the scattering of the radiation emitted by the source. Given the definition, $F_P(\boldsymbol{r}) \equiv \frac{\Gamma(\boldsymbol{r})}{\Gamma_0}$, we deduce spontaneous decay rates should also be proportional to radiated power, such that :

$$F_P(\boldsymbol{r}) = \frac{P(\boldsymbol{r})}{P_{e,0}}. \tag{23}$$

which is a ratio of classical emitted powers rather than quantum transition rates. However, since F_P is defined as ration of radiation rates, F_P only depends on the density of state ratios that we will find here.

(a) Given your calculations of the previous exercise, one can deduce that the LDOS of a dipole emitter at a position, $\boldsymbol{r}$, operating at a frequency, ω, and oriented along the direction, $\boldsymbol{n_p}$ and placed in a "scattering" environment described by, $\overleftrightarrow{\boldsymbol{G}}$, is:

$$\rho_{\text{LDOS}}(\boldsymbol{r},\omega) = \frac{6\varepsilon_b\mu_b\omega}{\pi c^2}\Im\left\{\boldsymbol{n_p} \cdot \overleftrightarrow{\boldsymbol{G}}(\boldsymbol{r},\boldsymbol{r}) \cdot \boldsymbol{n_p}\right\} , \tag{24}$$

Furthermore, following part k) of the previous exercise, the power emitted by the dipole in a locally modified environment is:

$$\langle P_e(\boldsymbol{r},\omega)\rangle_T = \rho_{\text{LDOS}}(\boldsymbol{r},\omega)|\boldsymbol{p}|^2\frac{\pi\omega^2}{12\epsilon_0\varepsilon_b} \tag{25}$$

From the above formulas, give the expression for F_P directly in terms of the Green's functions, $\overleftrightarrow{\boldsymbol{G}}$ and $\overleftrightarrow{\boldsymbol{g}}$.

(b) Take the expression derived in part a) above together with eq.(41) and the expression for $\Im\left\{\boldsymbol{n_p} \cdot \overleftrightarrow{\boldsymbol{g}}(\boldsymbol{r},\boldsymbol{r}) \cdot \boldsymbol{n_p}\right\}$ that we obtained in eq.(34) in order to obtain an expression for F_P written entirely in terms of $\overleftrightarrow{\boldsymbol{G}}_s$:

(c) The utility of all these manipulations becomes clear once you realize that the expression that you derived in part b) gives you a concrete method for calculating F_P provided that you have a numerical solver capable of calculating the field resulting from a small dipole emitter in the desired structured environment. Describe briefly, how you would go about this.