

# Light enpolarization by disordered media under partial polarized illumination: The role of cross-scattering coefficients

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**Abstract:** We show how disordered media allow to increase the local degree of polarization (DOP) of an arbitrary (partial) polarized incident beam. The role of cross-scattering coefficients is emphasized, together with the probability density functions (PDF) of the scattering DOP. The average DOP of scattering is calculated versus the incident illumination DOP.

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## 1. Introduction

Light polarization has been the focus of numerous studies for years [1–5], and unified theories today offer a rigorous mathematical treatment based on a matrix formalism. Hence partial polarization, temporal and spatial coherence can be simultaneously addressed for a better description of light properties. Most often partial polarization is investigated through temporal averages of square fields or correlations at a given location, which allows to define a local degree of polarization (DOP). Notice that similar effects and definitions can be obtained with spatial averages at different scales [6–8] that are not considered in this paper.

Due to the statistical behavior of phase delays associated with the optical field, its orientation may appear deterministic or quasi-random. Such random vector nature of light is usually and classically [1–5] quantified through the degree of polarization (DOP). For unpolarized light (DOP = 0), the randomness is maximum and the DOP cannot be modified when passing through an optical system without losses [9,10]; more exactly, a zero DOP of a beam at the entrance of an optical system remains zero at the system output, provided that the energy carried by the propagating beam is kept constant.

In this context enpolarization effects [9–11–12] may appear as unexpected results. This is indeed the case of the scattering process that was recently shown to increase the local DOP of light scattered from disordered media [11,12], even though the incident illumination was fully depolarized. More detail can be found in [11] where theory and experiment reveal high agreement, and where the statistics of the local DOP of scattering were shown to be  $\text{PDF}(u) = 3u^2$ , with PDF the probability density function. Furthermore an analytical demonstration followed in [12].

Such results on light enpolarization do not disagree with the fact that a lossless optical system would maintain the entrance DOP of a beam. Indeed enpolarization here is the result of a scattering process, which is connected with a loss process, in the same way as absorption, splitting (reflection or transmission), diffraction or scattering...

Other situations were previously discussed where enpolarization effects occur independently of a scattering process. In particular in ref [13] a relationship is given between the DOP and the coherence of an electromagnetic beam in a Young interference pattern, together with a validation by experiment. However in the present paper the incident and scattered waves are assumed to be fully coherent, so that enpolarization takes its origins in a different process.

In a general way enpolarization cannot occur for specular processes (reflection, transmission) and also vanishes for low scattering media predicted with perturbative (first-order) theories [14]. In fact the scattered-induced enpolarization effect is a local process and requires a high scattering medium to occur. It results from two processes:

- The first process lies in the arbitrary value of the energy ratio of the scattered polarization modes, due to the relative amplitude of the scattering coefficients on each polarization axis.
- The second process is less common and is related to an increase of mutual correlation between the polarization modes of scattering, which results from a linear combination of the incident modes on each polarization axis. This last property is specific of the scattering process and enforces itself with the value of cross-scattering coefficients; it will be further discussed throughout the next sections.

A number of preliminary results were already given in a recent paper gathering numerical, experimental and analytical data [11,12]. High scattering media were shown to be the seat of enpolarization effects when illuminated with totally unpolarized light. The local DOP of scattering took arbitrary values within the range [0,1] from one location of space to another, with a spatial average of 0.75 and a  $3u^2$  PDF law.

However all results in [11,12] are limited to a fully depolarized incident light, for which reason we here address the most general situation of an arbitrary degree of polarization for the incident beam. We emphasize new signatures, averages and statistics for the probability density function (PDF) of the DOP of scattering. The influence of the cross-scattering coefficients is investigated in detail. Applications concern the identification of living tissues in biomedical optics, but may also find specific interest in defense and earth observatory, precision optics and lightening...

## 2. Principles of enpolarization

### 2.1. Incident partial polarization

Basic results are first briefly recalled though can be found in numerous references [1–5]. We consider an incident quasi-parallel and quasi-monochromatic beam that propagates the

electromagnetic field  $\vec{E}_0 = \begin{cases} E_{0s}(t) \\ E_{0p}(t) \end{cases}$ , with  $E_{0s}$  and  $E_{0p}$  the polarization modes. These modes

are analytical signals and the polarization properties can be analyzed through the complex coherency matrix  $J_0$  given by [5]:

$$J_0 = \begin{pmatrix} \langle E_{0s} \bar{E}_{0s} \rangle & \langle E_{0s} \bar{E}_{0p} \rangle \\ \langle \bar{E}_{0s} E_{0p} \rangle & \langle E_{0p} \bar{E}_{0p} \rangle \end{pmatrix} \quad (1)$$

where  $\bar{E}$  is used for complex conjugation, and the brackets  $\langle \rangle$  are for temporal average. Within this formalism the incident degree of polarization ( $DOP_0$ ) can be directly calculated from the determinant (det) and the trace (tr) of the matrix, that is [5]:

$$DOP_0^2 = 1 - 4 \frac{\det[J_0]}{\{tr[J_0]\}^2} \quad (2)$$

Equation (2) can also be written versus the two parameters that are the mutual coherence  $\mu_0$  (or correlation) and the polarization ratio  $\beta_0$  of the incident field:

$$DOP_0^2 = 1 - 4 \frac{\beta_0}{[1 + \beta_0]^2} [1 - |\mu_0|^2] \quad (3)$$

with:

$$\beta_0 = \frac{\langle |E_{0p}|^2 \rangle}{\langle |E_{0s}|^2 \rangle} \quad \text{and} \quad \mu_0 = \frac{\langle E_{0s} \bar{E}_{0p} \rangle}{\sqrt{\langle |E_{0s}|^2 \rangle \langle |E_{0p}|^2 \rangle}} \quad (4)$$

Hence the correlation factor  $\mu$  and the polarization ratio  $\beta$  control the DOP value. Notice that these parameters vary with the choice of the axis while the DOP remains invariant for a

unitary transformation [15]; for this reason the DOP is a key parameter to describe light polarization.

Such formulae recall that there are two ways to fully polarize light, which consist in giving full correlation ( $\mu = 1$ ) to the modes, or in transferring the whole energy on one polarization axis ( $\beta = 0$  or  $1/\beta = 0$ ). Indeed from Eq. (3):

$$DOP = 1 \Rightarrow \beta = 0 \text{ or } \frac{1}{\beta} = 0 \text{ or } |\mu| = 1 \quad (5)$$

On the other hand, totally unpolarized light involves a unique couple of parameters ( $\beta = 1$  and  $\mu = 0$ ):

$$DOP = 0 \Rightarrow \beta = 1 \text{ and } \mu = 0 \quad (6)$$

From this last property (6) we notice that any correlation increase (due to a scattering process for instance- see next section) will make the DOP get departure from zero:

$$\mu \neq 0 \Rightarrow DOP \neq 0 \quad (7)$$

It is also useful to address the reciprocity of these remarks, that is:

a zero correlation does not guarantee unpolarized light:

$$\mu = 0 \Rightarrow DOP = \frac{1-\beta}{1+\beta} \neq 0 \text{ unless } \beta = 1 \quad (8)$$

a unity correlation guarantees full polarization:

$$\mu = 1 \Rightarrow DOP = 1 \quad (9)$$

full polarization is obtained for extreme values of the polarization ratio:

$$\beta = 0 \text{ or } \frac{1}{\beta} = 0 \Rightarrow DOP = 1 \quad (10)$$

when the polarization ratio is unity, the DOP is equal to the modulus of correlation:

$$\beta = 1 \Rightarrow DOP = |\mu| \quad (11)$$

At last in the case of partial polarization, different pairs ( $\mu_0, \beta_0$ ) of parameters may allow to keep a constant DOP with the condition:

$$\mu_0 = \mu_0(DOP_0, \beta_0) \quad (12)$$

However this parameter range is strongly reduced with the DOP value, which means that an arbitrary DOP cannot be reached with an arbitrary polarization ratio (or correlation).

## 2.2. The scattered field and the cross-scattering coefficients

When the incident beam lightens a disordered medium (rough surface or inhomogenous bulk), light is scattered at all directions of space and provides a stochastic electromagnetic beam [16]. At every direction the Jones matrix  $M = (v_{uv})$  allows to write the polarization modes ( $E_S, E_P$ ) of the scattered field  $E$  versus the polarization modes ( $E_{0S}, E_{0P}$ ) of the incident field  $E_0$ , with scalar terms  $v_{UV}$  that are the scattering coefficients:

$$\vec{E} = \begin{pmatrix} E_S \\ E_P \end{pmatrix} = \begin{pmatrix} v_{SS} & v_{PS} \\ v_{SP} & v_{PP} \end{pmatrix} \vec{E}_0 = \begin{pmatrix} v_{SS}E_{0S} + v_{PS}E_{0P} \\ v_{SP}E_{0S} + v_{PP}E_{0P} \end{pmatrix} \quad (13)$$

Notice that the scattering coefficients are here assumed to be static since the sample is not in motion. Moreover and following perturbative theories [14], these coefficients directly give the field at one direction in the far field, the region where all phenomena are here addressed. Such Eq. (13) first describes how each scattered polarization mode ( $E_S$  or  $E_P$ ) is the result of a combination of the two incident polarization modes ( $E_{0S}$  or  $E_{0P}$ ), due to the presence of cross-scattering coefficients ( $v_{SP}$  and  $v_{PS}$ ). In other words on each polarization axis (S or P) the scattering process performs a linear combination of the two initial random variables  $E_{0S}$  and  $E_{0P}$ , and the result is another couple of random variables  $E_S$  and  $E_P$  with new statistics and correlation. This remark directly announces why mutual coherence (and therefore the DOP) can be increased by a scattering process.

At this step we keep in mind that depolarization effects require the presence of cross-scattering coefficients ( $v_{SP}$  and  $v_{PS}$ ) that allow the linear combination to occur on each polarization axis. Moreover, the strength of these phenomena increase in average with the ratios  $v_{PS}/v_{SS}$  and  $v_{SP}/v_{PP}$ . Inversely, specular processes (ie: processes that do not involve optical cross-coefficients) such as reflection or transmission do not modify the modes correlation ( $\mu = \mu_0$ ) and can only change the DOP via the modification of the energy ratio  $\beta$  on the axis. In a similar way slightly disordered media cannot modify the correlation because perturbative theories [14] predict cross-scattering coefficients to vanish in the incidence plane. To summarize, depolarization effects require independent cross-scattering coefficients,

which occurs for high scattering samples with strong disorder. More detail is given in the next sections.

### 2.3. Polarization degree of the scattered field

Now that Eq. (13) is established, the polarization properties of the scattered field can be investigated through the coherency matrix  $J$  in a way similar to that of the incident field:

$$J = \begin{pmatrix} \langle E_S \overline{E_S} \rangle & \langle E_S \overline{E_P} \rangle \\ \langle \overline{E_S} E_P \rangle & \langle E_P \overline{E_P} \rangle \end{pmatrix} \quad (14)$$

with the relationships:

$$DOP^2 = 1 - 4 \frac{\det[J]}{\{tr[J]\}^2} = 1 - 4 \frac{\beta}{[1 + \beta]^2} [1 - |\mu|^2] \quad (15)$$

and where  $\beta$  is the polarization ratio and  $\mu$  the complex correlation coefficient of the scattered field:

$$\beta = \frac{\langle |E_P|^2 \rangle}{\langle |E_S|^2 \rangle} \quad \text{and} \quad \mu = \frac{\langle E_S \overline{E_P} \rangle}{\sqrt{\langle |E_S|^2 \rangle \langle |E_P|^2 \rangle}} \quad (16)$$

Notice here that in opposition to the incident beam, all parameters (matrix, DOP, correlation, polarization ratio, scalar coefficients...) vary with location (or direction), for which reason in the next sections we investigate different mappings, histograms and averages.

### 2.4. Relationship between input and output DOP

Relations of section 2.3 can be further developed with Eq. (13) to emphasize the analytical relationship between the scattered (output) DOP and the incident (input)  $DOP_0$ . The results are the following:

$$\beta(\mu_0, \beta_0) = \frac{|v_{SP}|^2 + \beta_0 |v_{PP}|^2 + 2\sqrt{\beta_0} \Re\{\mu_0 v_{SP} \overline{v_{PP}}\}}{|v_{SS}|^2 + \beta_0 |v_{PS}|^2 + 2\sqrt{\beta_0} \Re\{\mu_0 v_{SS} \overline{v_{PS}}\}} \quad (17)$$

$$\mu(\mu_0, \beta_0) = \frac{1}{\sqrt{\beta}} \frac{v_{SS} \overline{v_{SP}} + \beta_0 v_{PS} \overline{v_{PP}} + \sqrt{\beta_0} \{\mu_0 v_{SS} \overline{v_{PP}} + \overline{\mu_0} v_{PS} \overline{v_{SP}}\}}{|v_{SS}|^2 + \beta_0 |v_{PS}|^2 + 2\sqrt{\beta_0} \Re\{\mu_0 v_{SS} \overline{v_{PS}}\}} \quad (18)$$

Relations (17-18) now provide a way to calculate the scattering polarization parameters  $\beta$  and  $\mu$  versus those ( $\beta_0$  and  $\mu_0$ ) of the incident beam. From these values one can extract the DOP of the scattered light following relation (15), a procedure that we use in section 3 devoted to numerical calculation.

These formulae also emphasize the key role of cross-scattering coefficients. As an illustration, one can check the predictions of section 2.2 for specular or first-order scattering processes. Indeed in the absence of optical cross-coefficients ( $v_{SP} = v_{PS} = 0$ ), relations (17-18) are reduced to:

$$v_{SP} = v_{PS} = 0 \Rightarrow |\mu| = |\mu_0| \quad \text{and} \quad \beta = \beta_0 \frac{|v_{PP}|^2}{|v_{SS}|^2} \quad (19)$$

so that correlation is not modified and the DOP can only be changed via the energy ratio  $\beta$ .

Let us now comment relationships (17-18) in the most general case of an arbitrary scattering medium ( $v_{UV} \neq 0$ ), but for extreme values of the incident parameters  $\mu_0$  and  $\beta_0$ . When the incident light is fully polarized with the whole energy on one axis ( $\beta_0 = 0$ ), the two polarization modes of the scattered field become fully correlated and the scattered light remains fully polarized:

$$\beta_0 = 0 \Rightarrow \mu = \frac{\overline{v_{SP}/v_{SS}}}{|v_{SP}/v_{SS}|} \Rightarrow |\mu| = 1 \Rightarrow DOP = 1 \quad (20)$$

If now the incident light is totally unpolarized ( $\beta_0 = 1$  and  $\mu_0 = 0$ ), the results are:

$$\beta = \frac{|v_{SP}|^2 + |v_{PP}|^2}{|v_{SS}|^2 + |v_{PS}|^2} \quad \text{and} \quad \mu = \frac{1}{\sqrt{\beta}} \frac{v_{SP} \overline{v_{SS}} + v_{PP} \overline{v_{PS}}}{|v_{SS}|^2 + |v_{PS}|^2} \quad (21)$$

and we observe that the resulting correlation is no more zero, so that the DOP of the scattered light is necessarily increased ( $\mu \neq 0 \Rightarrow DOP \neq 0$ ), with unity values obtained at directions such as [11]:

$$DOP = 1 \Leftrightarrow v_{SP} v_{PS} = v_{SS} v_{PP} \quad (22)$$

This last property describes how highly disordered media allow to confer polarization degree to incident totally unpolarized light. Such phenomenon was recently predicted and measured with success [11,12] since a  $PDF(u) = 3u^2$  law was emphasized for the DOP probability density function, with a 0.75 DOP average. However until now all results were limited to a zero incident  $DOP_0$  and the question of generalization to an arbitrary incident  $DOP_0$  remains open, what is addressed with numerical data in the next section.

### 3. Numerical results on light enpolarization

#### 3.1. The scattering model

As discussed above, enpolarization effects are the result of cross-scattering coefficients. Therefore 1-dimensional electromagnetic models cannot be used for such investigation. Concerning 2D-models, we have at disposal different computer codes based on the integral method [17] (case of rough surfaces) and finite elements (case of inhomogeneous bulks) [18]; however the strength of enpolarization increases with the weight of the cross-coefficients, for which reason the samples should be highly disordered (high-slope surfaces, high index ratio inhomogeneities, small correlation lengths...), a situation where the computer codes are excessively time consuming.

All these reasons led us to use a well-known phenomenological phasor model [19] where each speckle pattern  $v_{uv}$  in the far field is assumed to be the Fourier Transform of an exponential function  $\exp[j\phi_{uv}(x,y)]$ , with  $\phi_{uv}(x,y)$  a random phase uniformly distributed within  $[0;2\pi]$ . With this model the scattering coefficients are independent numbers and the speckle is fully developed. Such phasor formalism has been used in many situations with success [19]; also, this model was recently used to predict enpolarization effects and revealed large agreement with experiment, provided that the media are highly disordered and the incident beam fully depolarized [11].

Following this approach we can now simulate the behavior of the scattering coefficients and calculate the speckle patterns and all other polarization parameters from Eq. (20-21), together with the associated probability density functions (PDF). The results are given in the next figures

Notice that the support of the exponential function is a square surface  $S = L^2$ , and is included within another square area of surface  $S_0 = L_0^2$ , with  $L < L_0$ . Therefore the intrinsic speckle resolution or grain size is given in the Fourier plane by  $\delta\sigma = 1/L$ , while the spectral step follows the Shannon/Niquist criteria:  $\Delta\sigma = 1/L_0 < 1/L$ . The result is that  $L_0/L$  is the number of data points within the speckle size, and must be chosen as an integer  $M < N$ , with  $N^2$  the total number of data points. Hence the speckle patterns that follow are plotted versus the vector spatial frequency  $\sigma = (\sigma_x, \sigma_y)$ , and were calculated with  $M = 16$  and  $N = 1024$ .

These speckle patterns could also be plotted versus the scattering directions  $(\theta, \psi)$  or locations  $M(\rho)$  within a receiver. Indeed the spatial frequency can also be written [14] as:

$$\vec{\sigma} = k \sin \theta (\cos \psi, \sin \psi) \quad (23)$$

with  $\theta$  the normal angle and  $\psi$  the polar angle and  $k = 2\pi/\lambda$  in air, with  $\lambda$  the illumination wavelength

Using spherical coordinates we find:

$$\vec{\rho} = \rho \left[ \sigma_x / k, \sigma_y / k, 1 - (\sigma / k)^2 \right] \quad (24)$$

with  $\rho = |\rho|$  and  $\sigma = |\sigma|$

This last relation allows to connect directions and positions in the far field.

#### 3.2. The DOP statistics of the scattered field

Figure 1 gives the speckle intensity patterns of scattering calculated when the  $DOP_0$  of the incident light increases from 0 to 1. Here the input  $DOP_0$  was calculated with zero correlation ( $\mu_0 = 0$ ) so that its value was controlled by the polarization ratio  $\beta_0$ .

In order to know the DOP of each speckle grain whose intensity is given in Fig. 1, we plotted in Fig. 2 the 2-dimensional maps of the local degree of polarization of scattering (DOP) for each incident  $DOP_0$ . We first observe at low and medium  $DOP_0$  values, that from one speckle grain to another, the scattered DOP may take arbitrary values within the range  $[0;1]$ . This proves that disordered media may locally increase the polarization of scattering, in regard to that of the incident beam. Also, as predicted the scattered light remains fully polarized ( $DOP = 1$ ) when the incident beam is fully polarized ( $DOP_0 = 1$ ).

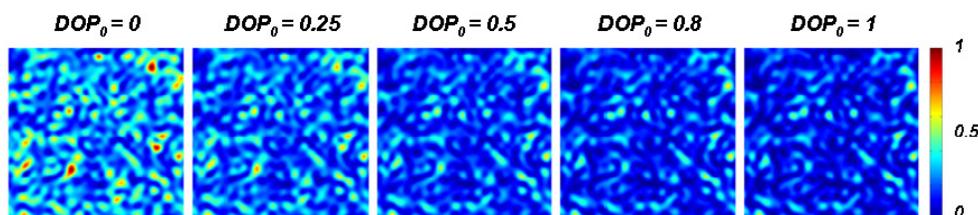


Fig. 1. Speckle intensity patterns of scattering versus the incident  $DOP_0$ .

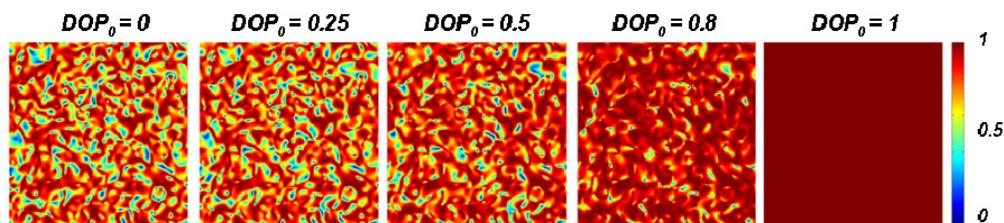


Fig. 2. Local degree of polarization (DOP) of the speckle patterns given in Fig. 1 versus the incident  $DOP_0$ . From left to right, the incident  $DOP_0$  increases from 0 to 1.

Now to complete the description of polarization for each  $DOP$  value, we use the Poincaré sphere given in Fig. 3 (bottom figures). Each point at the surface of the sphere characterizes a particular state of fully polarized light ( $DOP = 1$ ), while data within the sphere are specific of partially polarized light ( $DOP < 1$ ). Because 3D data in the bottom figures cannot easily be read within the spheres, we also plotted in Fig. 3 (top figures) the equatorial sections of the spheres, so that partial polarization can be easily detected; indeed data on the circles are for full polarization, while data inside the disk are for partial polarization (the center of circle is for totally unpolarized light).

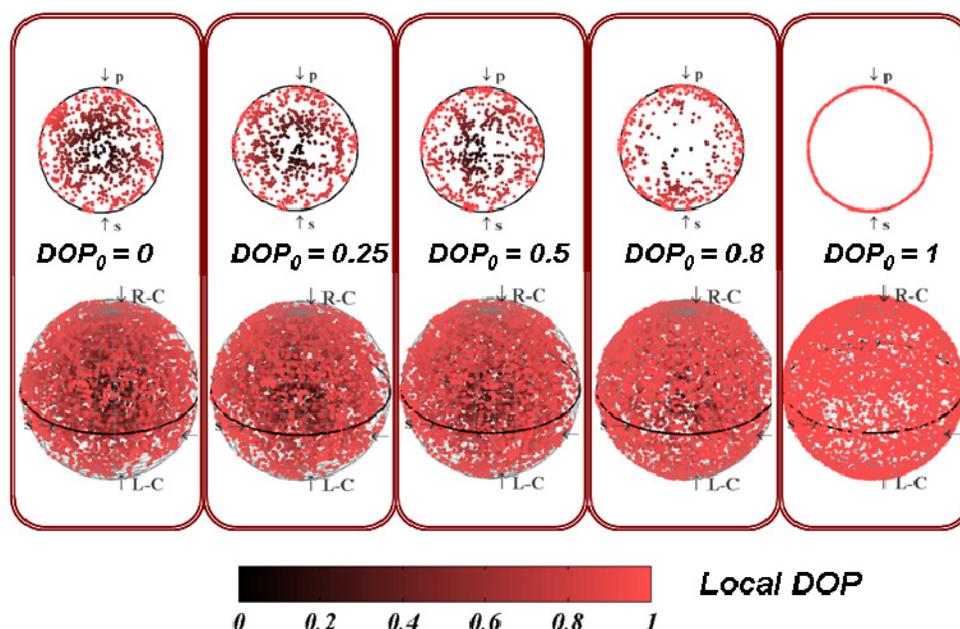


Fig. 3. Polarization states of the scattered light on the Poincaré spheres (bottom figures) and their equatorial sections (top figures). From left to right, the incident  $DOP_0$  increases from 0 to 1. The grey level is connected with the scattered  $DOP$  which measures the distance to the center of the sphere or disk.

In Fig. 3 from left to right the incident  $DOP_0$  is increased from 0 to 1. For the first value ( $DOP_0 = 0$ ) which corresponds to a fully unpolarized illumination, we observe that the data points spread over the whole disk, which means that the scattered light is not unpolarized in average and can take arbitrary  $DOP$  values within the range  $[0;1]$ ; in other words, the disordered medium may arbitrarily increase the incident polarization degree from one speckle grain to another. Then when the incident  $DOP_0$  is increased, the data points get closer to the circle and finally vanish within the disk for fully polarized incident light; this last result recalls that full polarized illumination creates fully polarized scattering.

To go further the PDF laws of the scattered  $DOP$  were calculated for each incident partial polarization  $DOP_0$ . The resulting variations are plotted in Fig. 4. The  $3u^2$  law plotted in red dashed line is recalled for the case of totally unpolarized incident light [11] that was analytically calculated [12]. We observe that all PDF curves are monotonic whatever the incident  $DOP_0$ . This means that for all incident  $DOP_0$  the most probable situation for scattering is full polarization, while the less probable is unpolarized scattering. Therefore the scattered light will be highly polarized in average. Notice that the scattered  $DOP$  naturally tends towards a Dirac function around  $DOP = 1$  when the incident light is fully polarized.

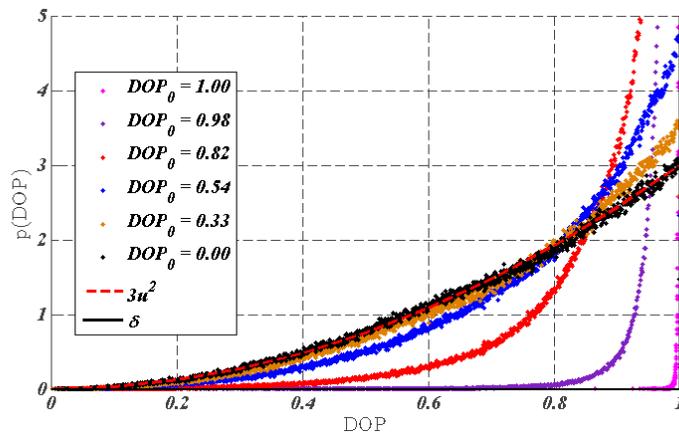


Fig. 4. Variations of the PDF laws of the scattered DOP for different incident  $DOP_0$ .

Finally Fig. 5 is given to summarize the results. It reveals the spatial average of the local DOP of scattering versus the polarization parameters ( $\mu_0, \beta_0$ ) that control the incident  $DOP_0$ . This average strictly increases with the incident  $DOP_0$ , so that the minimum value ( $DOP = 0.75$ ) is obtained for totally unpolarized light. Therefore the scattering DOP is higher than 0.75 whatever the incident polarization ( $0 < DOP_0 < 1$ ), which proves that light is strongly locally enpolarized by the scattering medium. These results are completed by those of Fig. 6 where the ratio of the scattered DOP to the incident  $DOP_0$  is plotted. This ratio is greater than 1.

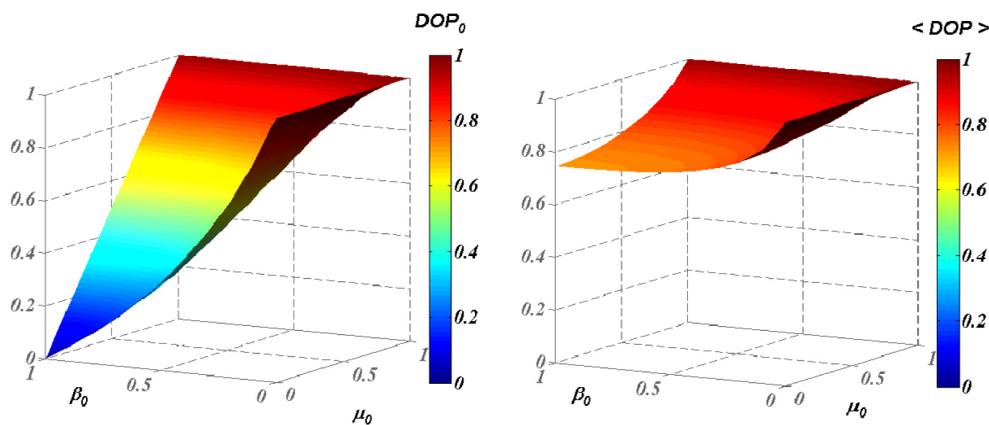


Fig. 5. Average of polarization degree of the incident light (left) and of the scattered light (right) versus the parameters that control the incident polarization (correlation  $\mu_0$  and polarization ratio  $\beta_0$ ).

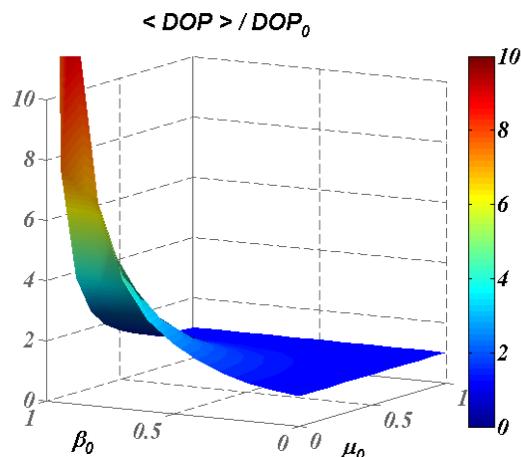


Fig. 6. Ratio of scattered DOP to incident  $DOP_0$ , versus the incident polarization parameters (correlation  $\mu_0$  and polarization ratio  $\beta_0$ ).

Until now we calculated the polarization degrees versus the two parameters that are correlation ( $\mu_0$ ) and polarization ( $\beta_0$ ) parameters. The reason is that the incident  $DOP_0$  can be reached with different sets ( $\mu_0, \beta_0$ ) of parameters leading to different intensity patterns, as given in relation (13); however we have checked that the average of the scattering DOP does not depend on these two parameters, but only on the incident  $DOP_0$ . Hence we were allowed to plot in Fig. 7 the variation of scattering DOP versus the incident  $DOP_0$ . Notice here that the fact that the average output DOP only depends on the input  $DOP_0$  does not prove that all statistics of the output DOP only depend on the input  $DOP_0$ .

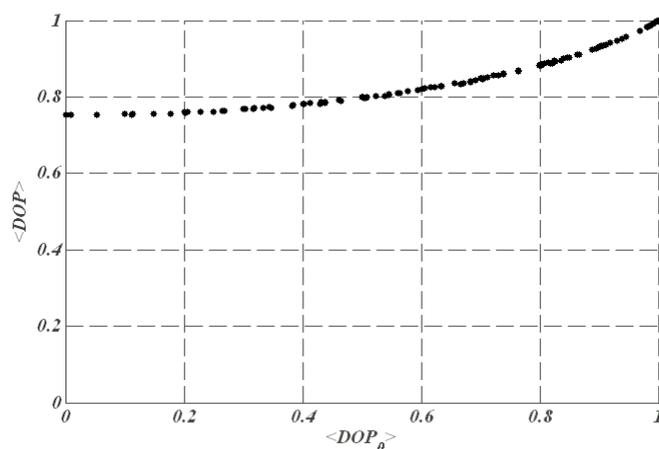


Fig. 7. Average scattered DOP plotted versus incident  $DOP_0$ .

#### 4. Conclusion

We have shown how disordered media allow to increase the local degree of polarization of the scattered light in the general case of a partially polarized incident beam. These enpolarization effects are noticeable since the scattered DOP lies within the range  $[0.75; 1]$  in average, whatever the incident  $DOP_0$ . These phenomena do not violate the entropy principles since scattering is specific of a loss process.

Most enpolarization phenomena are due to the presence of cross-scattering coefficients which create mutual correlation between the polarization modes of scattering, because off a linear combination of these modes on each polarization axis. As a consequence, such enpolarization effects vanish for specular processes and perturbative scattering.

Because enpolarization is a local effect, we also plotted the statistics of the polarization degree of scattering (DOP) versus space location or direction. The histograms were given for each incident polarization degree ( $DOP_0$ ) and can be used to emphasize new signatures when probing complex media [18]. Applications may concern remote sensing and biophotonics, defense, cosmetics and lightening.

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