

Tutorial: Foundations of Photonic Crystal Fibers

Main linear properties of microstructured optical fibers

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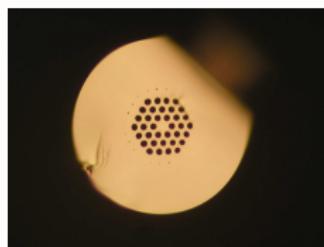
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- 3 Main properties of MOFs with solid-core and low refractive index inclusions
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- 5 Main properties of MOFs with hollow-core and low refractive index inclusions
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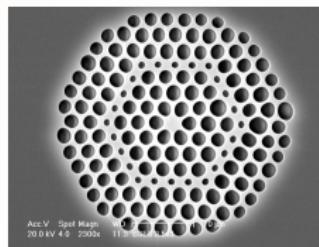
Introduction

Possible definition

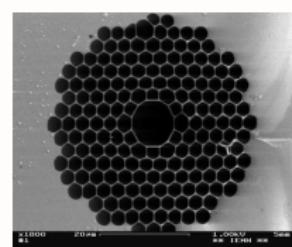
- A **microstructured optical fiber** is the generalization of an optical fiber: the core is surrounded by a microstructured cladding made of a set of inclusions with various shapes and refractive indices
 - The interactions of the electromagnetic field of the core with the microstructure → confinement and other properties of the fiber



Solid core MOF made of chalcogenide glass (EVC-ISCR, Université de Rennes/Perfos)



Solid core MOF made of
silica (X-Lim, Université de
Limoges)



Hollow core MOF made of silica (Phlam/IRCICA, Université de Lille I)

3 examples of MOFs

Introduction

MOFs have at least 3 advantages:

- The space of the geometric parameters is nearly infinite
 - A single material is required to fabricate them
 - Refractive index contrasts range from less than 1% up to 300%

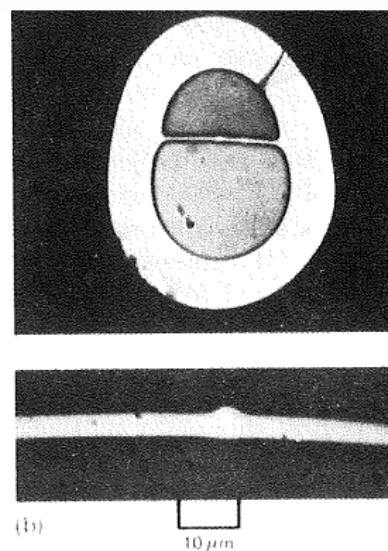
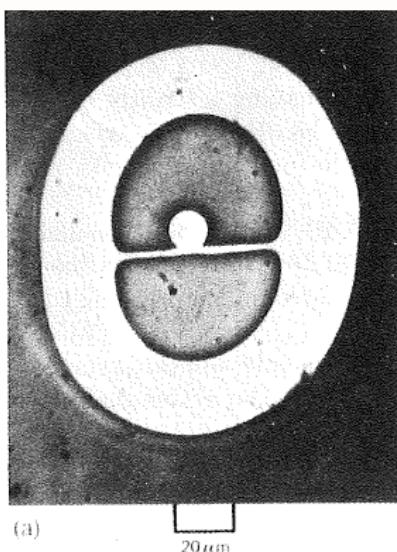
Introduction

Brief historical review

- The first MOFs were fabricated as soon as 1973

First experimental and theoretical study of MOFs:

Kaiser, Marcantili and Miller : *A New Optical Fiber* in *Bell Syst. Tech. J.*, vol. 52, p. 265-269, 1973.



MOFs made of Suprasil
2, @ 1.06 μ m:
 (a) $\mathcal{L}_{dB} \simeq 28$ dB/km
 (b) $\mathcal{L}_{dB} \simeq 55$ dB/km

A complementary article written by Kaiser and Astle was published in 1974 in the same journal.

FIG. 2—Photographs of an experimental (a) multimode SM fiber and (b) single mode SM fiber (top), with magnified core region (bottom).

Introduction I

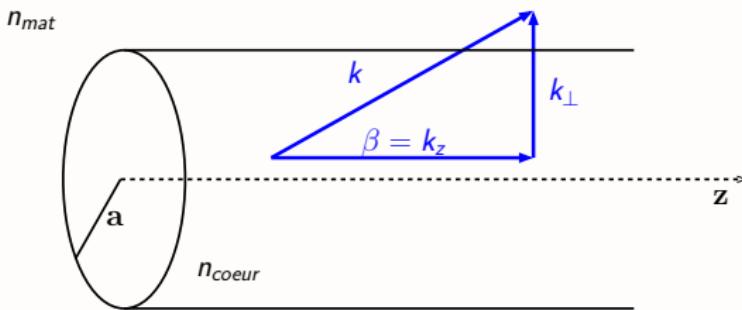
Brief and incomplete historical review

- **Suspended core fiber** by Kaiser, Marcatili and Miller in **1973**
 - The first article describing the **fabrication and characterization of a Photonic Crystal Fiber** was published in **1996** (*Optics Letters* by J.C. Knight, T. A. Birks, P. St. J. Russell, and D.M. Atkin)
 - The first propagation in a **hollow core PCF** has been published in **1999** by the team of P. St. J. Russell at the University of Bath. (*Science* by Cregan et al.)
 - The first guiding **solid core Bragg** fiber has been described in **2000** (X-Lim/University of Limoges).
 - The first guiding **hollow core Bragg** fiber has been described in **2002**.
 - The first **kagomé MOF** was shown in **2002** by F. Benabid et al. in Bath.
 - The first guiding **chalcogenide solid core MOF** was described in **2006** by the University of Rennes & PERFOS

Guiding in fibers

Guiding in conventional fibers

- A fiber is an excellent approximation of an invariant structure: $\lambda \sim 1\mu\text{m}$ and $L \gtrsim 1\text{m}$.
- One looks for linear solutions for the electromagnetic fields, **the modes**, of the form: $\mathbf{V}(r, \theta) \exp(-i(\omega t - \beta z))$.
- One solves Maxwell's equations.
- One solves **the homogeneous problem** (no incident field).



Step-index fiber and decomposition of the wave vector. $k_0 = \frac{2\pi}{\lambda_{vide}}$.

The propagation constant β is the key parameter of modes.
effective indice $n_{eff} \equiv \beta/k_0$

Guiding in fibers

Modes and guiding conditions

- A **mode** is defined as the solution:
 - ▶ of the propagation equation in the different parts of the fiber
 - ▶ taking into account continuity relations at the different interfaces
 - ▶ taking into account the boundary conditions at infinity
- Conditions to get **guided modes** (lossless modes with zero Dirichlet conditions at infinity)
 - ▶ In a homogeneous medium n_{medium} , one gets $n_{eff} = \beta/k_0 < n_{medium}$ (plane waves, continuum)
 - ▶ To get a guided mode in the fiber core n_{core} : this mode should not be guided in the matrix surrounding the core, i.e. $\beta/k_0 > n_{mat}$

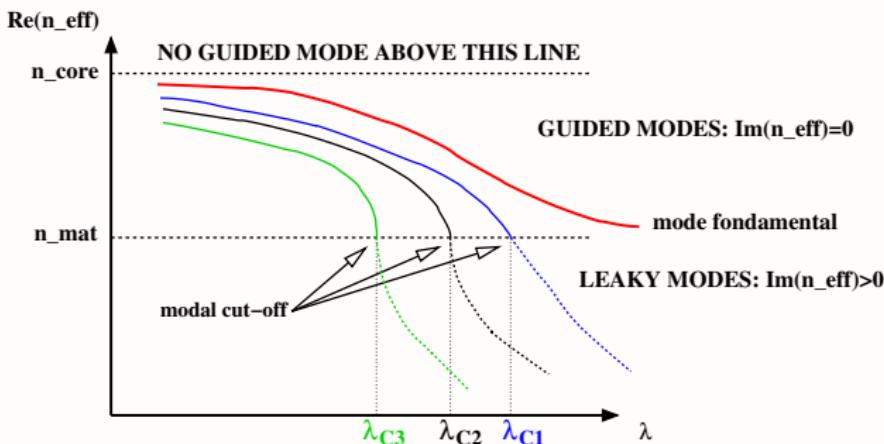
For a step-index fiber with n_{core} for the core and n_{mat} for the matrix, one gets the following bracketing:

$$n_{mat} < \beta/k_0 = n_{eff} < n_{core}$$

Guiding in fibers

Differences between conventional and microstructured fibers

For lossless materials i.e. $\Im m(n_{core}) = \Im m(n_{matrix}) = 0$



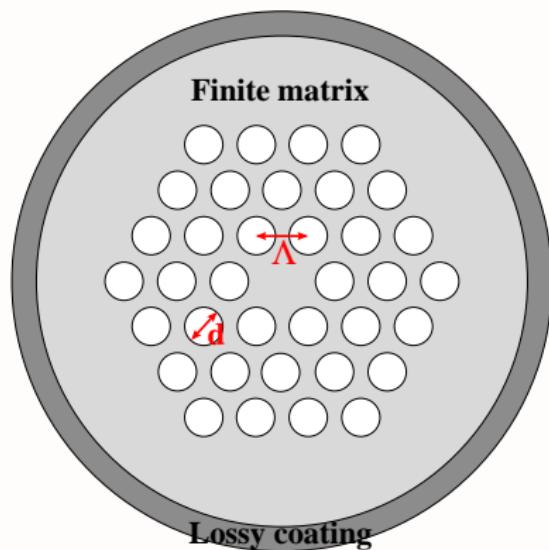
Schematic dispersion curves $\text{Re}(n_{\text{eff}})$ for the first modes of a step-index fiber

- Modes still exist after their cutoff wavelengths λ_C : they become **leaky modes**
 - Guiding losses are proportional to $\Im m(n_{eff}) = \Im m(\beta/k_0)$

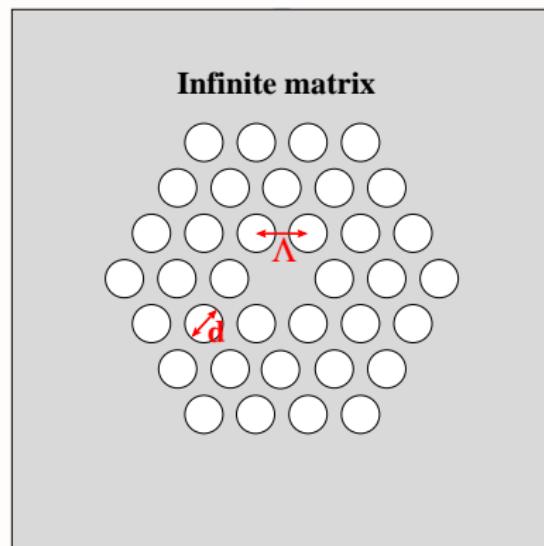
Guiding in fibers

For a microstructured optical fiber (MOF) with a solid core and low refractive index inclusions

Λ = lattice pitch d = inclusion diameter N_r = number of inclusion rings



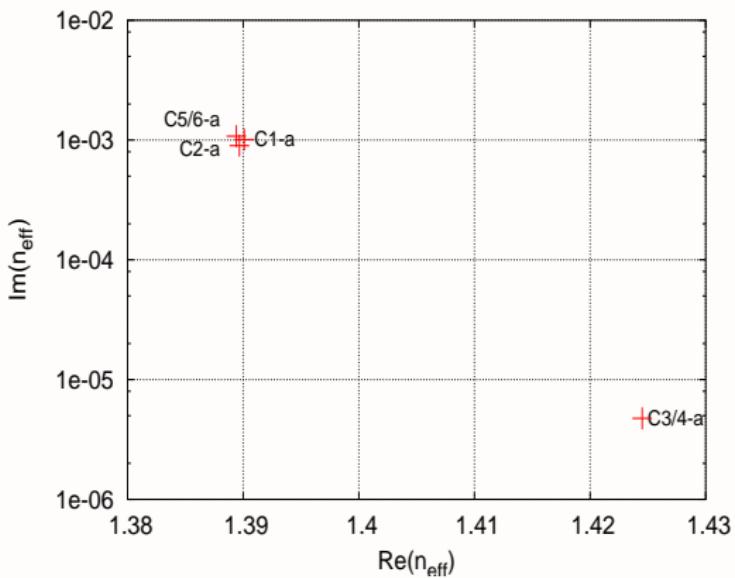
Experimental configuration



Configuration used in modelling

Introduction

For a microstructured optical fiber (MOF) with a solid core and low refractive index inclusions: $n_{core} = n_{mat} > n_{inc}$



Complex effective indices of the first MOF modes

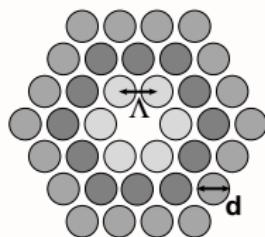
- The modes are not perfectly guided even if $\Im m(n_{core}) = \Im m(n_{mat}) = 0$, they are **leaky modes**: $\Im m(n_{eff}) > 0$

Main properties of MOFs with solid-core (SC) and low refractive index inclusions: $n_{core} = n_{mat} > n_{inc}$
 Guiding losses \mathcal{L} of the fundamental mode

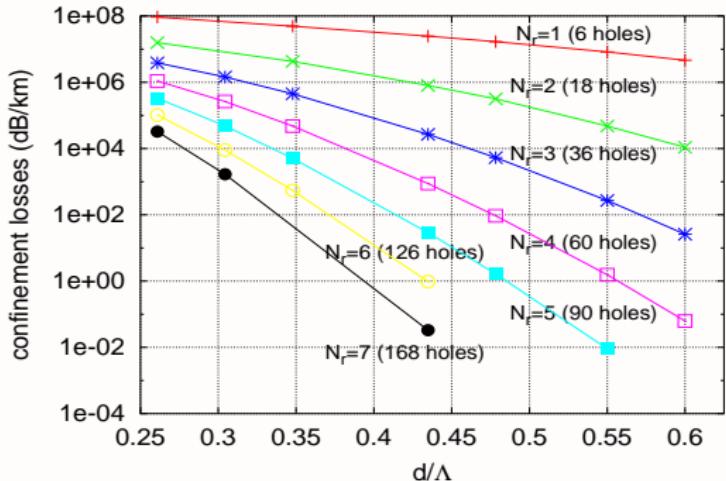
Guiding losses \mathcal{L} in dB/m for λ in m :

$$\mathcal{L} = \frac{20}{\ln(10)} \frac{2\pi}{\lambda} \Im m(n_{\text{eff}})$$

Main properties of MOFs with solid-core (SC) and low refractive index inclusions: $n_{core} = n_{mat} > n_{inc}$
 Guiding losses \mathcal{L} of the fundamental mode



$$N_r = 3$$



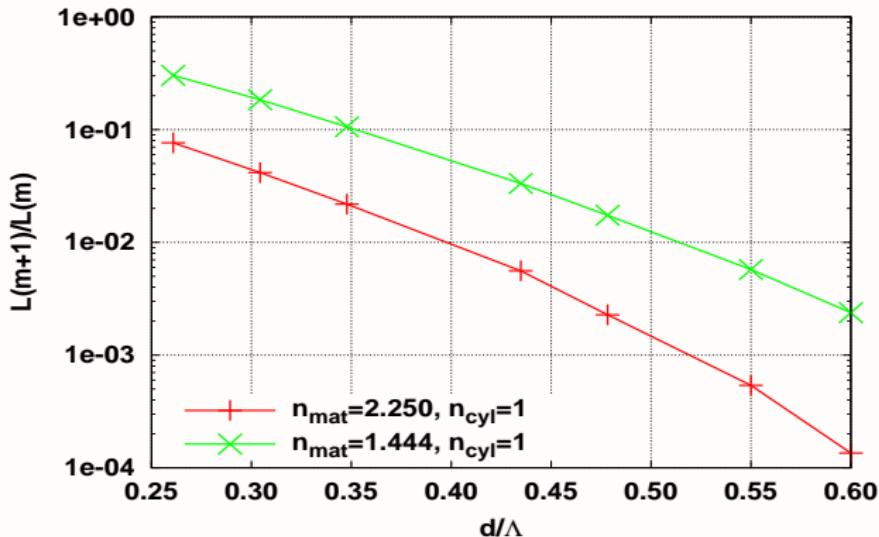
Losses in dB/km for $\Lambda = 2.3 \mu m$, $\lambda = 1.55 \mu m$, et $n_{mat} = 1.444$.

Dependency of \mathcal{L}

- \mathcal{L} decays exponentially when N_r increases.
 - The bigger d/Λ , the more \mathcal{L} decreases.

Main properties of SC-MOFs with low index inclusions

Impact of the matrix refractive index



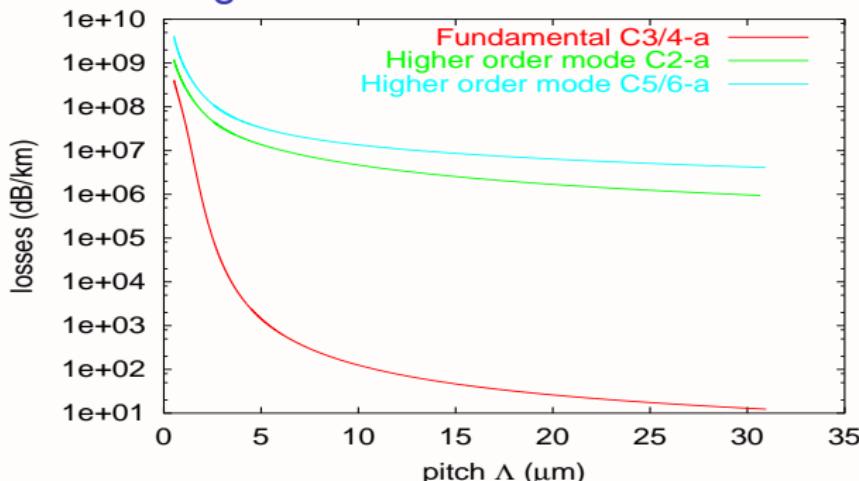
Ratio $L(m+1)/L(m)$ of the guiding losses for a MOF with $m+1$ hole rings and the ones for a FOM with m hole rings, as a function of d/Λ for 2 values of the matrix refractive index

For high index glasses

3-5 rings of holes are sufficient to obtain low guiding losses → material losses are the limiting factor.

Main properties of SC-MOFs with low index inclusions

How to define the single mode behaviour?



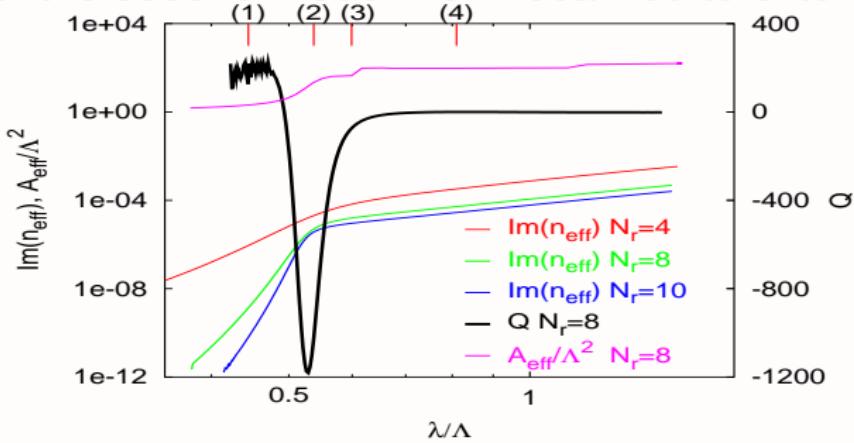
Losses of the "first" three modes for $d/\Lambda = 0.4$, $N_r = 3$, $\lambda = 1.55 \mu\text{m}$

Issues

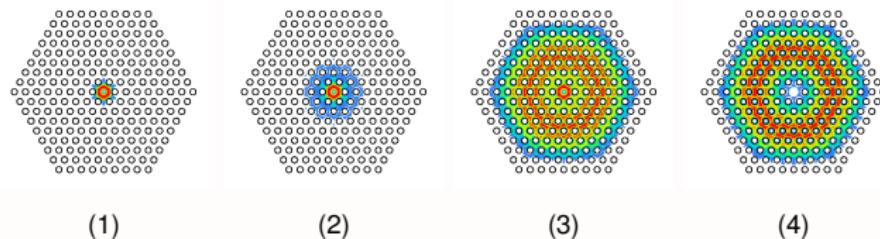
- If one considers only losses: no qualitative differences between the fundamental mode and the second ones
 - If one introduces an absolute threshold: the single-modedness depends on the fiber length

Main properties of SC-MOFs with low index inclusions

Transition of the second mode: from localized to extended



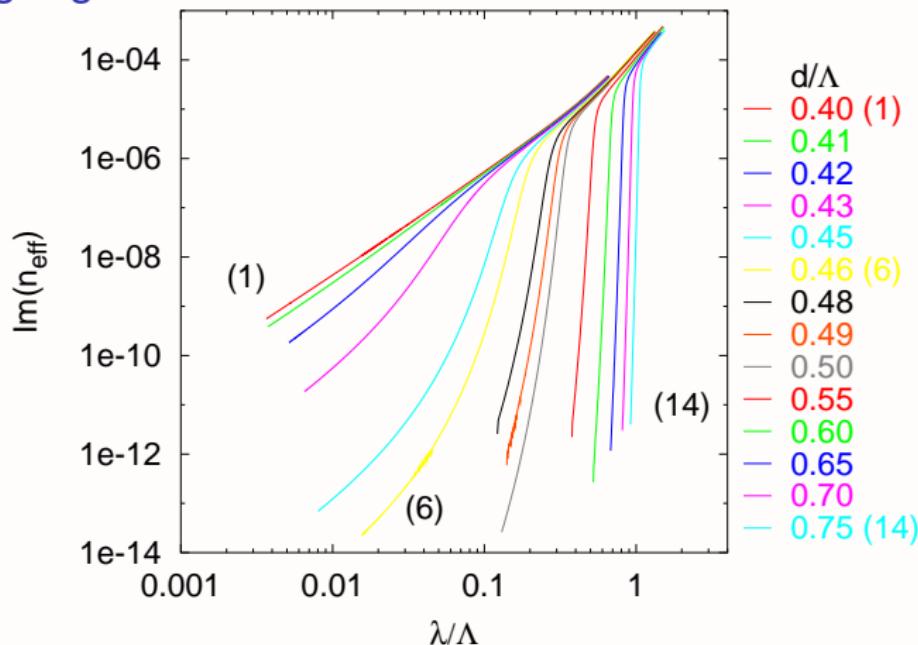
Transition of the 2nd mode, $d/\Lambda = 0.55$



Modulus of the longitudinal component of the Poynting vector for the 2nd mode

Main properties of SC-MOFs with low index inclusions

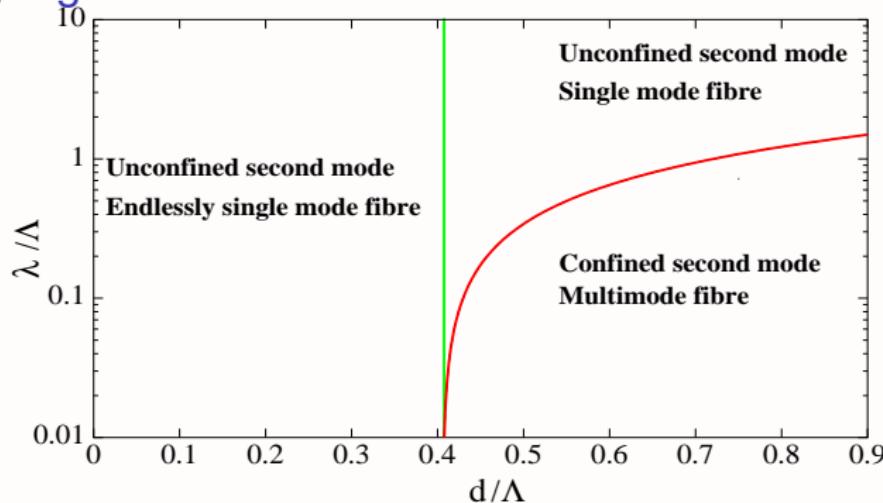
A phase diagram for the 2nd mode: toward the definition of operating regimes in SC-MOFs



$\Im m(n_{eff})$ as a function of λ/Λ for $N_r = 8$ and $\lambda = 1.55 \mu m$.

Main properties of SC-MOFs with low index inclusions

A phase diagram for the 2nd mode: toward the definition of operating regimes in SC-MOFs



Phase diagram defined by the minimum position of \mathcal{Q}

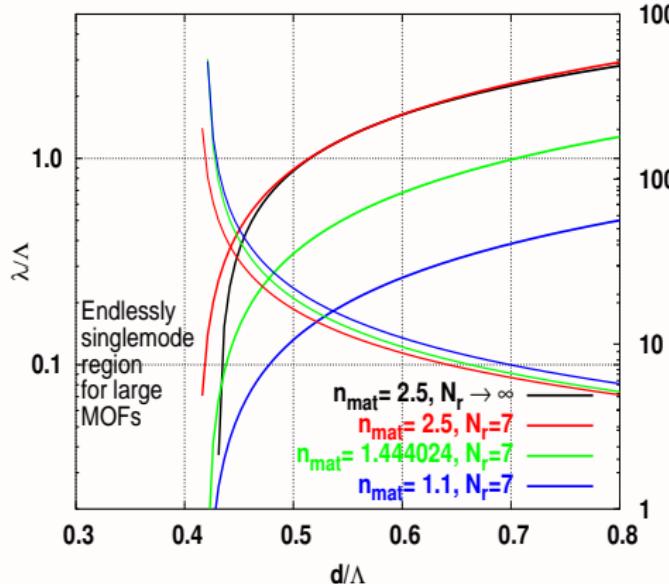
Equation of the 2nd mode locus (red curve):

$$(\lambda/\Lambda)_{\text{S.M.}} \simeq 2.8((d/\Lambda) - 0.41)^{0.9}$$

- Experimental validations by Folkenberg *et al.* (Opt. Lett., 2003) and by Labonté *et al.* (Opt. Lett., 2006).

Main properties of SC-MOFs with low index inclusions

Generalized phase diagram and scaling law for the 2nd mode



Phase diagram for three values of n_{mat} obtained from simulations (left scale).

Generalized phase diagram (right scale): with the invariant ν .

- Numerical simulations: $(d/\Lambda)_{critical} \in [0.420, 0.425]$ for large MOFs.

Within the scalar approximation:

$$\nu = 2\pi \frac{\Lambda}{\lambda} (n_{\text{mat}}^2 - n_{\text{inc}}^2)^{1/2}$$

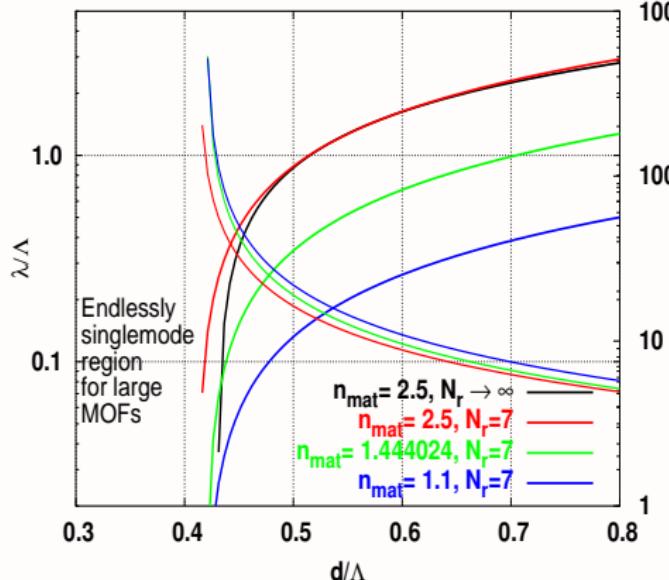
> is an invariant.

We get a **scaling law**:

$$\frac{(\lambda/\Lambda)'_{\text{S.M.}}}{(n'_{\text{mat}} - n'_{\text{inc}})^{1/2}} = \frac{(\lambda/\Lambda)_{\text{S.M.}}}{(n^2_{\text{mat}} - n^2_{\text{inc}})^{1/2}}$$

Main properties of SC-MOFs with low index inclusions

Generalized phase diagram and scaling law for the 2nd mode



Phase diagram for three values of n_{mat} obtained from simulations (left scale).

Generalized phase diagram (right scale): with the invariant ν .

- The extent of the 'endlessly singlemode region' is only preserved for large N_r .
 - The position of the 'singlemode region' is nearly fixed for $N_r \gtrsim 4$.

Within the scalar approximation:

$$\nu = 2\pi \frac{\Lambda}{\lambda} (n_{\text{mat}}^2 - n_{\text{inc}}^2)^{1/2}$$

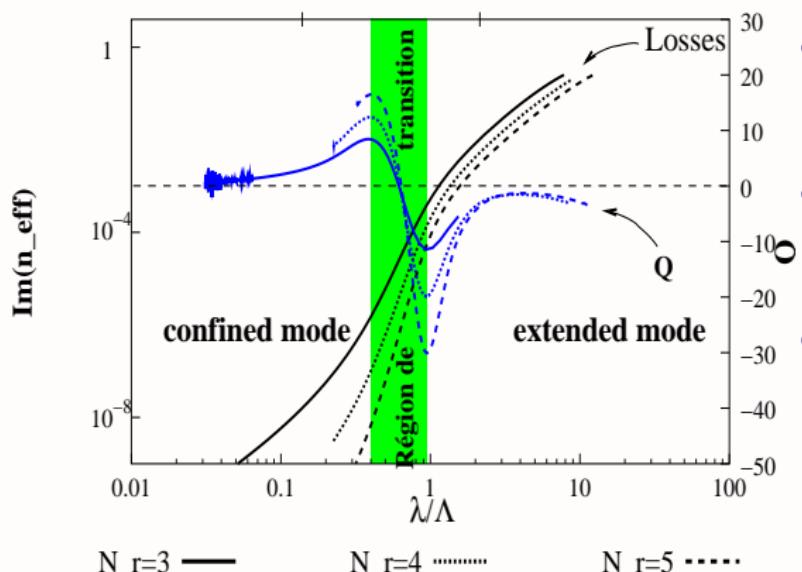
> is an invariant.

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Main properties of SC-MOFs with low index inclusions

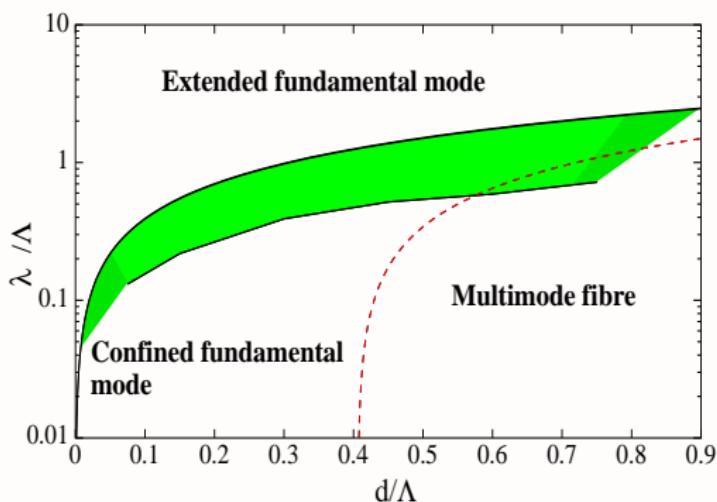
A phase diagram for the fundamental mode



- No delocalization from the core, transition is different from the 2^{nd} mode one
 - In the **transition region**: **remarkable properties** for SC-FOMs
 - Outside it: one finds regimes that can be obtained using conventional W-profile fibers (through 2 asymptotic models).

Main properties of SC-MOFs with low index inclusions

A phase diagram for the fundamental mode



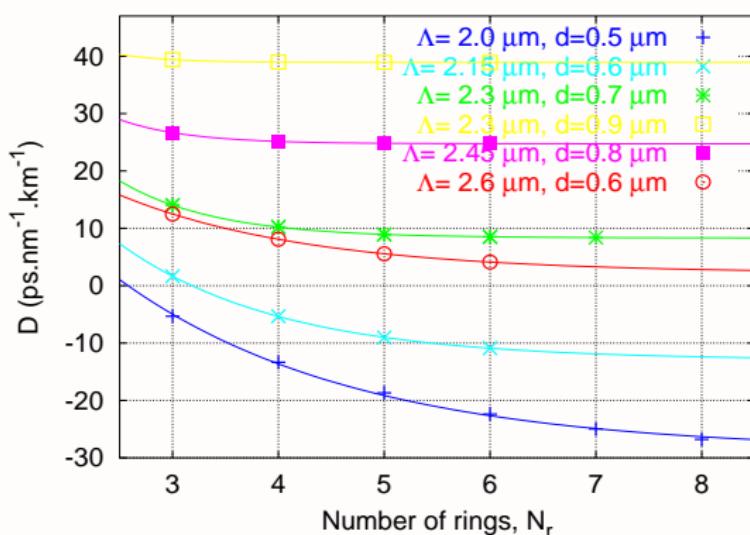
- No delocalization from the core, transition is different from the 2^{nd} mode one
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- Confined fundamental mode: fiber properties converge with N_r .
 - Unconfined fundamental mode: fiber properties do not converge with N_r .

Main properties of SC-MOFs with low index inclusions

Chromatic dispersion D_c

- $D_c = -\frac{\lambda}{c} \frac{\partial^2 \Re e(n_{\text{eff}})}{\partial \lambda^2}$
 - $D_c \simeq D_W + D_{mat}$
 - $\Delta\tau = D_c \Delta\lambda \Delta I$
 - D_c plays a key role in nonlinear pulse propagation

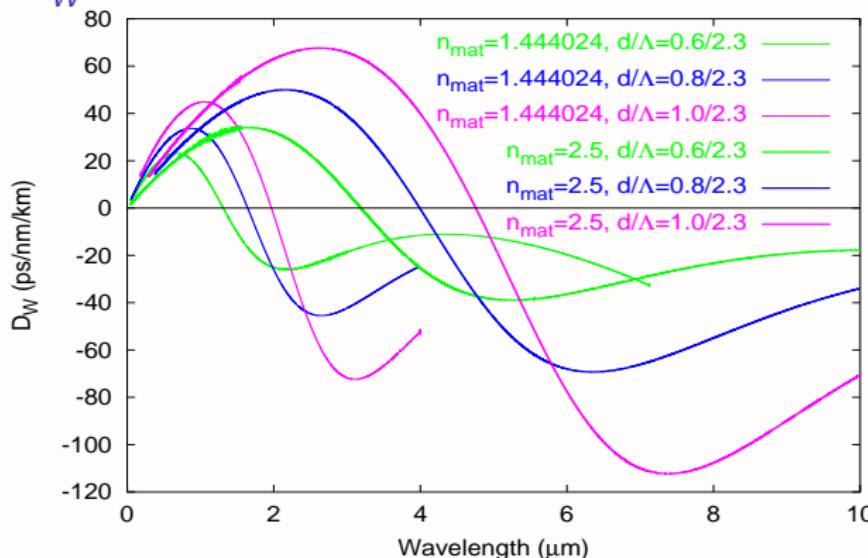


Chromatic dispersion for silica SC-MOFs taking into account material dispersion

- Chromatic dispersion D_c can strongly depend on the number of inclusion rings N_r .

Main properties of SC-MOFs with low index inclusions

Impact of matrix refractive index on waveguide chromatic dispersion D_W

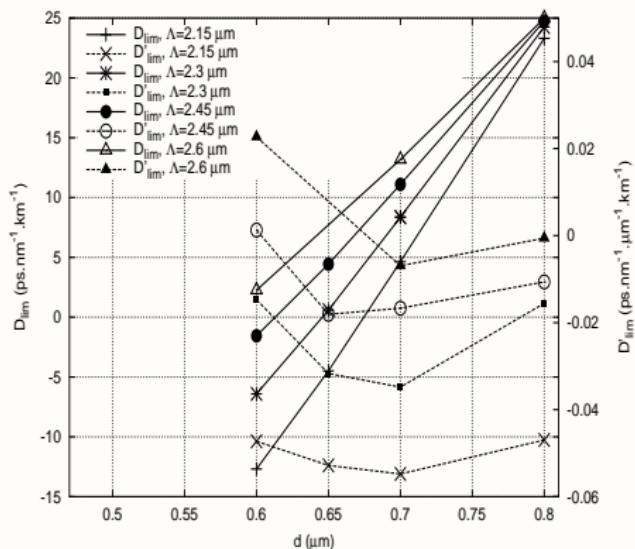


$n_{\text{mat}}=1.444024$ (thin line) et $n_{\text{mat}}=2.5$ (thick line) for several values of d/Λ ratio for SC-MOFs with $N_r = 3$.

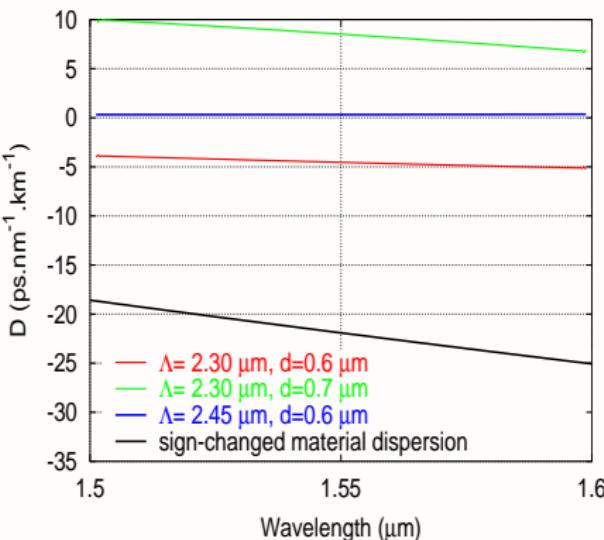
- Shift toward longer wavelengths of D_W extrema
- Increase of the amplitude of D_W extrema

Main properties of SC-MOFs with low index inclusions

1st example: to get a flat chromatic dispersion



Abacus to define optimum parameters to get flat chromatic dispersion at 1.55 μm



Example of ultra-flattened chromatic dispersion

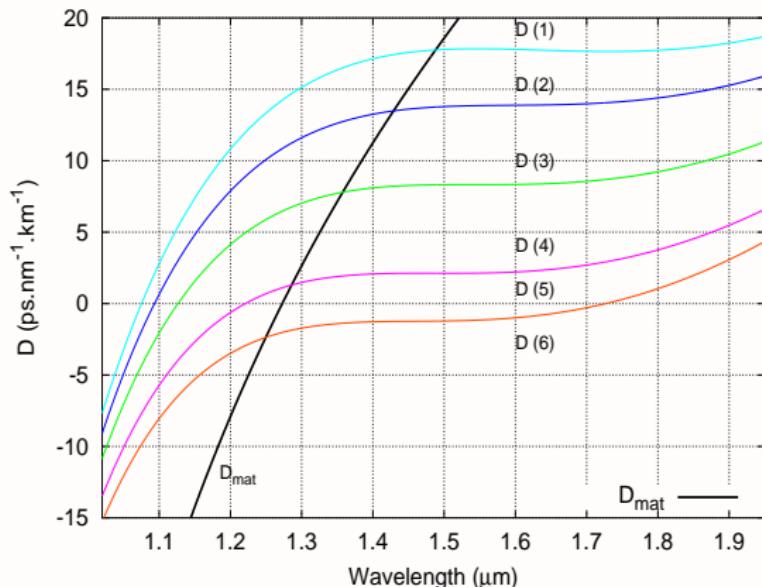
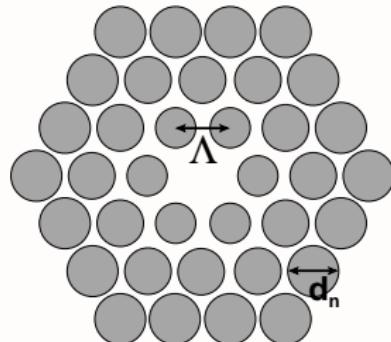
- An usual SC-FOM provides an ultra-flattened chromatic dispersion but it requires **18 hole rings** to get $\mathcal{L} < 1 \text{ dB/km}$.

Main properties of SC-MOFs with low index inclusions

2nd example: to get a flat chromatic dispersion with a new profile

- Idea of a refractive index gradient
 - Generalization of a scaling law on D_W (*Ferrando 2001*)
 - Optimization

→ new SC-MOF profile



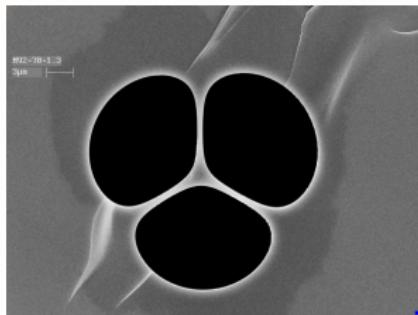
- Only 7 rings of holes with 3 different diameters are needed to obtain an ultra-flattened chromatic dispersion with $\mathcal{L} < 0.2 \text{ dB/km}$

Main properties of SC-MOFs with low index inclusions

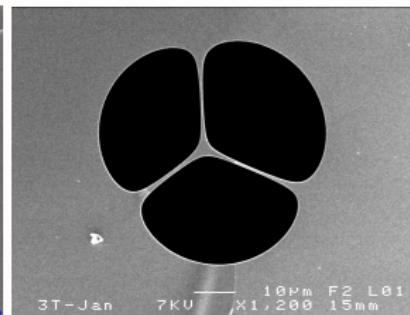
3rd example: to adjust a zero dispersion wavelength

- The D_C management is limited with usual SC-MOFs
- To generate supercontinua : $\lambda_{pump} \simeq \lambda_{ZDW}$

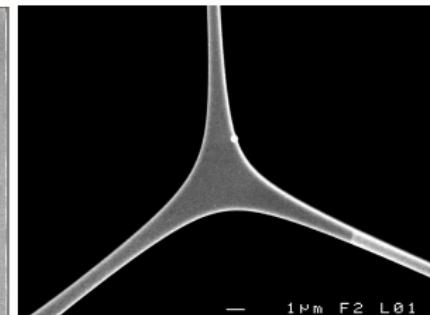
→ MOF with a suspended core



As_2S_3 , $\varnothing_{core} = 2.3 \mu\text{m}$



As_2Se_3 , $\varnothing_{core} = 3.5 \mu\text{m}$

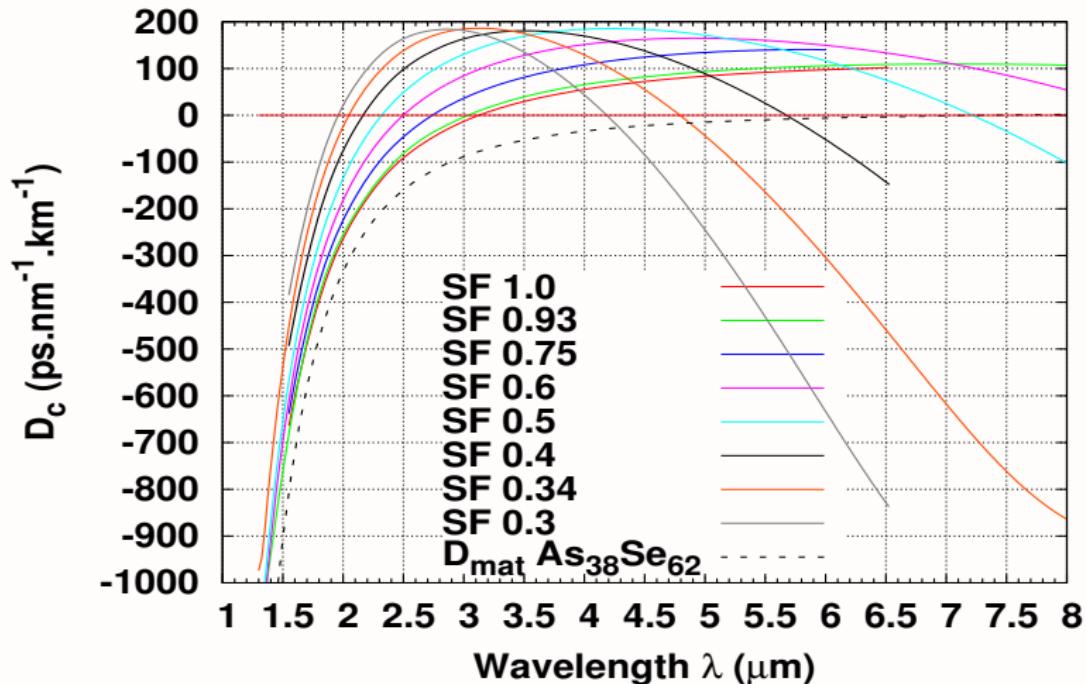


Zoom of the core region

As_2S_3 : ICB, Univ. of Burgundy, Dijon; As_2Se_3 : EVC-ISCR, Univ. of Rennes & PERFOS

Main properties of SC-MOFs with low index inclusions

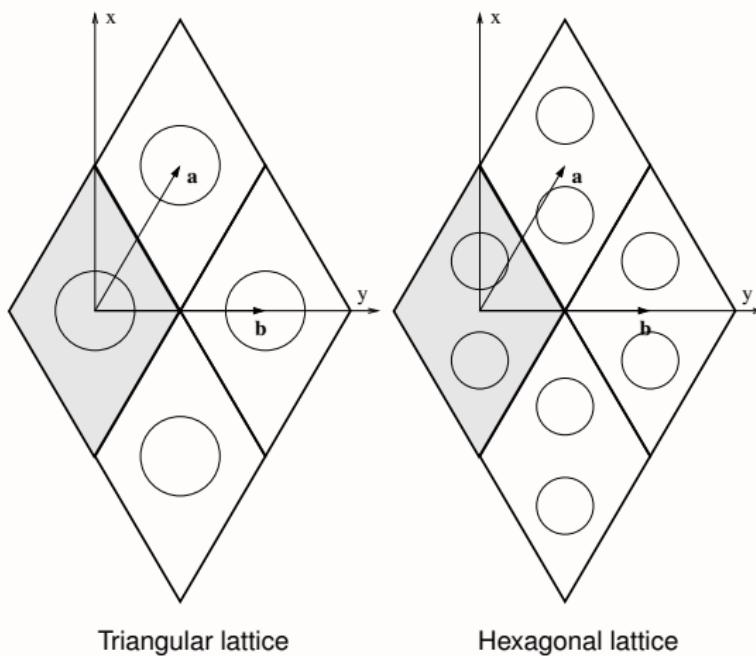
3rd example: to adjust a zero dispersion wavelength



D_c as a function of core \emptyset : Scaling Factor= 1.0 → 3.48 μm SF = 0.3 → 1.044 μm
dashed line: As_2Se_3 material dispersion

Guiding mechanism: analysis using band diagram

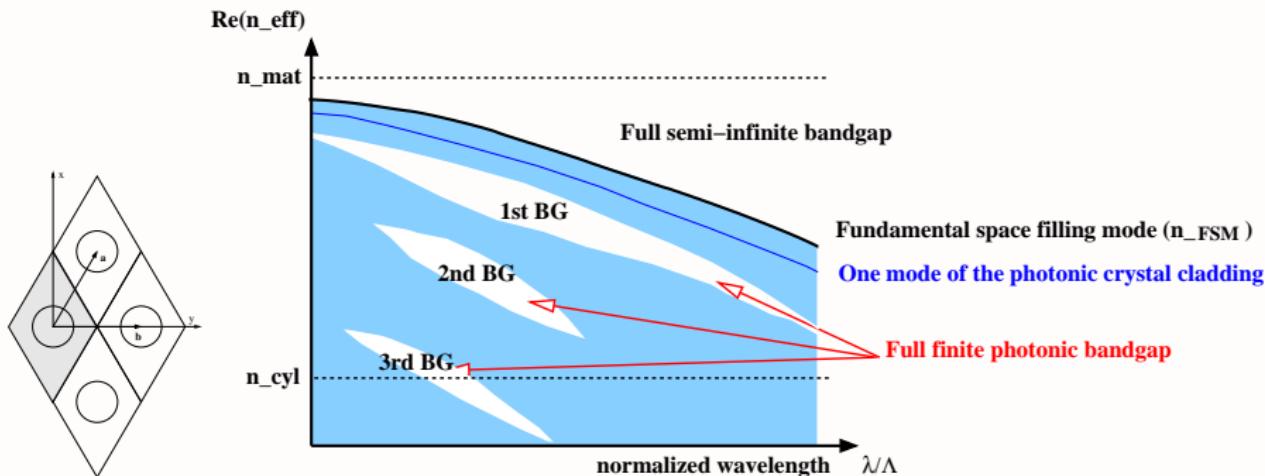
Periodic structures : real lattices



Representation of some cells with the lattice vectors $\mathbf{a} = \Lambda \mathbf{e}_x$ and $\mathbf{b} = \frac{\Lambda}{2} \mathbf{e}_x + \frac{\Lambda\sqrt{3}}{2} \mathbf{e}_y$.

Guiding mechanism: analysis using band diagrams

Band diagrams



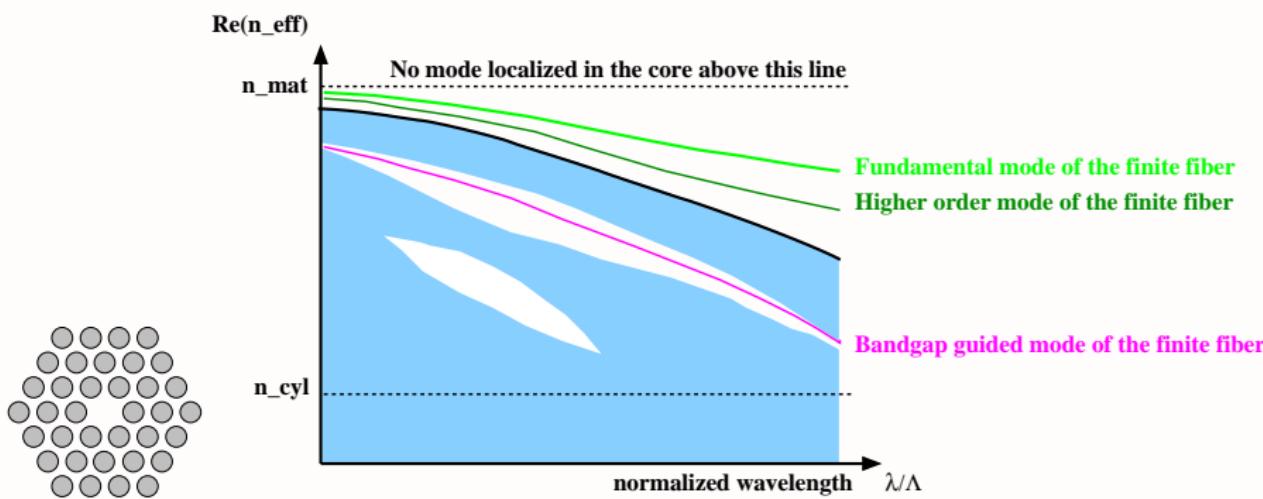
Band diagram for a lattice of low refractive index inclusions n_{cyl} in a high index matrix

$$n_{mat} > n_{cyl}.$$

- Modes of the periodic structure are Bloch modes: they do not have guiding losses.

Guiding mechanism: analysis using band diagram

Comparison between modes of periodic structures and finite size MOFs

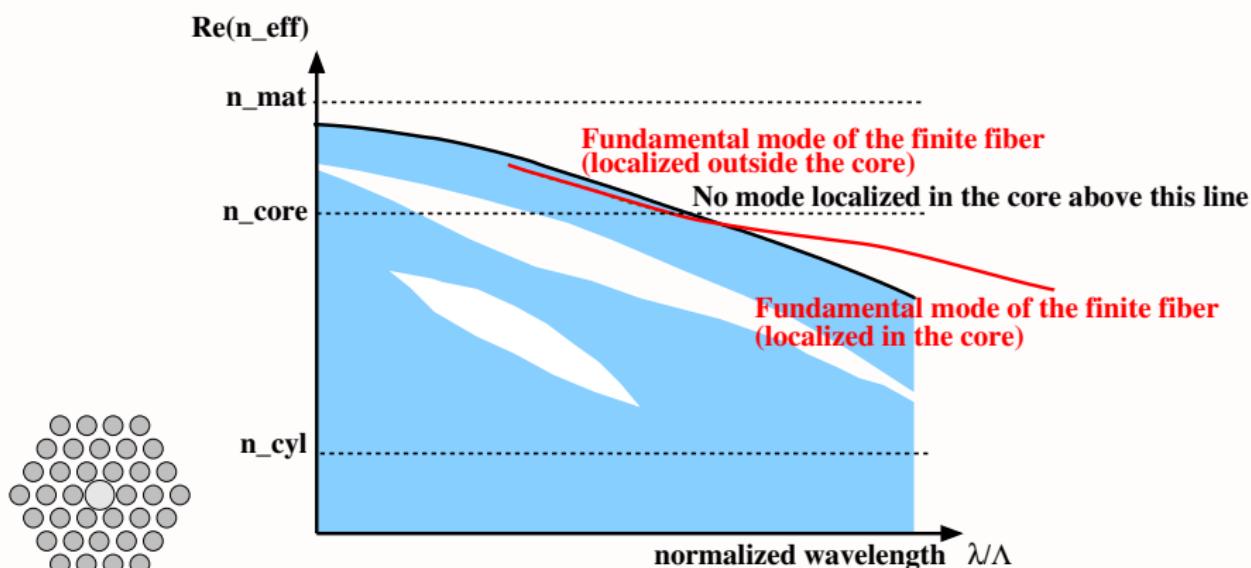


Band diagram for a lattice of inclusions in a matrix such that $n_{cyl} < n_{mat}$ and dispersion curves of several modes from a finite size SC-MOF $n_{core} = n_{mat} > n_{cyl}$.

- The modes of the finite size structure (the '*true*' SC-MOF) are leaky...

Guiding mechanism: analysis using band diagram

Back to the guiding mechanism: special case $n_{cyl} < n_{core} < n_{mat}$



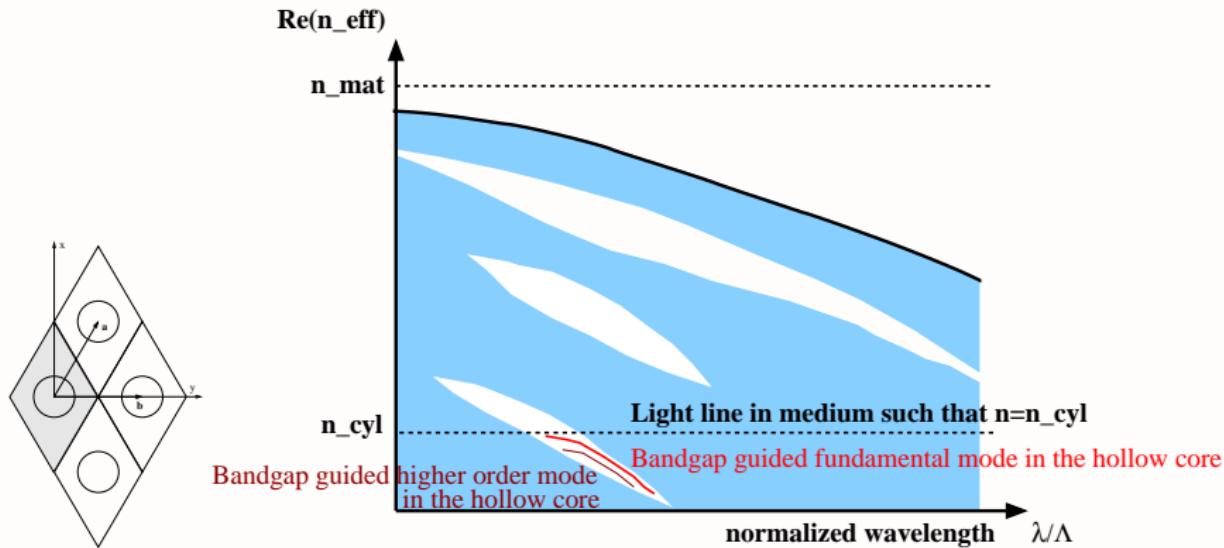
Band diagram of a periodic array of inclusions with n_{cyl} in a matrix n_{mat} and the dispersion curve of the fundamental mode of the finite size MOF with a n_{core} core.

The fundamental mode has a **transition**: when λ decreases, delocalization from the low index core to the higher index region surrounding it

Main properties of MOFs with hollow-core (HC-MOFs) and low refractive index inclusions

Guiding mechanism: analysis using band diagram for

$$n_{core} = n_{cyl} < n_{mat}$$

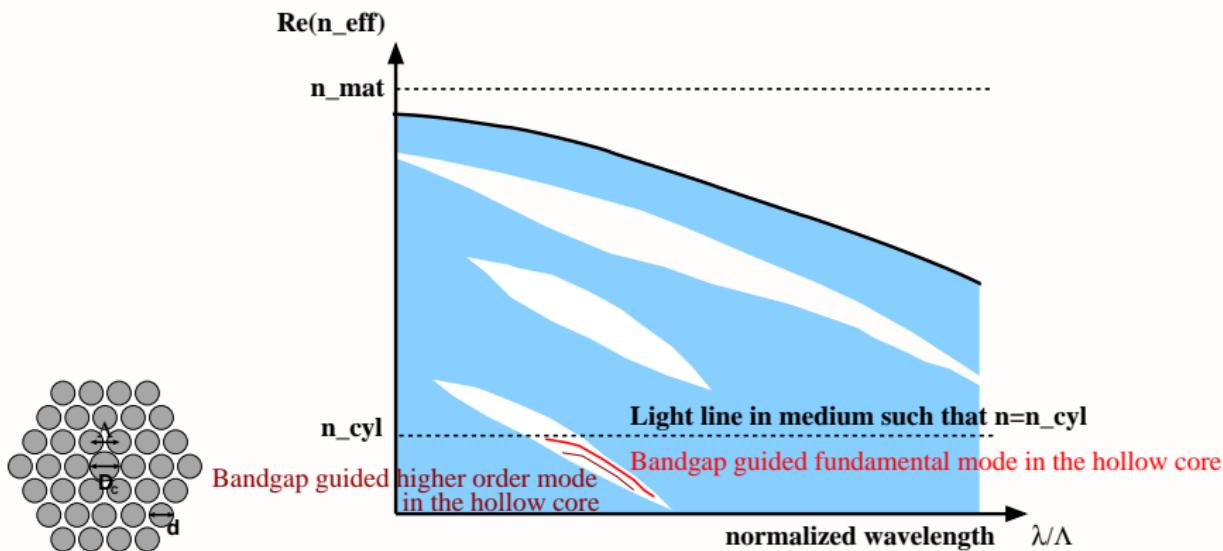


Band diagram for a lattice of inclusions n_{cyl} in a matrix such that $n_{mat} > n_{cyl}$ and dispersion curves of several modes from a finite size HC-MOF.

Main properties of MOFs with hollow-core (HC-MOFs) and low refractive index inclusions

Guiding mechanism: analysis using band diagram for

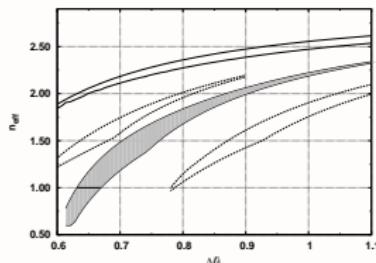
$$n_{core} = n_{cyl} < n_{mat}$$



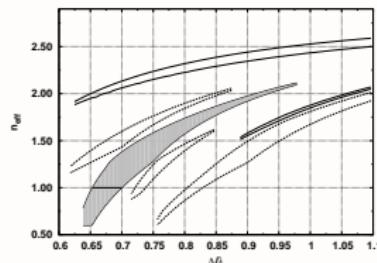
Band diagram for a lattice of inclusions n_{cyl} in a matrix such that $n_{mat} > n_{cyl}$ and dispersion curves of several modes from a finite size HC-MOF.

Main properties of HC-MOFs with low-index inclusions

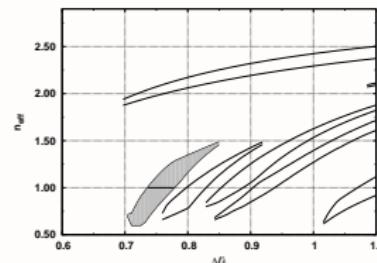
Forbidden photonic bandgap: choice of d/Λ ratio



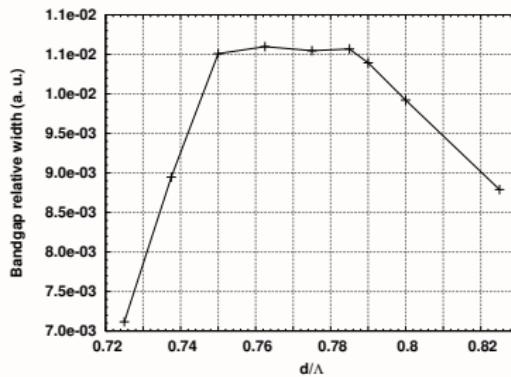
$$d/\Lambda = 0.7375$$



$$d/\Lambda = 0.7625$$



$$d/\Lambda = 0.825$$

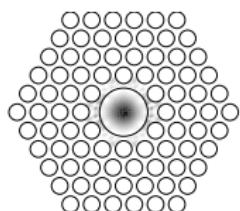


Relative width of the main PBG

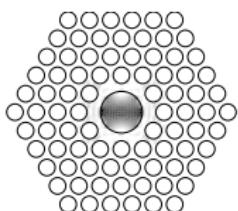
For a triangular lattice of circular air-hole in matrix with $n_{mat} = 2.9095$.

Main properties of HC-MOFs with low-index inclusions

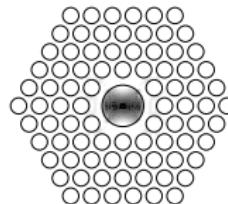
Finite size effects: core size



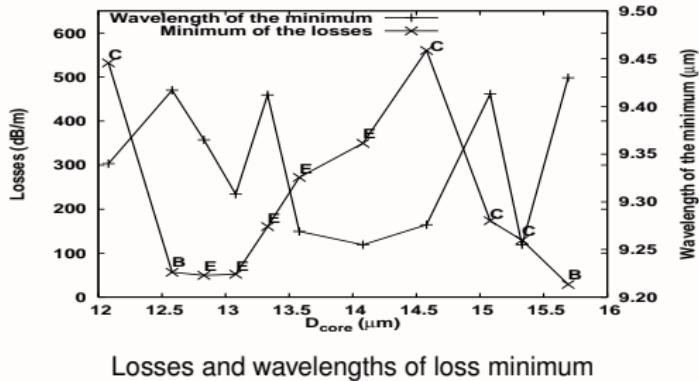
D = 14.58 μm



D = 12.83 μm



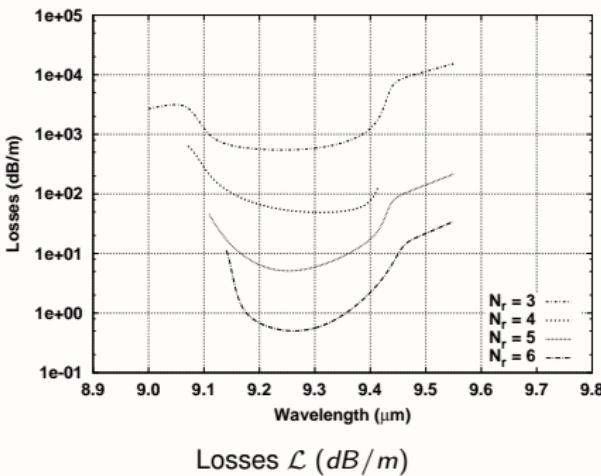
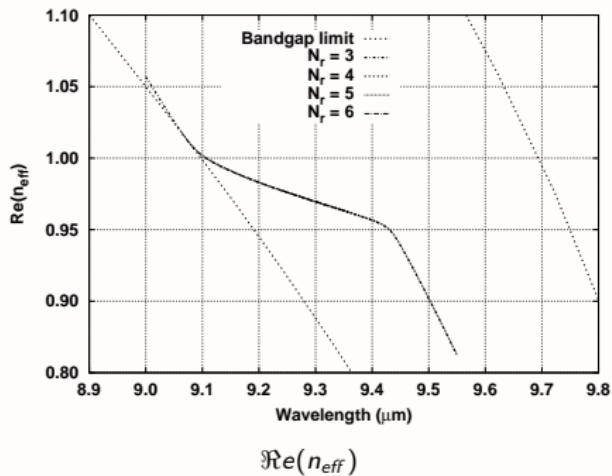
D = 12.58 μm



→ Impact of **surface modes**

Main properties of HC-MOFs with low-index inclusions

Finite size effects: N_r and surface modes influence

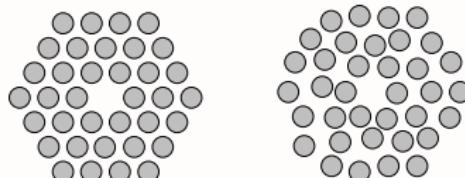


Dependency of \mathcal{L}

- \mathcal{L} decreases exponentially when N_r increases.
 - \mathcal{L} has strong variations at some wavelengths due to surface modes.
 - The size and the shape of the core play a crucial role.

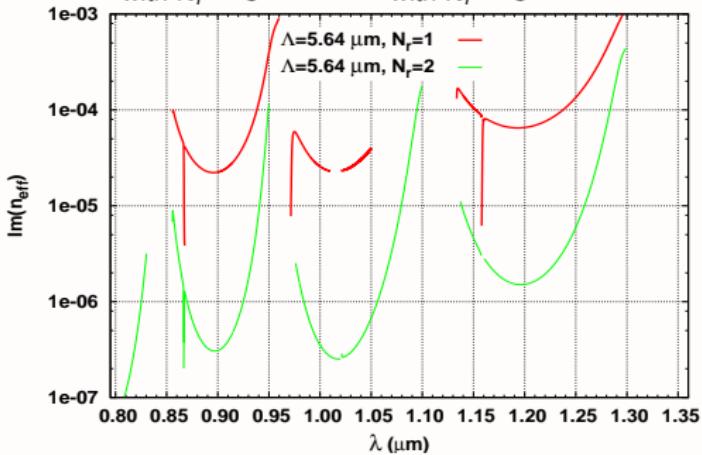
Main properties of MOFs with solid-core and high refractive index inclusions: $n_{mat} < n_{cyl}$

Guiding mechanism: results for the finite-size ARROW MOF



regular MOF
with $N_r = 3$

disordered MOF with $N_r = 3$

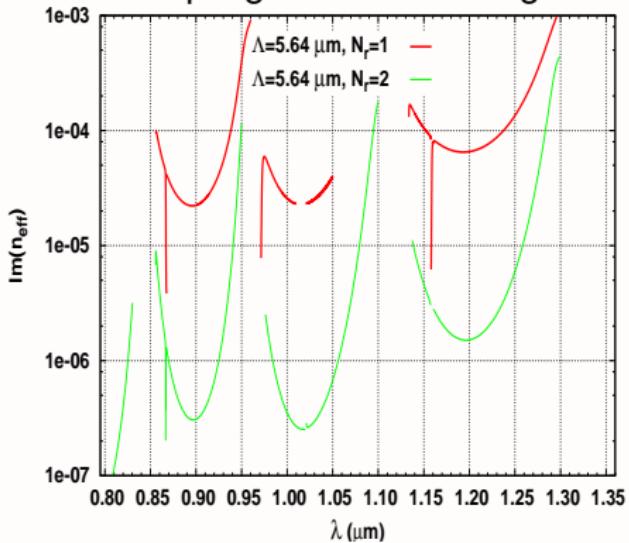


Loss of the core localized mode

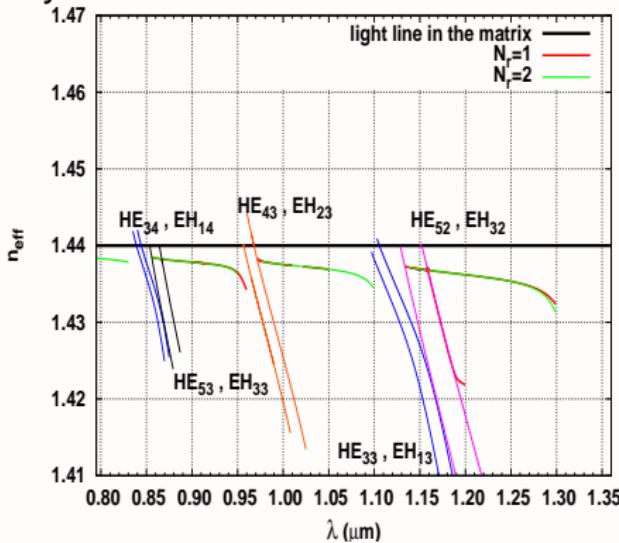
Main properties of MOFs with solid-core and high refractive index inclusions: $n_{mat} < n_{cyl}$

Guiding mechanism: ARROW model (planar waveguide: Duguay 1986)
 Confined in fiber core $\Rightarrow n_{\text{eff}} \lesssim n_{\text{mat}}$

- An isolated cylinder with high index n_{cyl} in a low index matrix n_{mat}
 - Cutoffs of the guided modes of the cylinder → high loss regions
 - No coupling between the high index cylinders



Loss of the core localized mode

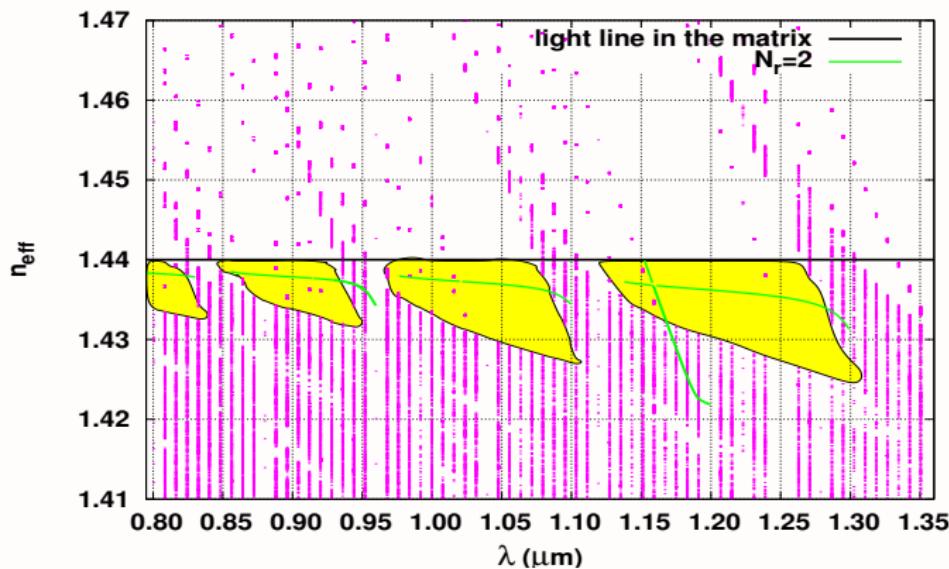


Real parts of the effectives indices $\text{Re}(n_{\text{eff}})$

Main properties of MOFs with solid-core and high refractive index inclusions

Guiding mechanism: periodic structures

- A lattice of cylinders with high index n_{cyl} in a low index matrix n_{mat} : **photonic bandgaps** of the lattice → low loss regions
 - Finite size effects are fully neglected.

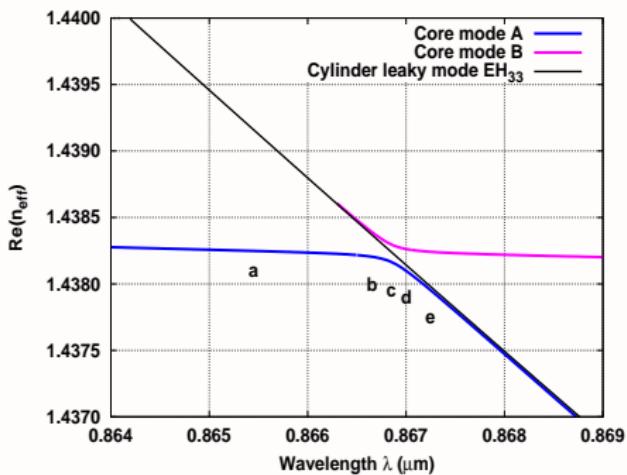


$\text{Re}(n_{eff})$ of the core localized modes and of Bloch modes of the periodic structures

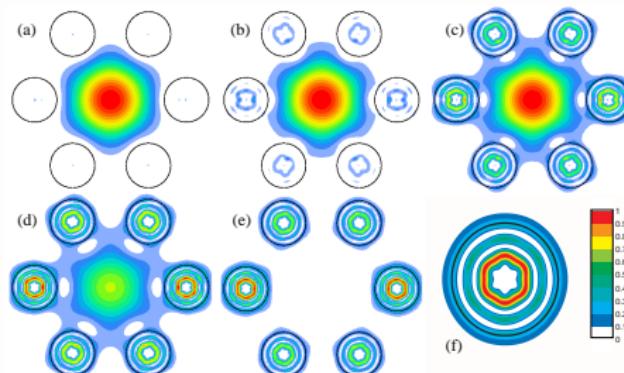
Main properties of SC-MOFs with high index inclusions

Guiding mechanism: avoided-crossing between leaky modes

- We consider the leaky modes of the SC-MOF and the ones of an isolated cylinder → **avoided-crossing** between leaky modes



Avoided-crossing between 2 leaky modes: $\text{Re}(n_{\text{eff}})$



Field maps of the delocalization transition

Models

All the 3 simplified models bring insights on the behaviour of the core localized modes of this type of MOFs → help to obtain specific properties

Conclusions

- The **wide optogeometric parameter space** available for MOFs allows a strong **management of their linear properties**
- These linear properties control the properties of:
 - bend losses
 - nonlinear pulse propagation**: D_c , β_n , A_{eff} , ...
 - coupling with acoustic waves: field profile, β_n , ...
- MOFs do not replace conventional fibers but complete them.

More complex MOFs are already obtained:

- hybrid fibers with different materials
- fibers with graded index inclusions
- fibres with metal parts
- twisted fibers
- tapers
- ...

To design and then to use efficiently these waveguides, one must:

- understand their modal properties
- model precisely both periodic and finite size structures

Joint book on MOFs



F. Zolla, G. Renversez, A. Nicolet,
B. Kuhlmeijer, S. Guenneau, D. Felbacq,
A. Argyros, and S. Leon-Saval.

Foundations of Photonic Crystal Fibres.

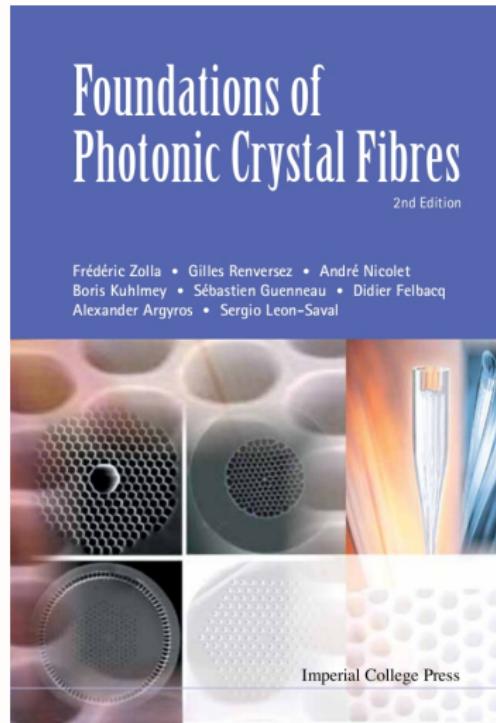
Imperial College Press, London,

2nd edition, 2012

511 pages

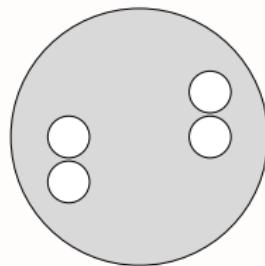
ISBN-10: 1848167288

ISBN-13: 978-1848167285.



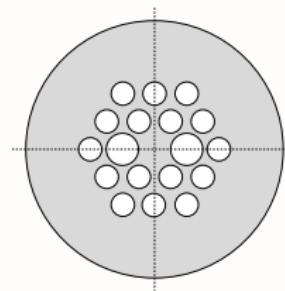
Méthodes pour étudier les FOM I

Symétries des FOM et de leurs modes

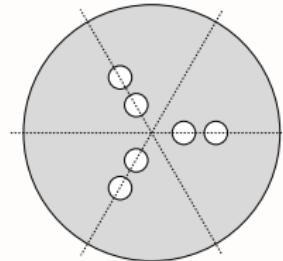


C_2 (pas de plan de symétrie)

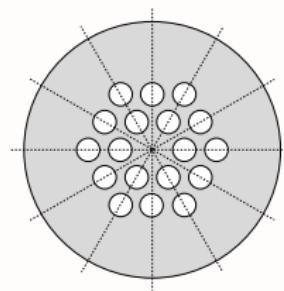
Ces structures sont invariantes par rotation de $2\pi/n$.



C_{2v} (2 plans de symétrie)



C_{3v} (3 plans de symétrie)

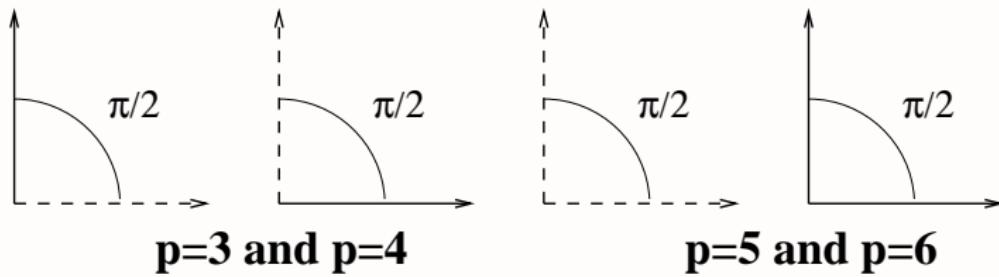
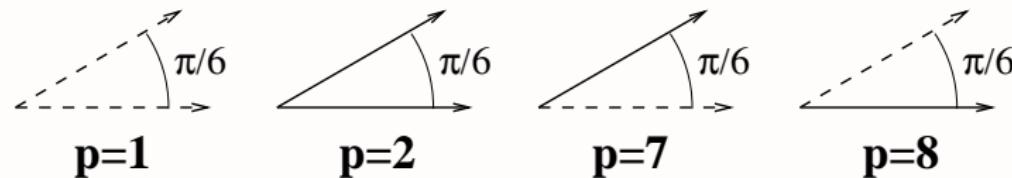


C_{6v} (6 plans de symétrie)

Méthodes pour étudier les FOM

Symétries des modes et dégénérescences pour le cas C_{6v}

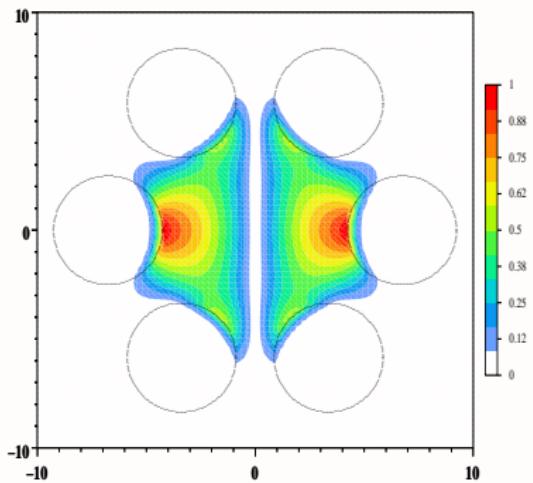
8 classes et 2 secteurs irréductibles géométriques (McIsaac 1975)



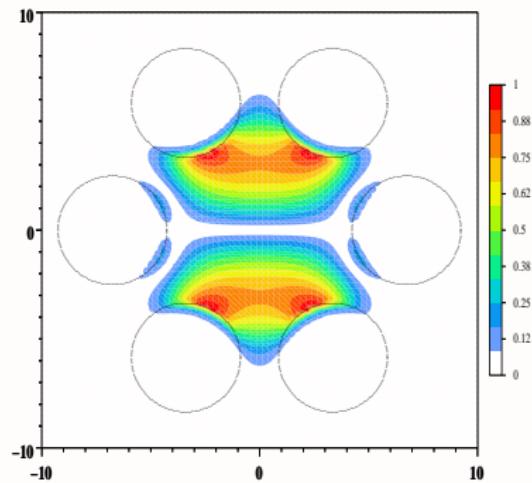
Les 8 classes de symétrie avec les secteurs géométriques irréductibles associés pour le cas C_{6v} . Ligne continue $\rightarrow E_z = 0$. Ligne pointillée $\rightarrow H_z = 0$.

Fundamental mode profiles

$|E_z|$ profiles du mode fondamental pour le cas C_{6v}



$|E_z|$ pour $p = 3$

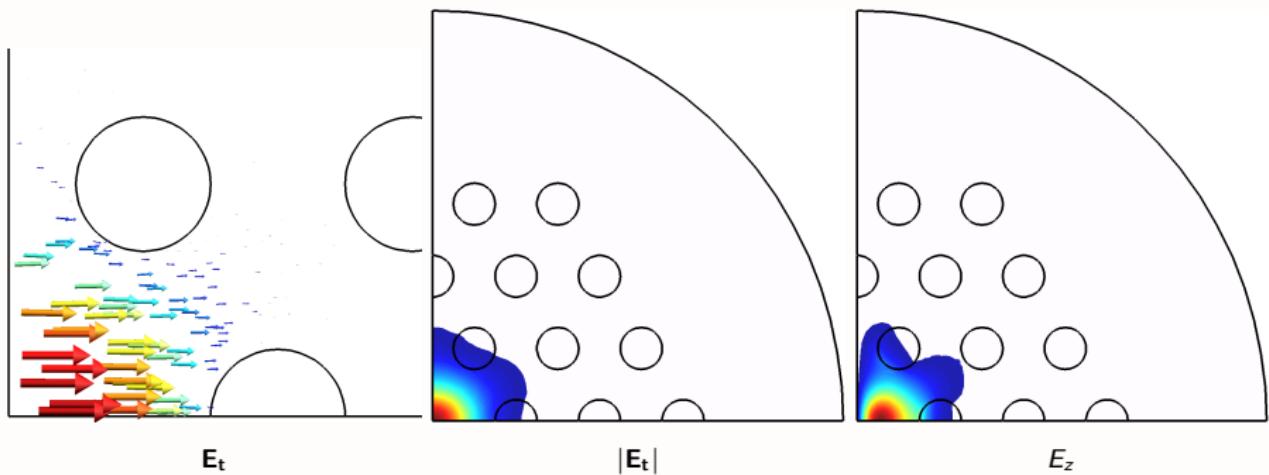


$|E_z|$ pour $p = 4$

E_z components pour les classes dégénérées $p = 3$ et $p = 4$.

Fundamental mode profiles

profiles du mode fondamental pour le cas C_{6v}



Champ électrique \mathbf{E} pour la classe dégénérées $p = 3$

Méthodes pour étudier les FOM

Structures de taille finie : points communs aux méthodes numériques

- ➊ On cherche des solutions pour **les modes**, sous la forme:
 $\mathbf{V}(r, \theta) \exp(-i(\omega t - \beta z))$ ou $\mathbf{V}(x, y) \exp(-i(\omega t - \beta z))$.
- ➋ On résout **rigoureusement** les équations de **Maxwell**.
- ➌ On pose le problème comme un problème de diffraction →
le problème homogène pour obtenir les modes.
- ➍ $\beta \in \mathbb{C}$ ou $\in \mathbb{R}$, β et les champs sont les inconnues.
- ➎ On modélise des **structures de taille finie**.
- ➏ On peut tenir compte des **symétries spatiales** du profil transverse des FOM.
- ➐ Aucune information *a priori* n'est requise sur les champs.
- ➑ Les dispersions matérielles peuvent être considérées.

Méthodes pour étudier les FOM

Structures périodiques : points communs aux méthodes numériques

- ➊ On cherche des solutions pour **les modes**, sous la forme:
 $\mathbf{V}(r, \theta) \exp(-i(\omega t - \beta z))$ ou $\mathbf{V}(x, y) \exp(-i(\omega t - \beta z))$.
- ➋ On résout **rigoureusement** les équations de **Maxwell**.
- ➌ On pose le problème comme un problème de diffraction →
le problème homogène pour obtenir les modes.
- ➍ $\beta \in \mathbb{R}$, β et les champs sont les inconnues.
- ➎ On impose des **conditions de périodicité**.
- ➏ Aucune information *a priori* n'est requise sur les champs.
- ➐ Les dispersions matérielles peuvent être considérées*.



Des méthodes numériques pour étudier les FOM

La méthode multipolaire (1)

Quelques équations

- Dans la matrice les champs vérifient l'équation de Helmholtz.
 - On sait exprimer les composantes transverses en fonction des deux **composantes longitudinales des champs**
 - Développement de ces composantes E_z et H_z en série de Fourier-Bessel autour de chaque inclusion:

$$\begin{cases} E_z(r, \theta, z) &= \sum_{n \in \mathbb{Z}} [A_n^E J_n(k_\perp r) + B_n^E H_n^{(1)}(k_\perp r)] e^{in\theta} e^{i\beta z} \\ H_z(r, \theta, z) &= \sum_{n \in \mathbb{Z}} [A_n^H J_n(k_\perp r) + B_n^H H_n^{(1)}(k_\perp r)] e^{in\theta} e^{i\beta z} \end{cases}$$

- Construction des vecteurs \mathbf{A}_l et \mathbf{B}_l avec les coef. A_n et B_n de l'inclusion l



Des méthodes numériques pour étudier les FOM

La méthode multipolaire (2)

Matrice de diffraction d'une inclusion S_1

- $\mathbf{A}_I \simeq$ champ incident et $\mathbf{B}_I \simeq$ champ diffracté par l'inclusion /
 - $\mathbf{B}_I = \mathbf{S}_I \mathbf{A}_I$
 - Si l'inclusion est circulaire alors \mathbf{S}_I est connue analytiquement en fonction des J_n et $H_n^{(1)}$

Bilan des champs, identité de Rayleigh

- Matrice de changement de base $\mathcal{H}_{j,I}$ (théorème d'addition de Graf)
 - Champ incident sur l'inclusion I = champ diffracté par toutes les autres inclusions + le champ incident direct:

$$\mathbf{A}_I = \sum_{j \neq I} \mathcal{H}_{j,I} \mathbf{B}_j + \mathcal{H}_{0,I} \mathbf{B}_0$$

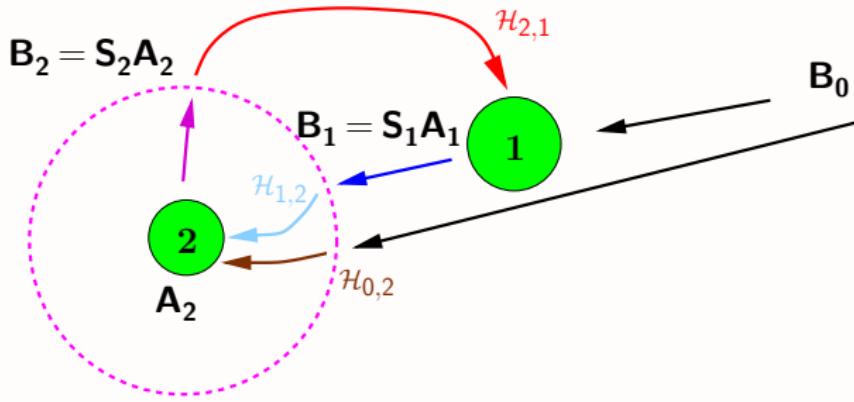


Schéma de principe de la méthode multipolaire: auto-cohérence des champs incidents et diffractés par l'ensemble des inclusions

Equation matricielle de la méthode

$$\left\{ \begin{array}{l} \mathbf{B}_1 = \mathbf{S}_1(\sum_{j \neq 1} \mathcal{H}_{j,1} \mathbf{B}_j + \mathcal{H}_{0,1} \mathbf{B}_0) \\ \mathbf{B}_2 = \mathbf{S}_2(\sum_{j \neq 2} \mathcal{H}_{j,2} \mathbf{B}_j + \mathcal{H}_{0,2} \mathbf{B}_0) \\ \dots \end{array} \right. \quad \text{soit} \quad \mathcal{M} \begin{bmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \\ \vdots \end{bmatrix} = \mathcal{D} \begin{bmatrix} \mathbf{B}_0 \\ \mathbf{B}_0 \\ \vdots \end{bmatrix}$$

Des méthodes numériques pour étudier les FOM

La méthode multipolaire (4)

La matrice \mathcal{M} et les modes

- \mathcal{M} dépend de λ , de β , et des paramètres des inclusions
- Un mode correspond à un champ propagé sans sources extérieures ($\mathbf{B}_0 = 0$):

$$\mathcal{M} \begin{bmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \\ \dots \end{bmatrix} = 0 \text{ avec } \mathbf{B}_l \text{ non nuls}$$

$\det \mathcal{M}(\beta) = 0$ (équation de dispersion d'une FOM)

- β étant déterminé, on calcule les champs via les vecteurs propres de \mathcal{M}

Aspect pratique mais crucial

Le problème modal est nettement plus difficile numériquement que le problème de diffraction.

La recherche efficace dans \mathbb{C} des β tel que $\det(\mathcal{M}(\beta)) = 0$ est non triviale ...

Des méthodes numériques pour étudier les FOM

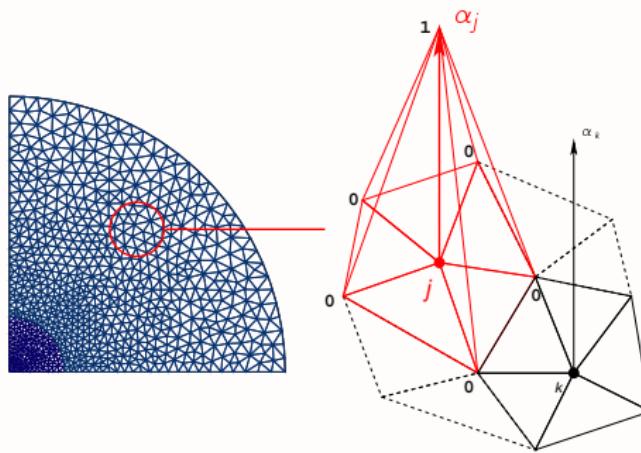
La méthode de type éléments finis (1)

Principes

- ➊ discrétisation spatiale: maillage et fonction de forme
 $\psi(x, y) \rightsquigarrow \sum_{j=1}^{\text{nb él.}} \psi_j \alpha_j(x, y)$
- ➋ formulation faible des équations de Maxwell: formulation intégrale
- ➌ résolution numérique impliquant uniquement des matrices creuses

Des méthodes numériques pour étudier les FOM

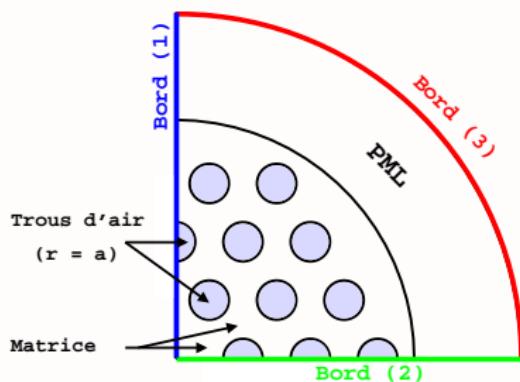
La méthode de type éléments finis (2)



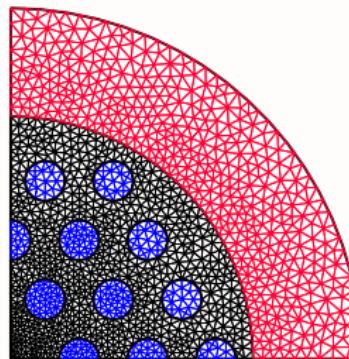
Maillage sur un quart de section de fibre et fonctions de forme dans le cas scalaire

Des méthodes numériques pour étudier les FOM I

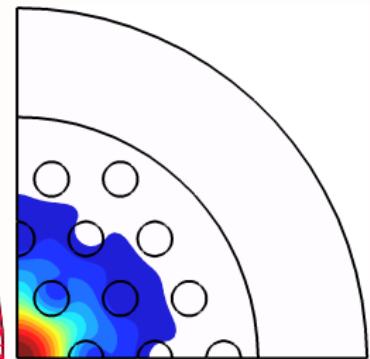
La méthode de type éléments finis (3)



Géométrie



Maillage associé

 $|S_z|$ pour le mode fondamental

pour une fibre optique microstructurée à cœur plein

Difficulté technique

- description de la couche parfaitement adaptée 'Perfect Matching Layer' (PML)

Des méthodes numériques pour étudier les FOM

Les trois méthodes numériques rigoureuses maîtrisées
ont été conçues et implémentées au sein du laboratoire.

- La méthode multipolaire est la méthode de référence.
- L'institut Fresnel est le seul laboratoire à disposé de ces 3 outils de niveau mondial.

Des méthodes numériques pour étudier les FOM

Les trois méthodes numériques rigoureuses maîtrisées

Méthode Multipolaire (MM)

Temps de calcul:

Le plus faible

Precision:

La meilleure sur $\Re(n_{eff})$
et sur $\Im m(n_{eff})$

Avantages:

1. Calculs semi-analytiques pour des inclusions circulaires
 2. Nombre de couches d'inclusions élevé
-

Limitations:

1. Milieux homogènes et isotropes
2. Les inclusions inscrites dans des cercles non-sécants
3. Pour inclusion non-circulaire:
fournir la matrice S

Méthode différentielle (MD)

Temps de calcul:

> MM

Precision:

> MEF mais < MM

Avantages:

1. Géométrie arbitraire
 2. Milieux inhomogènes et anisotropes
 3. Mise en place des symétries
-

Limitations:

1. Nombre de couches d'inclusions
2. Intégration numérique
3. Codage des structures

Méthode des éléments finis avec gmsh/getdp (MEF)

Temps de calcul:

> MM, \lesssim MD

Precision:

< MM, \lesssim MD pour $\Im m(n_{eff})$

Avantages:

1. Géométrie arbitraire
 2. Milieux inhomogènes et anisotropes
 3. Codage des structures
-

Limitations:

1. Modes à pertes \Rightarrow PML
2. Plusieurs paramètres internes
3. Nombre de couches d'inclusions

Des méthodes numériques pour étudier les FOM

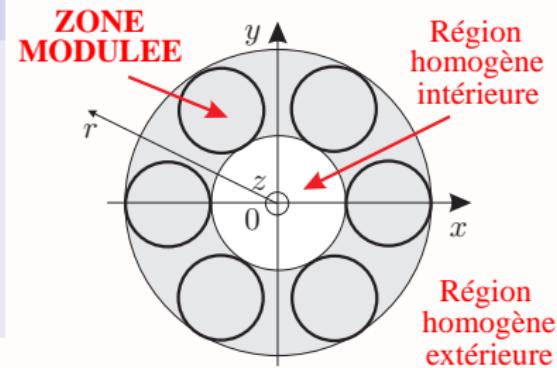
La méthode différentielle (en bref)

Principes

- ① méthode différentielle en coordonnées cylindriques (r, θ, z) et en conique
- ② utilisation des règles correctes de factorisation
- ③ utilisation de l'algorithme S
- ④ problème de diffraction \rightarrow problème homogène

Partage de l'espace en trois régions

- 2 régions homogènes
expressions explicites des champs
- 1 région intermédiaire: la zone modulée
système différentiel du 1^{er} ordre en r à intégrer numériquement



Après calcul des matrices $S^{(i)}$ à travers la zone modulée

Matrice de diffraction S de la zone modulée:

$$\mathbf{S}^{-1} \mathbf{B} = \mathbf{A}$$

Des méthodes numériques pour étudier les FOM

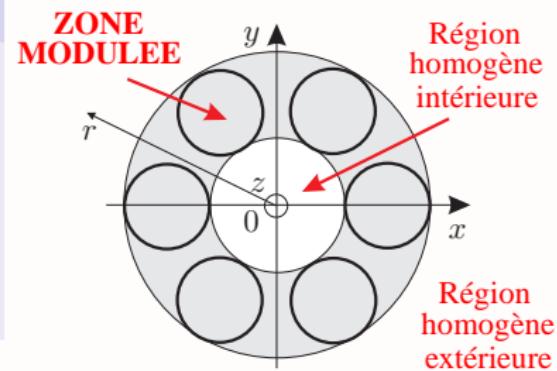
La méthode différentielle (1) (Boyer, Popov, Nevière)

Principes

- ① méthode différentielle en coordonnées cylindriques (r, θ, z)
- ② utilisation des règles correctes de factorisation
- ③ utilisation de l'algorithme S

Partage de l'espace en trois régions

- 2 régions homogènes
expressions explicites des champs
- 1 région intermédiaire: la zone modulée
système différentiel à intégrer
numériquement



Des méthodes numériques pour étudier les FOM

La méthode différentielle (2)

Dans la zone modulée:

- $\mathbf{D}(r, \theta, z) = \epsilon(r, \theta)\mathbf{E}(r, \theta, z)$
- développements de Fourier de $\begin{cases} \mathbf{D}(r, \theta, z) \\ \mathbf{E}(r, \theta, z) \end{cases}$
- développement de Fourier de $\epsilon(r, \theta)$

Règles de factorisation (L. Li (1996))

- **Règle directe:** $[\mathbf{D}_T] = [\epsilon] [\mathbf{E}_T]$
- **Règle inverse:** $[\mathbf{D}_N] = \left[\begin{smallmatrix} 1 \\ \epsilon \end{smallmatrix} \right]^{-1} [\mathbf{E}_N]$
- Expression de $[\mathbf{E}_T]$ et $[\mathbf{E}_N]$ en fonction de $[\mathbf{E}]$
 $\Rightarrow [\mathbf{D}] = Q_\epsilon [\mathbf{E}]$ (la matrice Q_ϵ n'est pas simple)

Notation: $[\epsilon] = \begin{pmatrix} & & \\ & \vdots & \\ & \epsilon_{n-m} & \dots \end{pmatrix}$

Des méthodes numériques pour étudier les FOM

La méthode différentielle (3)

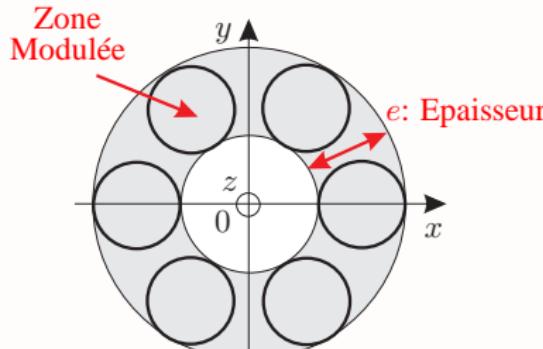
Intégration numérique à travers la zone modulée

Matrice de transmission T de la zone modulée

$$\mathbf{U} = \mathbf{T}\mathbf{V} \quad \text{avec} \quad \left\{ \begin{array}{l} \mathbf{U}: \text{Coef. de Fourier des champs} \\ \text{dans la région homogène extérieure} \\ \mathbf{V}: \text{Coef. de Fourier des champs} \\ \text{dans la région homogène intérieure} \end{array} \right.$$

Si e trop grand

- ➊ Divergence de certains éléments de la matrice T
- ➋ Contaminations numériques

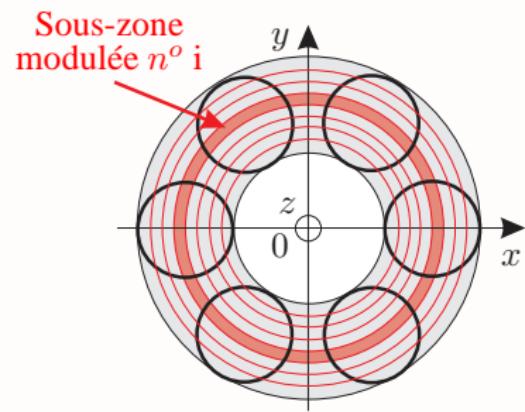


Des méthodes numériques pour étudier les FOM

La méthode différentielle (4)

Principe de l'algorithme S

- Subdivision de la zone modulée en sous-zones modulées
- Pour la sous-zone modulée $n^o i$:
 - Intégration:**
Arrêt avant contaminations numériques
 \Rightarrow matrice $T^{(i)}$ bien conditionnée
 - Calcul itératif:** $S^{(i)} = f(S^{(i-1)}, T^{(i)})$
dont les éléments sont bornés



Après calcul des matrices $S^{(i)}$ à travers la zone modulée

Matrice de diffraction S de la zone modulée:

$$\mathbf{B} = \mathbf{SA} \quad \text{avec} \quad \left\{ \begin{array}{l} \text{B: Coefficient de Fourier des champs diffractés} \\ \text{A: Coefficient de Fourier des champs incidents} \end{array} \right.$$

Des méthodes numériques pour étudier les FOM

La méthode de type éléments finis: cas vectoriel (1)

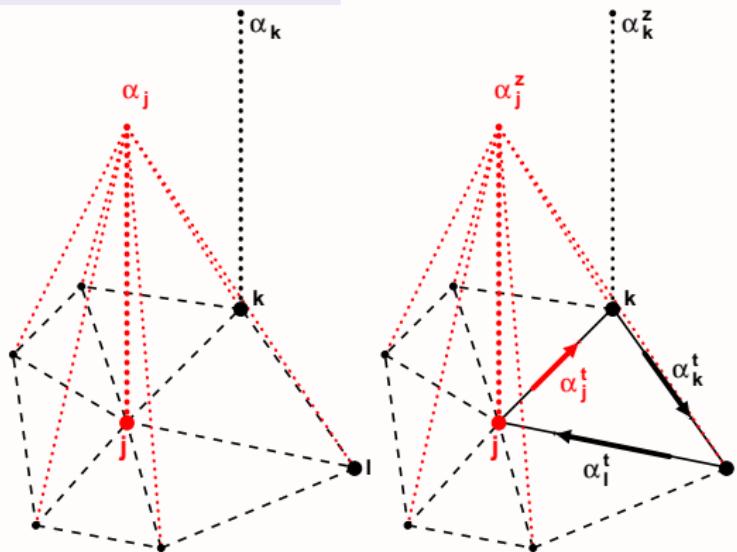
L'équation vectorielle

Directement issue des équations de Maxwell:

$$\nabla \times (\nabla \times \mathbf{E}) = k_0^2 \epsilon_r(x, y)$$

Forme des solutions recherchées

$$\mathbf{E} = (\mathbf{E}_t + E_z \mathbf{e}_z) e^{i\beta z - i\omega t}$$

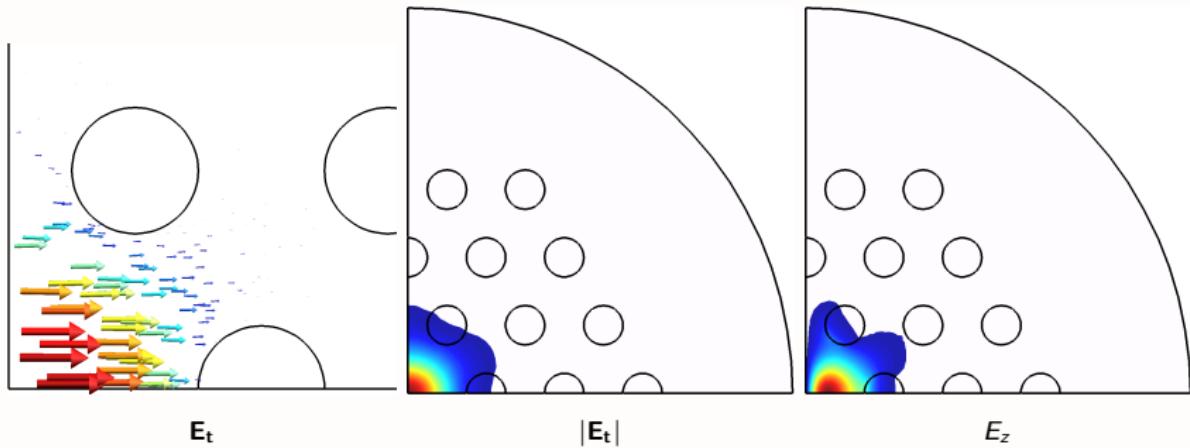


Fonction de forme scalaire

Fonctions de forme vectorielles

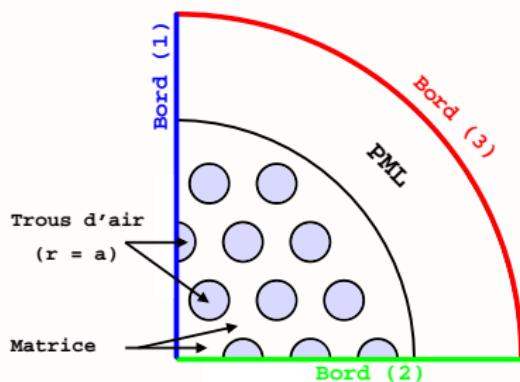
Des méthodes numériques pour étudier les FOM

La méthode de type éléments finis: cas vectoriel (2)

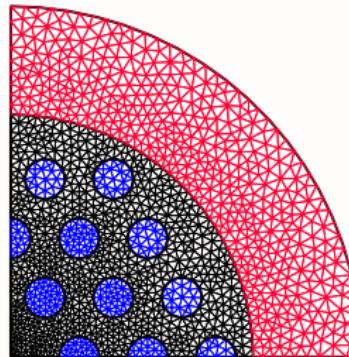


Des méthodes numériques pour étudier les FOM

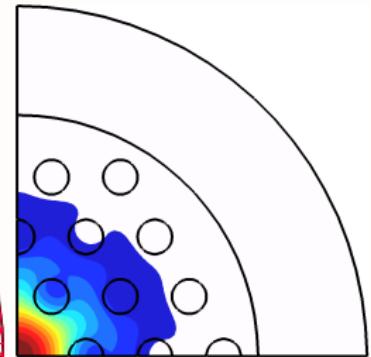
La méthode de type éléments finis (3)



Géométrie



Maillage associé

 $|S_z|$ pour le mode fondamental

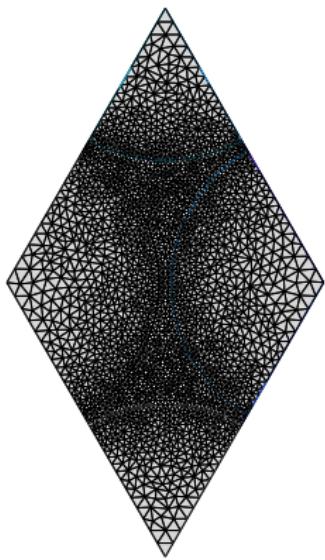
Pour une fibre optique microstructurée à cœur plein

Difficulté technique

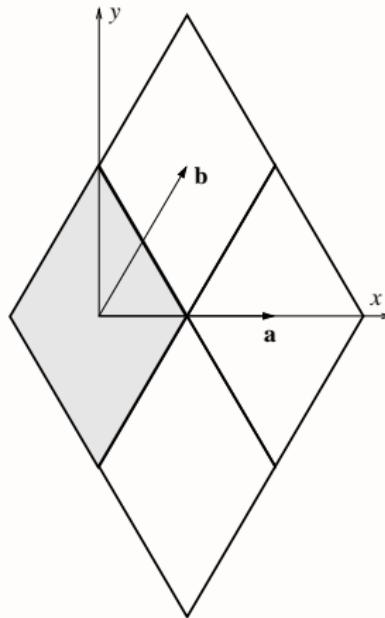
- description de la couche parfaitement adaptée 'Perfect Matching Layer' (PML)

Numerical methods to study MOFs

Computing band diagram using the finite element method (1)



Meshing of a basic rhombic cell with 4628 triangles.

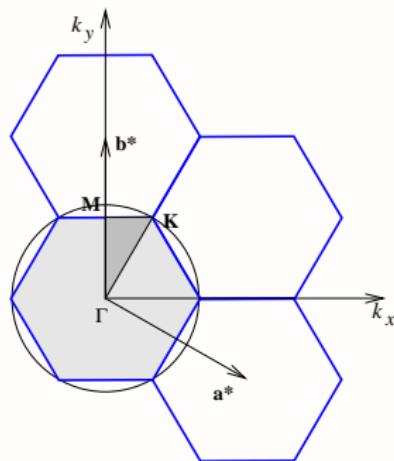


Representation of some lattice cells with the lattice vectors $\mathbf{a} = \Delta \mathbf{e}_x$ and $\mathbf{b} = \frac{\Delta}{2} \mathbf{e}_x + \frac{\Delta\sqrt{3}}{2} \mathbf{e}_y$.

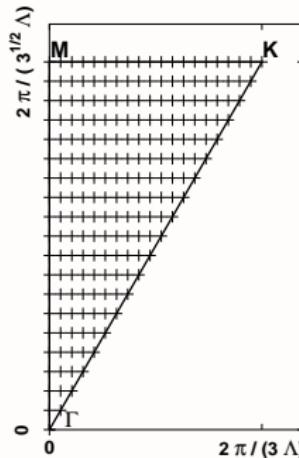
Numerical methods to study MOFs

Computing band diagram using the finite element method (2)

Two-dimensional periodic structure: the triangular lattice of circular inclusions. The basic cell is rhombic with a side length Λ with a circular air inclusion with a radius $R = 0.48\Lambda$.



Some cells of the reciprocal lattice with the lattice vectors



Example of sampling points in the irreducible part of the Brillouin zone (IBZ) for a triangular lattice. 210 points are used in this case to sample in 2D the IBZ.

Des méthodes numériques pour étudier les FOM

La méthode multipolaire (1)

Quelques équations

- Dans la matrice les champs vérifient l'équation de Helmholtz.
- On sait exprimer les composantes transverses en fonction des deux **composantes longitudinales des champs**
- Développement de ces composantes E_z et H_z en série de Fourier-Bessel autour de chaque inclusion:

$$\begin{cases} E_z(r, \theta, z) &= \sum_{n \in \mathbb{Z}} [A_n^E J_n(k_\perp r) + B_n^E H_n^{(1)}(k_\perp r)] e^{in\theta} e^{i\beta z} \\ H_z(r, \theta, z) &= \sum_{n \in \mathbb{Z}} [A_n^H J_n(k_\perp r) + B_n^H H_n^{(1)}(k_\perp r)] e^{in\theta} e^{i\beta z} \end{cases}$$

- Construction des vecteurs \mathbf{A}_I et \mathbf{B}_I avec les coef. A_n et B_n de l'inclusion I

Des méthodes numériques pour étudier les FOM

La méthode multipolaire (2)

Matrice de diffraction d'une inclusion S_I

- $\mathbf{A}_I \simeq$ champ incident et $\mathbf{B}_I \simeq$ champ diffracté par l'inclusion /
- $\mathbf{B}_I = \mathbf{S}_I \mathbf{A}_I$
- Si l'inclusion est circulaire alors \mathbf{S}_I est connue analytiquement en fonction des J_n et $H_n^{(1)}$

Bilan des champs, identité de Rayleigh

- Matrice de changement de base $\mathcal{H}_{j,I}$ (théorème d'addition de Graf)
- Champ incident sur l'inclusion I = champ diffracté par toutes les autres inclusions + le champ incident direct:

$$\mathbf{A}_I = \sum_{j \neq I} \mathcal{H}_{j,I} \mathbf{B}_j + \mathcal{H}_{0,I} \mathbf{B}_0$$

Des méthodes numériques pour étudier les FOM

La méthode multipolaire (3)

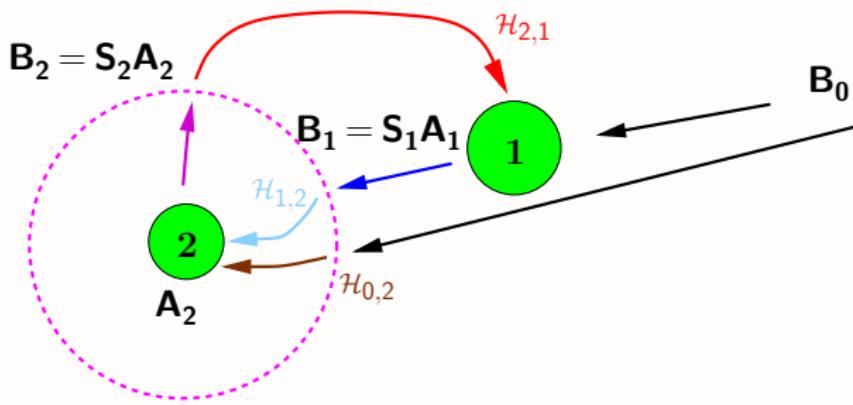


Schéma de principe de la méthode multipolaire: auto-cohérence des champs incidents et diffractés par l'ensemble des inclusions

Equation matricielle de la méthode

$$\begin{cases} \mathbf{B}_1 = \mathbf{S}_1 \left(\sum_{j \neq 1} \mathcal{H}_{j,1} \mathbf{B}_j + \mathcal{H}_{0,1} \mathbf{B}_0 \right) \\ \mathbf{B}_2 = \mathbf{S}_2 \left(\sum_{j \neq 2} \mathcal{H}_{j,2} \mathbf{B}_j + \mathcal{H}_{0,2} \mathbf{B}_0 \right) \\ \dots \end{cases} \quad \text{soit} \quad \mathcal{M} \begin{bmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \\ \vdots \end{bmatrix} = \mathcal{D} \begin{bmatrix} \mathbf{B}_0 \\ \mathbf{B}_0 \\ \vdots \end{bmatrix}$$

Des méthodes numériques pour étudier les FOM

La méthode multipolaire (4)

La matrice \mathcal{M} et les modes

- \mathcal{M} dépend de λ , de β , et des paramètres des inclusions
- Un mode correspond à un champ propagé sans sources extérieures ($\mathbf{B}_0 = 0$):

$$\mathcal{M} \begin{bmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \\ \dots \end{bmatrix} = 0 \text{ avec } \mathbf{B}_l \text{ non nuls}$$

$\det \mathcal{M}(\beta) = 0$ (équation de dispersion d'une FOM)

- β étant déterminé, on calcule les champs via les vecteurs propres de \mathcal{M}

Aspect pratique mais crucial

Le problème modal est nettement plus difficile numériquement que le problème de diffraction.

La recherche efficace dans \mathbb{C} des β tel que $\det(\mathcal{M}(\beta)) = 0$ est non triviale ...

Numerical methods to study MOFs

Comparison between the 3 numerical methods

- consider finite size structures
- get the losses
- take into account symmetries
- take into account material dispersion

All these 3 methods have been developed and implemented at the lab.

- The MM and the FEM are well suited to be adapted to study periodic structures : compute band diagrams

Numerical methods to study MOFs

Comparison between the 3 numerical methods

Méthode Multipolaire (MM)

Temps de calcul:

Le plus faible

Précision:

La meilleure sur $\Re e(n_{eff})$
et sur $\Im m(n_{eff})$

Avantages:

1. Calculs semi-analytiques pour des inclusions circulaires
 2. Nombre de couches d'inclusions élevé
-

Limitations:

1. Milieux homogènes et isotropes
2. Les inclusions inscrites dans des cercles non-sécants
3. Pour inclusion non-circulaire:

Méthode différentielle (MD)

Temps de calcul:

> MM

Précision:

> MEF mais < MM

Avantages:

1. Géométrie arbitraire
 2. Milieux inhomogènes et anisotropes
 3. Mise en place des symétries
-

Limitations:

1. Nombre de couches d'inclusions
2. Intégration numérique
3. Codage des structures

Méthode des éléments finis avec gmsh/getdp (MEF)

Temps de calcul:

> MM, \lesssim MD

Précision:

< MM, \lesssim MD pour $\Im m(n_{eff})$

Avantages:

1. Géométrie arbitraire
 2. Milieux inhomogènes et anisotropes
 3. Codage des structures
-

Limitations:

1. Modes à pertes \Rightarrow PML
2. Plusieurs paramètres internes
3. Nombre de couches d'inclusions