# Antiresonant reflecting optical waveguide microstructured fibers revisited: a new analysis based on leaky mode coupling

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**Abstract:** Using two different modal methods, the multipole method and the more recent fast Fourier factorization method, we exhibit and explain a core mode transition induced by avoided crossing between a core localized leaky mode and an high-index cylinder leaky mode in anti-resonant reflecting optical waveguide microstructured optical fibers (ARROW MOFs). Due to its wavelength selectivity and to the leaky nature of the involved modes, this transition does not seem to have already been described in detail and analyzed as done in this work in spite of several already published studies on core mode dispersion properties. The main properties of this transition are also described. We also revisit the already mentioned cut-off phenomena limiting the transmission band in ARROW MOFs in terms of mode coupling between the core mode and one or several high-index cylinder modes.



**OCIS codes:** (060.2400) Fiber properties, (060.2280) Fiber design and fabrication, (230.3990) Microstructure devices

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## 1. Introduction

Microstructured optical fibers (MOFs) consist of a core region surrounded by a finite lattice of inclusions embedded in the fiber matrix. In the case in which the core region refractive index is lower than the inclusion region one, we may get a photonic bandgap fiber. In this article, we focus on this kind of MOF, and more precisely on antiresonant reflecting optical waveguide (ARROW) MOFs. These MOFs are obtained when the inclusions are made of higher refractive index material than the matrix refractive index. These high refractive index inclusions MOFs have already been proposed for designing tunable photonic devices [1]. These fibers may be obtained by filling the air holes with an high-index liquid [1]. More recently, all-solid silica-based ARROW MOF have been fabricated [2, 3]. The guiding properties of these MOFs are mainly determined by the individual properties of high refractive index inclusions rather than their positions and number [4, 5]. It has also been stated that the location of the spectral transmission minima are associated with the cut-off wavelengths of the high-index inclusion guided modes.

In what follows we describe and explain a new phenomenon occuring in these fibers [6], which have not been already described in detail. This phenomenon is the core mode sharp transition induced by an avoided crossing with a leaky mode of the high refractive index cylinders. First, we illustrate with some details this transition on an already studied structure using two different modal methods and we compare it to similar phenomena occuring in another kind of MOFs. Secondly, we study the avoided crossing transition versus the inclusion lattice pitch and the refractive index contrast between the cylinders and the matrix. Before the conclusion, we discuss the results obtained in our study.

## 2. First numerical example of an avoided crossing between leaky modes

The ARROW MOF we consider is the one described in the innovatory article [4] written by White and his colleagues. It is a one ring $C_{6v}$ MOF with circular inclusions with diameters $d$ equal to 3.315 $\mu m$, cylinder refractive index $n_{cyl} = 1.8$, matrix refractive index $n_{mat} = 1.44$, and the pitch $\Lambda$ being set to 5.64 $\mu m$. Computing the core mode dispersion curves we observe that several discontinuities seem to have been overlooked in the Fig. 2(a) of Ref. [4] even if the authors wrote at the end of their article, as a strong physical insight, ”Note that some of the smaller details of the loss spectrum are also predited accurately by the scattering ratio for the single cylinder”. One such discontinuities is shown in Fig. 1. These curves are obtained with the well-established multipole method [7, 8] which was also used in reference [4] and other ARROW MOF publications. These observed discontinuities are not computational artefacts since we also obtain them with another fully different modal method, the fast Fourier factorization method which was developed to overcome the known limitations of the multipole method [9].

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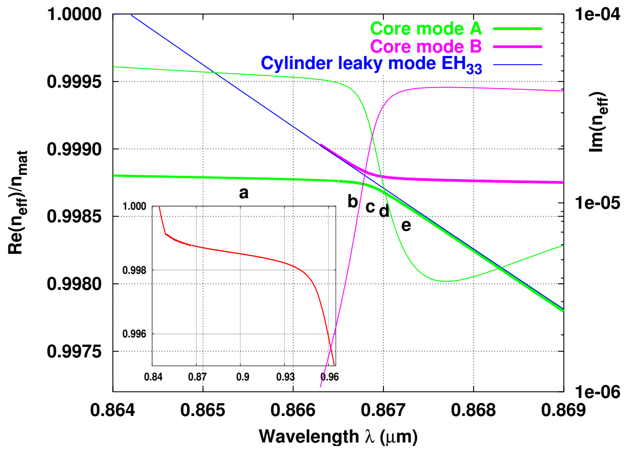


Fig. 1. Normalized real part of the effective index, $\Re e(n_{\text{eff}})$, of the ARROW MOF core mode versus the wavelength around the transition. The asymptot for the core mode curves A and B is the real part of $n_{\text{eff}}$ for leaky mode $EH_{33}$ of the isolated high-index cylinder. The imaginary part, $\Im m(n_{\text{eff}})$, of the core modes A and B is also given (thin curves, right *y*-scale). Inset: overall view of the dispersion curve (without the discontinuities) in the corresponding transmission band.

For both methods, all the dispersion curves are computed using an efficient extrapolation based scheme [7, 8].

Figure 2 shows the *z*-component of the Poynting vector modulus for several increasing wavelengths around the observed transition in the core mode dispersion curve shown in Fig. 1. We clearly see that the core mode undergoes a spatial transition from the core fiber to the high-index inclusions. In the present case, we can note that this transition is abrupt, the typical wavelength scale being approximately 1.0 $10^{-3}$ $\mu m$. In the vicinity of the transition wavelength and for neighbouring values of the effective index real part, on can check that there is no guided mode of a single high index cylinders in a infinite matrix undergoing a cut-off [10]. Nevertheless, it is possible to find leaky modes for such isolated cylinder *i.e.*, mode with a non null effective index imaginary part. Consequently, we can analyze the core mode transition associated with the observed discontinuities in Fig. 1 as an avoided crossing between the ARROW MOF core localized leaky mode and a defect leaky mode generated by the set of high-index cylinders. To our knowledge, this is the first time such phenomenon [5] is described in details and clearly analyzed in ARROW MOFs in terms of leaky mode coupling. Some consequences of this effect have already been observed on the modal effective area and field distribution [5]. Nevertheless, these consequences were analyzed in terms of guided mode cut-off mainly because the plane wave method used is not able to tackle the imaginary part of the modal effective index occuring in the finite size structure and also because of the spectral sharpness of the phenomenon in the studied configuration.

## 3. Some properties of these avoided crossings between leaky modes

Avoided crossings between defect and core modes have already been oberved and studied in Bragg MOFs [11] or in hollow core band-gap MOFs [12]. Since, in our study, the observed defect modes are linked to the isolated cylinder leaky modes, the avoided crossings will occur in ARROW MOFs whatever the cylinder optogeometric parameters are (including the ones associated with the single guided mode regime). We can also note that both the observed avoided

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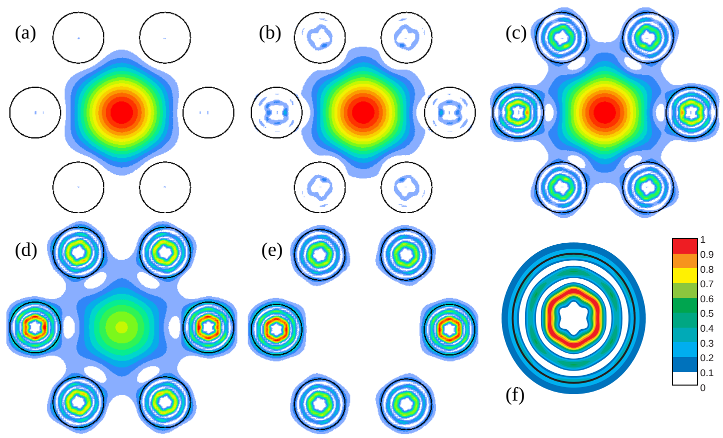


Fig. 2. Modulus distribution of the $z$-component of the Poynting vector across the transition of core mode A. (a) $\lambda$=0.8654 $\mu$m, (b) $\lambda$=0.8666 $\mu$m, (c) $\lambda$=0.8668 $\mu$m, (d) $\lambda$=0.8669 $\mu$m, (e) $\lambda$=0.8672 $\mu$m (see Fig. 1 for their respective positions in the $\Re e(n_{\mathrm{eff}})(\lambda)$ dispersion curve), (f) Modulus distribution of the $z$-component of the Poynting vector for the leaky mode $EH_{33}$ of an isolated high-index inclusion (border shown by the black circle), $n_{eff}$= 1.438142 + i 9.121 $10^{-7}$ and $\lambda$=0.867$\mu$m.

crossings with the isolated cylinder leaky modes and the behaviour of the core modes in ARROW band gap lower edges (short wavelength) are similar phenomena: a structure core mode transition induced by the coupling with the existing leaky modes of the confining elements *i.e.* the high refractive index cylinders. To illustrate these properties, we give the dispersion curves showing two other avoided crossings in Fig. 3, one associated with the isolated cylinder leaky mode $HE_{53}$ and another one associated with the $EH_{14}$ cylinder leaky mode.

The dispersion curves concerning the lower transition of the core mode is associated to the avoided crossing with the isolated cylinder leaky mode $EH_{14}$. The $EH_{14}$ leaky mode has much higher losses than the two others leaky modes ($HE_{53}$ and $EH_{33}$) below the common cut-off limit given by the matrix refractive index (data not shown). These higher losses of the $EH_{14}$ leaky mode together with its proximity of the cut-off limit explain why this mode is associated with the ARROW band gap lower edge. For the upper edges (long wavelength) of ARROW band gaps (see as an example the leftmost curve in Fig. 3), we observe that the involved modes in the avoided crossings are usually multiple cylinder leaky modes coming from the coupling between some modes of the isolated high refractive index cylinders (data not shown). Such modes are more clearly defined in ARROW MOFs with several rings of high-index inclusions than in the one ring MOF shown in Fig. 2 since the proportion of outer ring cylinders over the entire cylinder set decreases, which induces that the perturbative coupling terms between the isolated cylinder leaky modes become more homogeneous across the high-index inclusion region (a semi-analytical treatment of these issues is currently under investigation).

In order to quantify the avoided crossing properties we introduce the quantity $\mathscr{R} = \partial^2 \Re e(n_{eff})/\partial\lambda^2$. As it can be seen in Fig. 3 and more precisely in Fig. 4, the spectral sharpness of the transition decreases with the pitch for a fixed $d$ value (even if a factor $1/\Lambda^2$ is taken into account to compensate for the core size change). In the same way, the strength of the transition (quantified by $max(|\mathscr{R}|)$) decreases with the pitch. The product of the two quantities staying nearly constant in the studied range of parameters. These properties can be explained as follows in terms of coupling between the high refractive index cylinder modes and the core localized mode: a decrease of the pitch induces a stronger coupling between these modes, and conse-

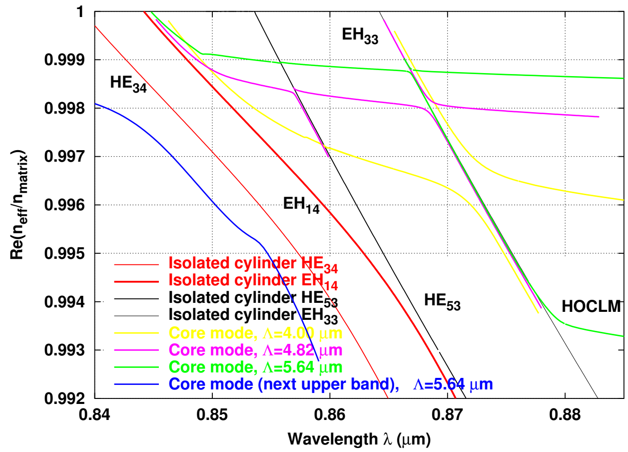


Fig. 3. Avoided crossings between the structure core mode and leaky modes of high refractive index cylinder ($EH_{14}$, $HE_{53}$, $EH_{33}$) ; the cylinder parameters and the refractive indices are the same as the ones used previously. Dispersion curves for two smaller values of the pitch $\Lambda$ are also given. HOCLM means higher order core localized mode. Due to its sharpness, the avoided crossing for $\Lambda = 5,64\mu$m with the $HE_{53}$ mode is not well resolved in this plot.

quently the coupling spreads over a greater spectral range and its maximum intensity decreases. This kind of behaviour for the modal interaction is similar to the one already described by Engeness *et al.* [11]. in Bragg MOF. We have also studied the influence of the refractive index contrast on the transition (see Fig. 4), and as expected, a contrast decrease induces a decrease of its strength and an increase of its relative spectral range. In Bragg MOF avoided crossings

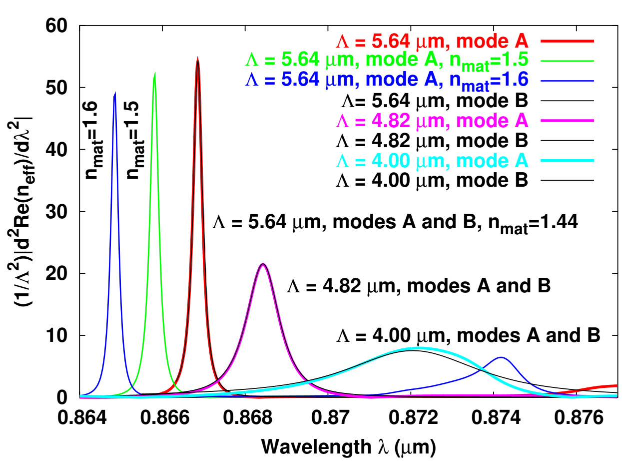


Fig. 4. $|\mathscr{R}|/\Lambda^2$ (as defined in the text) versus the wavelength for core modes A and B for three pitch values around the avoided crossing with leaky mode $EH_{33}$ ($d$ is kept constant). The influence of the refractive index contrast is also given. The curves for $n_{mat}$= 1.5 and 1.6 have been translated along the $x$-axis to make them visible.

may occurs for all the core modes that have the same azimuthal symmetry and polarization as the defect mode [11]. Consequently, some symmetry selection rules may be applied to select

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the avoided crossings. On the contrary, in ARROW MOFs since the modes are expressed as a Fourier-Bessel series a single azimuthal number of the mode is not defined [8] therefore one can not expect the kind of selection rules mentioned above (other selection rules should exist due to the fiber $C_{6v}$ symmetry, these rules are under investigation).

From the above argument, we can expect that avoided crossing with leaky modes is a common phenomenon in ARROW MOFs, in fact we have already observed them in the described MOF but in different transmission bands and also in other ARROW MOF configurations. One of the consequences of these avoided crossings can be seen in Fig. 3 in which the A core mode with $\Lambda = 5.64\,\mu$m first undergoes the transition described in Fig. 2, and then follows the isolated cylinder dispersion curve, and finally undergoes a continuous transition into an higher order core localized mode (see the right bottom of the graph).

## 4. Discussion

As it can be easily seen in Fig. 3, the observed avoided crossings do not occur at the cut-off wavelengths of the guided modes of the isolated high-index cylinder. The reason is simple: in ARROW MOFs we must consider the core localized leaky modes with $\Re e(n_{\mathrm{eff}})/n_{\mathrm{mat}} < 1$, so the avoided crossings occur with the high-index cylinder modes leaky modes.

Besides, the gap between the average wavelength of an avoided crossing and the cut-off wavelength of the high-index cylinder leaky mode may be notably bigger than the wavelength bandwitdh of the avoided crossing (see Fig. 4).

Concerning the experimental observation of these numerically demonstrated avoided crossings, experimental teams should be able to realize them since these phenomena, with narrow wavelength bandwidths, are now clearly defined and localized.

We have not use the usual LP notation for the high-index cylinder leaky modes since the weak guidance approximation is not valid in the present study due to the large index contrast ($n_{mat} = 1.44$ and $n_{cyl} = 1.8$). As example, it can be easily seen in Fig. 3 for the core mode associated with $\Lambda = 4.82\,\mu$m, the avoided crossing with the $HE_{53}$ leaky mode and the one with the $EH_{33}$ leaky mode are not similar. They have not the same bandwidth whereas they are represented by the unique $LP_{43}$ leaky mode in the weak guidance approximation (see also the leaky modes $HE_{34}$ and $EH_{14}$, and their $LP_{24}$ counterpart). As already mentioned at the end of the previous section, this difference between the two avoided crossings may be linked to selection rules exiting in the coupling between the core localized leaky mode of a $C_{6v}$ waveguide and the high-index cylinder leaky modes.

With the weak approximation hypothesis, we tried to follow a previous work [13] in which we used the scaling law proposed by Birks and his colleagues [14] in order to obtain semi-analytical results. Nevertheless, we have not yet succeeded to find a good agreement between the full-vectorial numerical results and the scaling law predictions.

## 5. Conclusion

In conclusion, we have given a new and unified analysis of ARROW MOF modal behavior based on leaky mode coupling. This analysis has the advantage to complete the usual definition of bandgaps in such MOFs and also to be valid and accurate for phenomenons even more selective in wavelength like the avoided crossings shown in Fig. 3. The described avoided crossings between the leaky modes of the high-index cylinders composing the ARROW confining structure and core localized modes must surely be taken into account in applications involving high power transmission or nonlinear effects especially in the high-index cylinders, even if these avoided crossings have a narrow bandwitdh.

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