

# Improved nonlinear slot waveguides

Integrated nonlinear plasmonics using slot waveguides:  
stationary states, bifurcation, stability, and time evolution

M. M. R. Elsawy and G. Renversez

Université d'Aix-Marseille and Institut Fresnel (CNRS), France  
[gilles.renversez@fresnel.fr](mailto:gilles.renversez@fresnel.fr)

**Collaborators:** Y. Kartashov (ICFO & Russian academy of Sciences), A. Rodriguez  
(Princeton Univ.), and F. Ye (Shanghai Jiao Tong Univ.)

V. Nazabal (CNRS ISCR, Univ. de Rennes I), J.-M. Fedeli (CEA-LETI, Grenoble) and M.  
Chauvet (CNRS FEMTO-ST, Univ. de Franche-Comté)

**Former PhD student:** W. Walasik (Univ. of New York at Buffalo)

META'16 conference, SP29



# Outline

- 1 What is a plasmon–soliton?
- 2 Motivations and context
- 3 Simple nonlinear slot waveguides
- 4 Improved nonlinear slots using dielectric buffer layers
- 5 Slot with a metamaterial nonlinear core
- 6 Conclusion

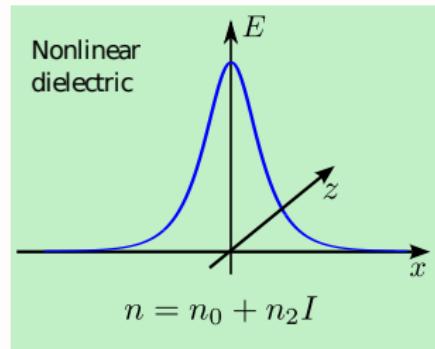
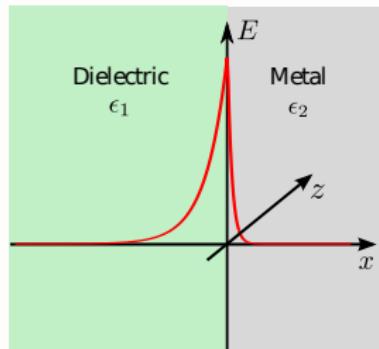
# Plasmon–soliton wave building blocks

Surface plasmon polariton

Spatial optical soliton

Solution to a linear wave equation

Solution to a nonlinear wave equation



Propagation constant

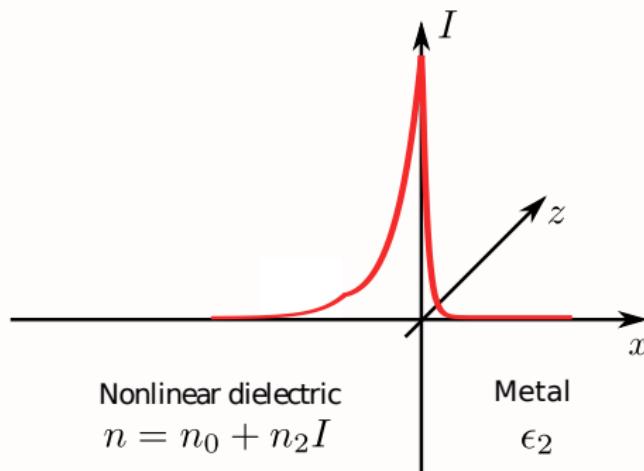
$$\beta_p = k_0 \sqrt{\frac{\epsilon_1 \epsilon_2}{\epsilon_1 + \epsilon_2}}$$

Propagation constant

$$\beta_s = k_0 n_0 \sqrt{1 + \frac{n_2 I}{n_0}}$$

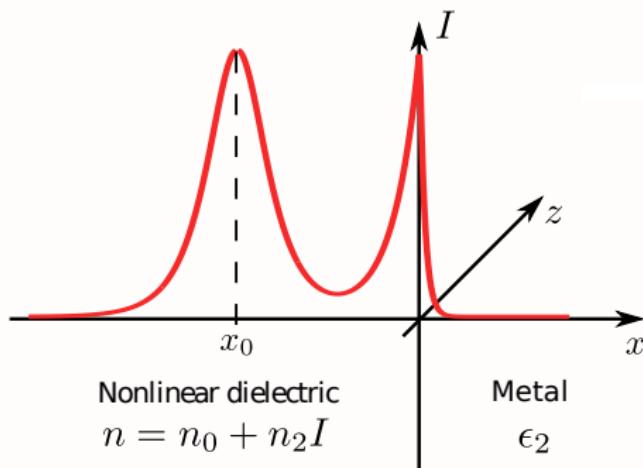
## What is a plasmon–soliton wave?

A nonlinear optical wave combining a spatial soliton and a plasmon field with a single propagation constant

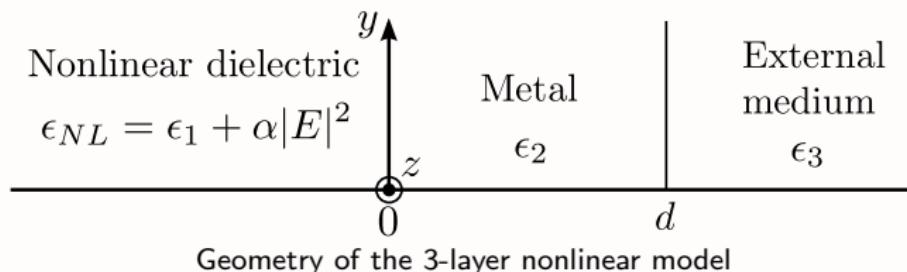


## What is a plasmon–soliton wave?

A nonlinear optical wave combining a spatial soliton and a plasmon field with a single propagation constant



## Motivation — Plasmon-soliton coupling in the semi-infinite NL region case



- Seminal articles:



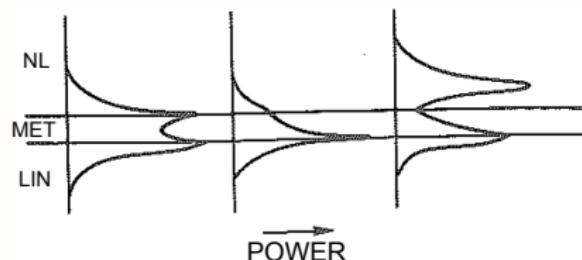
V. M. Agranovich *et al.*

Nonlinear surface polaritons.  
*Sov. Phys. JETP*, 32(8):512,  
1980.



J. Ariyasu *et al.*

Nonlinear surface polaritons  
guided by metal films.  
*J. Appl. Phys.*, 58(7):2460,  
1985.



# Motivation — Plasmon-soliton coupling in the semi-infinite NL region case

## More recent articles:

- Using the 'interaction picture' approach:

 K. Y. Bliokh, Y. P. Bliokh, and A. Ferrando.  
Resonant plasmon-soliton interaction.  
*Phys. Rev. A*, 79:41803, 2009.

 C. Milián, D. E. Ceballos-Herrera, D. V. Skryabin, and A. Ferrando.  
Soliton-plasmon resonances as Maxwell nonlinear bound states.  
*Opt. Lett.*, 37(20):4221–4223, 2012.

- Starting from nonlinear Schrödinger's equation:

 A. Baron, T. B. Hoang, C. Fang, M. H. Mikkelsen, and D. R. Smith.  
Ultrafast self-action of surface-plasmon polaritons at an air/metal interface.  
*Phys. Rev. B*, 91, 195412, 2015

# Motivation — Plasmon-soliton coupling in the semi-infinite NL region case

## More recent articles:

- Starting from Maxwell's equations:



A. R. Davoyan, I. V. Shadrivov, and Y. S. Kivshar.

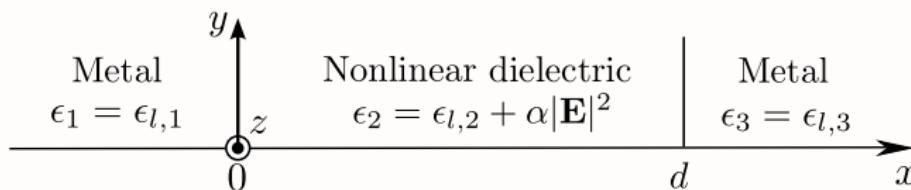
Self-focusing and spatial plasmon-polariton solitons.

*Opt. Express*, 16(24):21732–21737, 2009.

-W. Walasik, V. Nazabal, M. Chauvet, Y. Kartashov, and G. Renversez,  
*Low-power plasmon-soliton in realistic nonlinear planar structures*, *Opt. Lett.*,  
37(22): 4579, (2012)

-W. Walasik, G. Renversez, and Y. Kartashov,  
*Stationary plasmon-soliton waves in nonlinear planar structures: modeling and properties*. *Phys. Rev. A*, 89: 023816, (2014)

## Nonlinear slot waveguide — Introduction



**Linear case:**



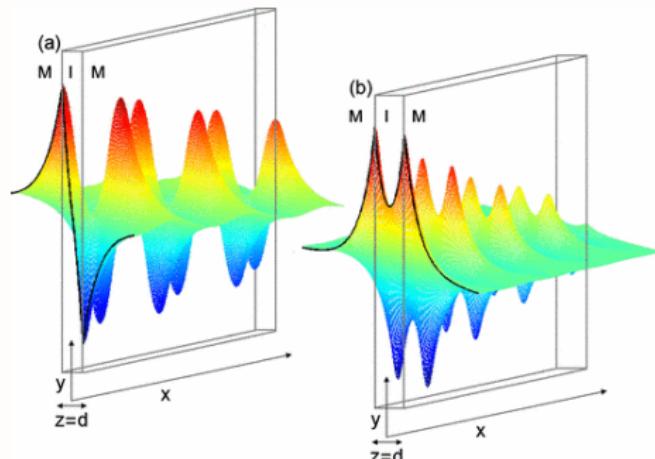
V. R. Almeida *et al.*

Guiding and confining light in void nanostructure,  
*Opt. Lett.*, 29:1209-1211, (2004)

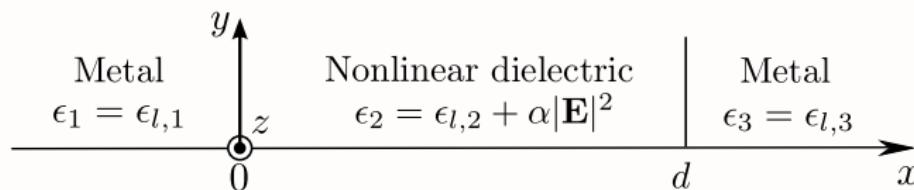


J. A. Dionne *et al.*

Plasmon slot waveguides: Towards  
chip-scale propagation with  
subwavelength-scale localization,  
*Phys. Rev. B*, 73(3):035407,  
(2006)



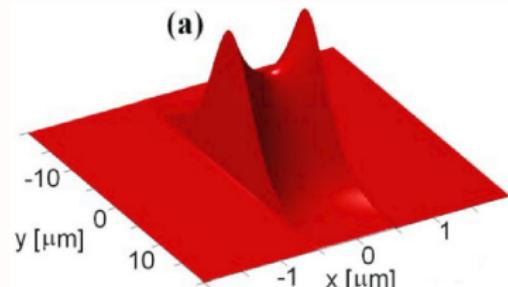
## Nonlinear slot waveguide — Introduction



### Nonlinear case:

E. Feigenbaum and M. Orenstein.  
Plasmon–soliton.  
*Opt. Lett.*, 32(6):674, (2007)

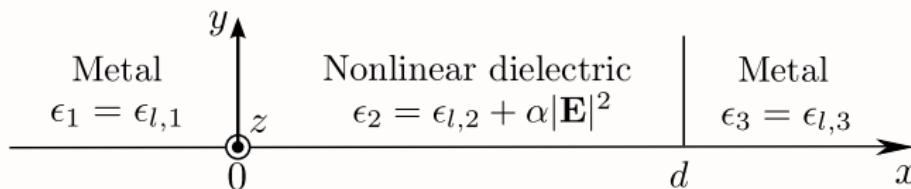
A. Davoyan, I. Shadrivov, and Y. Kivshar,  
Nonlinear plasmonic slot  
waveguides,  
*Opt. Express*, 16(26), (2008)



**No experimental results on plasmon–soliton in nonlinear slot**

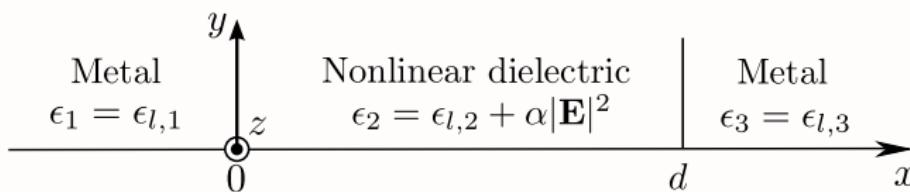
**Too high nonlinear index change  $\Delta n = n_2 l$**

## Nonlinear slot waveguide — Introduction



- **Waveguide configuration**
- **Subwavelength focusing**
- **Control the solutions with the power**
- **Peculiar nonlinear effects**

## Nonlinear slot waveguide — Models



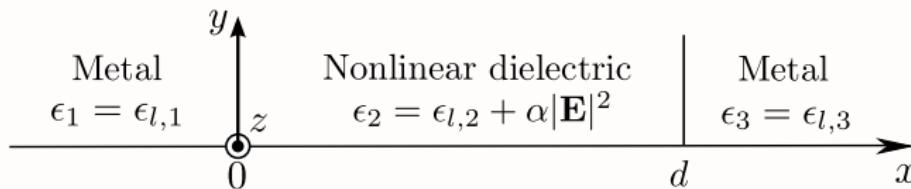
Focusing Kerr effect

Approach: the spatial dependency of transverse field components is kept

"Modal" nonlinear solutions of Maxwell's equations for TM stationary waves  
using field continuity conditions in 1D structure

Both  $n_{\text{eff}}$  and field profiles that depend on total power  $P_{\text{tot}}$  are computed

## Nonlinear slot waveguide — Models



### Common hypotheses to our two models

- **Stationary solutions of Maxwell's equations:**

$$\begin{Bmatrix} \mathbf{E}(x, z, t) \\ \mathbf{H}(x, z, t) \end{Bmatrix} = \begin{Bmatrix} \mathbf{E}_{\text{NL}}(x) \\ \mathbf{H}_{\text{NL}}(x) \end{Bmatrix} \exp[i(\beta_{\text{NL}} k_0 z - \omega t)]$$

$k_0 = 2\pi/\lambda$ , and  $\beta_{\text{NL}}$  is the effective index  $n_{\text{eff}}$  of this nonlinear wave

- **TM waves:**

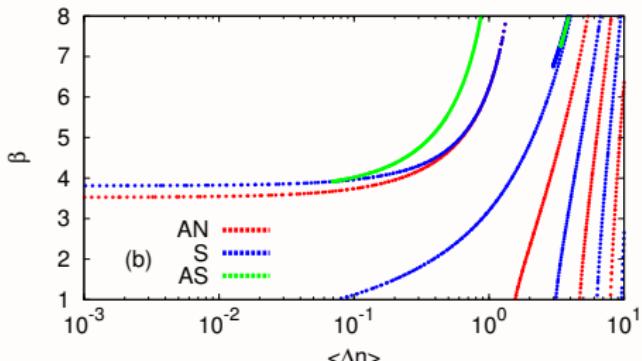
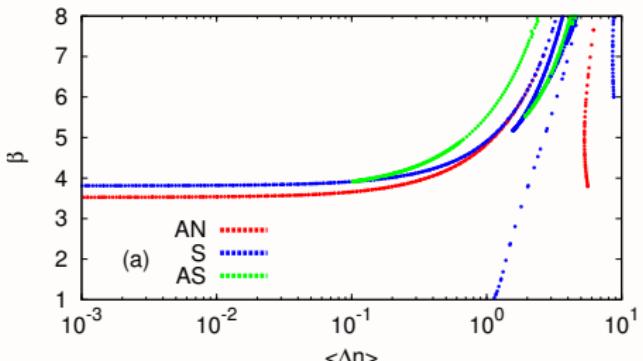
$$\mathbf{E} = [E_x, 0, E_z] \text{ and } \mathbf{H} = [0, H_y, 0]$$

- **Kerr nonlinearity**

- Maxwell's equations + boundary conditions

→ **Nonlinear dispersion relation**

## Nonlinear slot waveguide — Models



### Jacobi Elliptic function based Model (JEM)

Extension to slot configuration of  
W. Chen and A. A. Maradudin  
*J. Opt. Soc. Am. B, 5, 529 (1988)*

- + Low nonlinearity depending only on the transverse electric field
- + Analytical formulas for field shapes and nonlinear dispersion relation

### Finite Element Method (FEM)

Adaptation to slot configuration of:  
F. Drouart, G. Renversez, et al.  
*J. Opt. A: Pure Appl. Opt., 10, 125101J (2008)*

- + Exact treatment of Kerr-type nonlinearity
- Field shapes and dispersion curves obtained numerically

## Nonlinear slot waveguide — Models

### Jacobi Elliptic function based Model (JEM)

Extension to slot configuration of  
W. Chen and A. A. Maradudin  
*J. Opt. Soc. Am. B, 5, 529 (1988)*

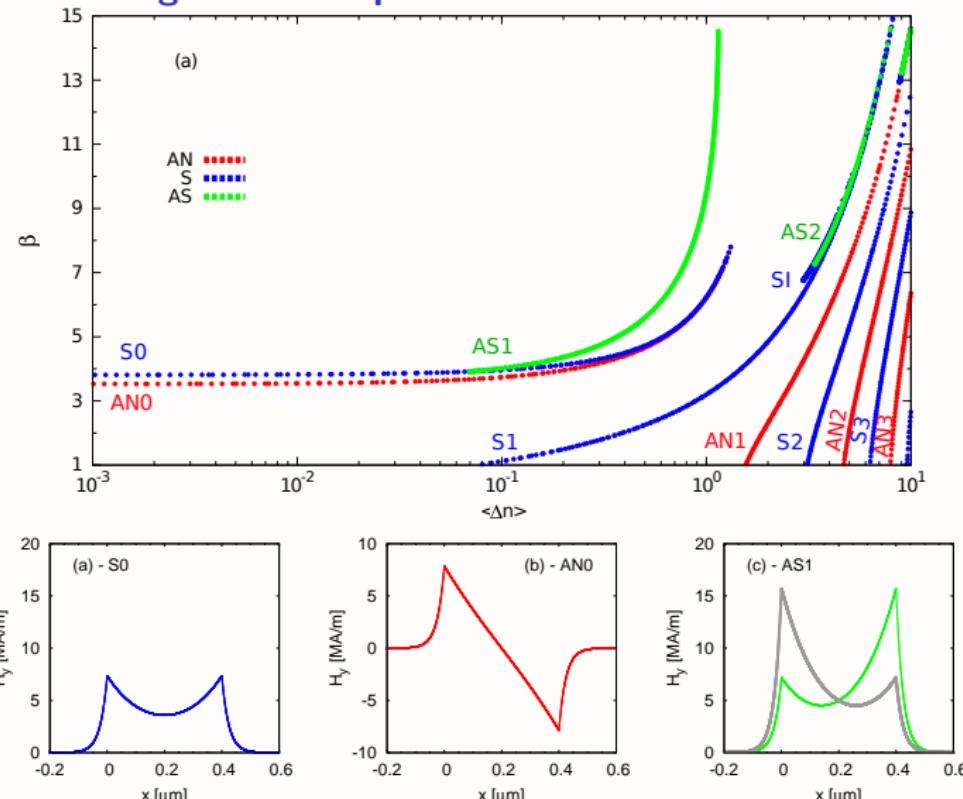
### Finite Element Method (FEM)

Adaptation to slot configuration of:  
F. Drouart, G. Renversez, et al.  
*J. Opt. A: Pure Appl. Opt., 10, 125101J*  
(2008)

- Low nonlinearity depending only on the transverse electric field
- + Analytical formulas for field shapes and nonlinear dispersion relation

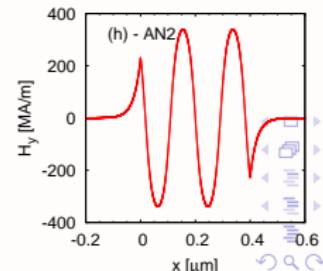
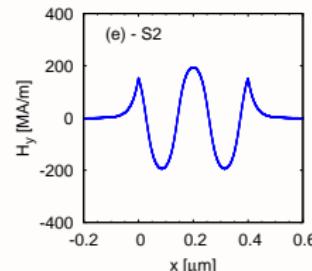
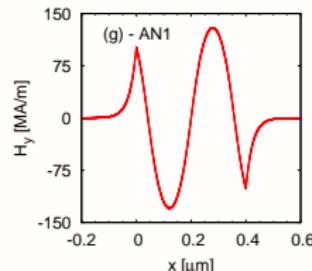
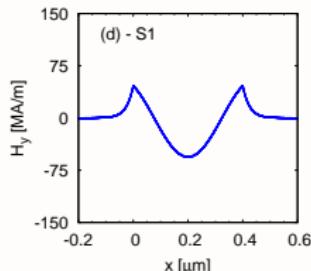
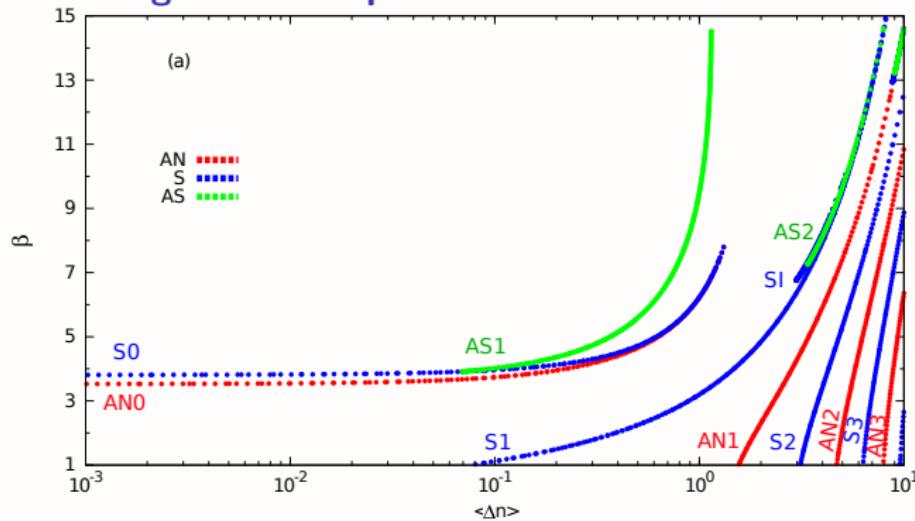
- + Exact treatment of Kerr-type nonlinearity
- Field shapes and dispersion curves obtained numerically

## Nonlinear slot waveguide — Dispersion relations

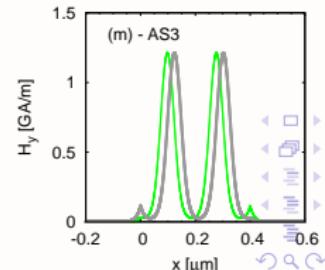
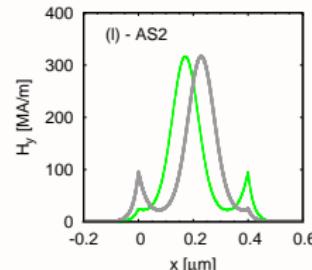
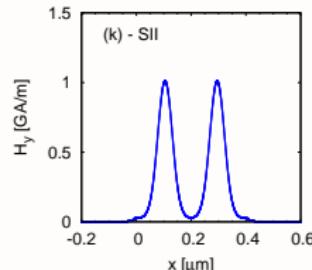
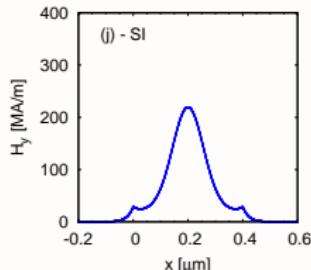
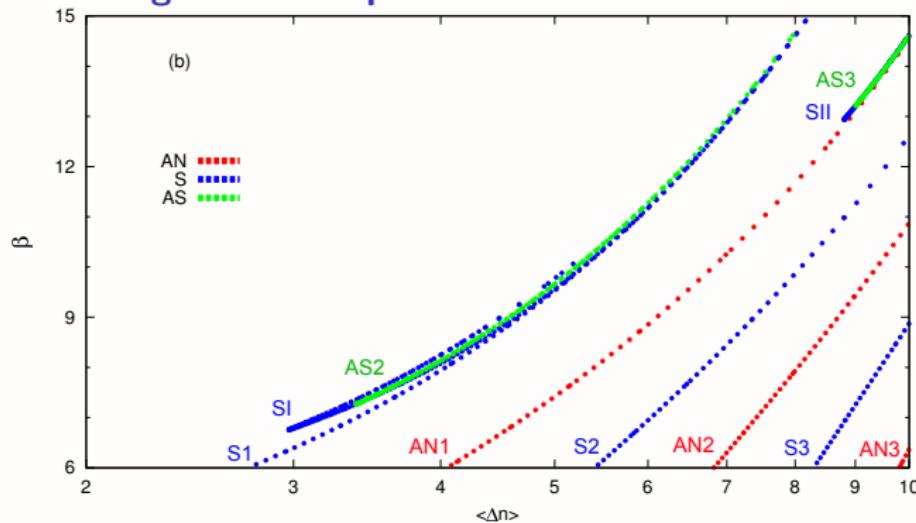


A. Davoyan, I. Shadrivov, and Y. Kivshar, Nonlinear plasmonic slot waveguides, *Opt. Express*, 16(26), 2008

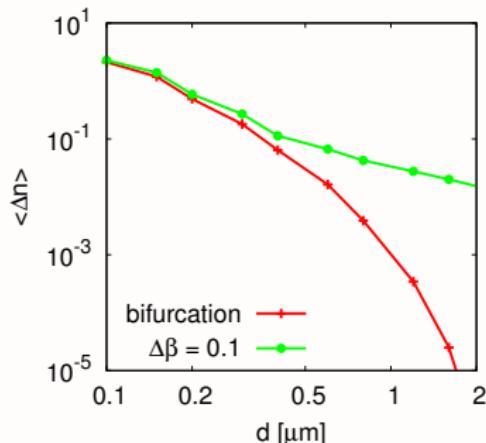
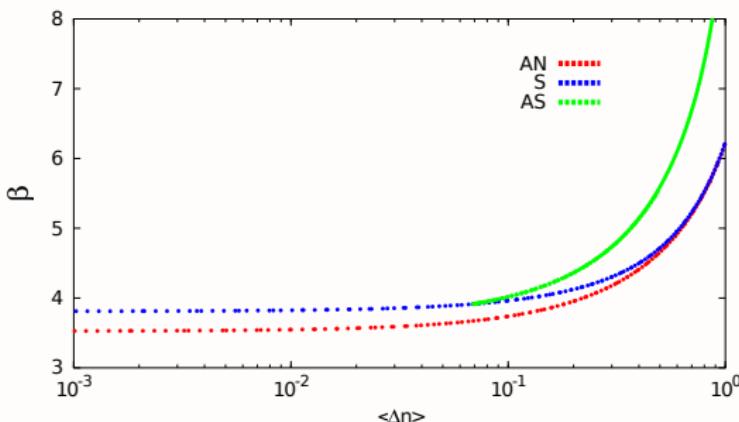
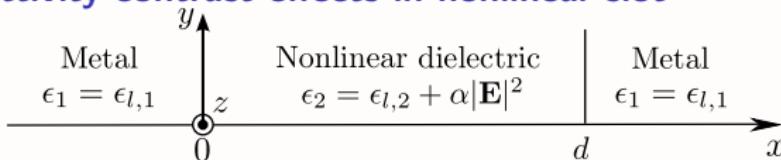
## Nonlinear slot waveguide — Dispersion relations



## Nonlinear slot waveguide — Dispersion relations

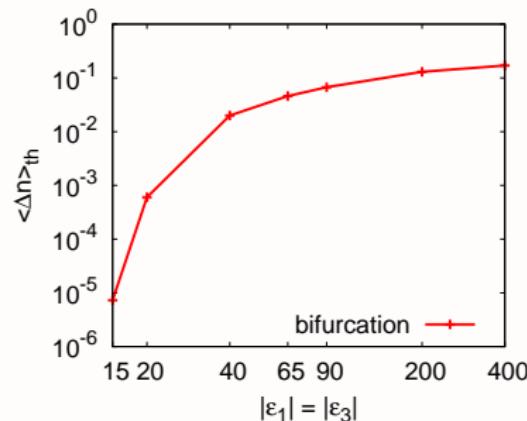
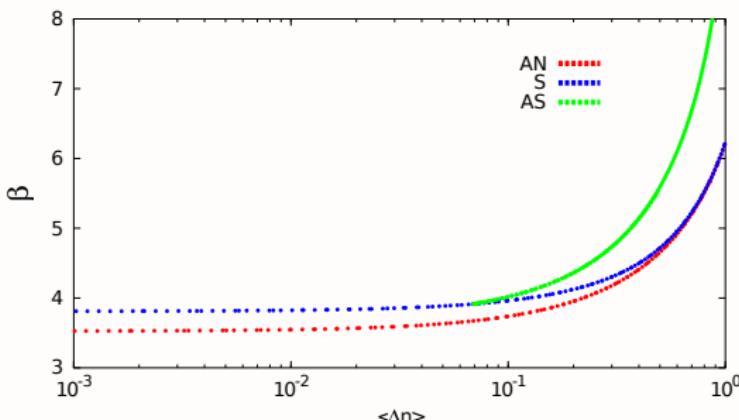
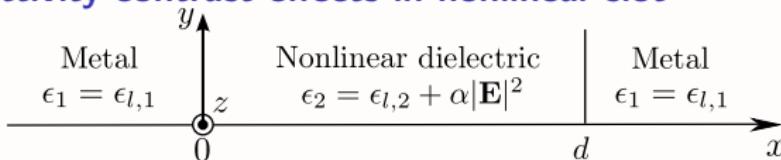


## Size and permittivity contrast effects in nonlinear slot



- Bifurcation — spontaneous symmetry breaking
- Asymmetric modes in symmetric structures
- Parameter rules to lower power needed for nonlinear effects

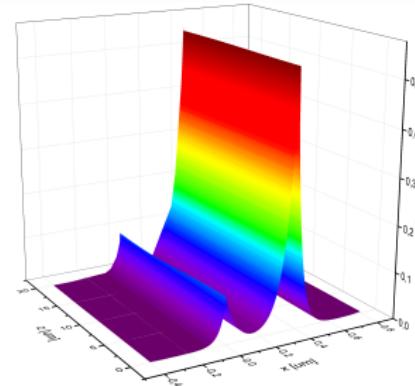
## Size and permittivity contrast effects in nonlinear slot



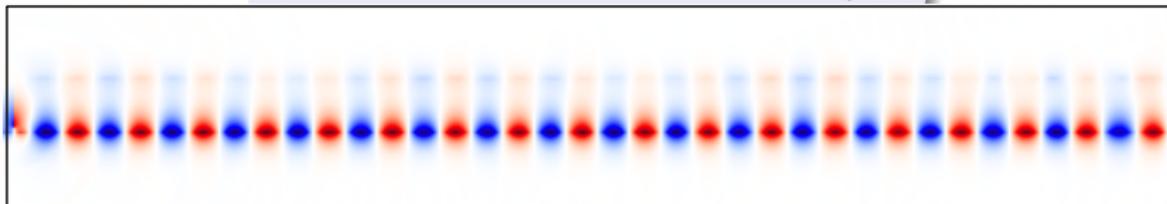
- Bifurcation — spontaneous symmetry breaking
- Asymmetric modes in symmetric structures
- Parameter rules to lower power needed for nonlinear effects

## Stability analysis: summary of results from our 2 numerical methods

Temporal propagation simulation over  $15\lambda$  for the asymmetric mode  $|\mathbf{E}|$  (comsol based)



Temporal propagation simulation over  $15\lambda$   
MEEP full vector nonlinear FDTD  
simulation for the asymmetric mode  $H_y$



- stability of the asymmetric mode

## Intermediate conclusions

- Two semi-analytical models for **nonlinear slot waveguide** configuration with a finite size nonlinear core
- Prediction of the existence of **higher order modes** in nonlinear slot waveguides
- Study of size and permittivity contrast effects on **bifurcation threshold**  
→ ways to reduce it
- **Stability study** of plasmon–solitons using two numerical methods  
→ stable asymmetric mode

-W. Walasik, A. Rodriguez, G. Renversez, *Symmetric Plasmonic Slot Waveguides with a Nonlinear Dielectric Core: Bifurcations, Size Effects, and Higher Order Modes*, *Plasmonics*, **10**, 33, (2015)

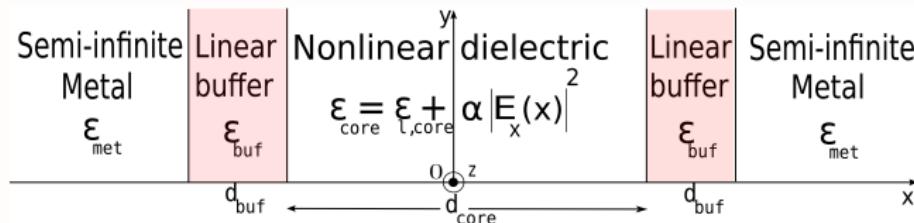
-W. Walasik, G. Renversez, *Plasmon–soliton waves in planar slot waveguides: I. Modeling*. *Phys. Rev. A*, **93**: 013825, (2016)

-W. Walasik, G. Renversez, F. Ye, *Plasmon–soliton waves in planar slot waveguides: II. Results for stationary waves and stability analysis*. *Phys. Rev. A*, **93**: 013825, (2016)

## Intermediate conclusions

- Two semi-analytical models for **nonlinear slot waveguide** configuration with a finite size nonlinear core
- Prediction of the existence of **higher order modes** in nonlinear slot waveguides
- Study of size and permittivity contrast effects on **bifurcation threshold**  
→ ways to reduce it
- **Stability study** of plasmon–solitons using two numerical methods  
→ stable asymmetric mode
- **But losses and bifurcation threshold are still high for realistic and useful parameters.**

## Nonlinear slot waveguide configurations — Improved structures



- Reduce the overall losses
- Modify the type of solutions

Realistic parameters:

- $\lambda = 1.55 \mu\text{m}$
- $d_{core} = 400 \text{ nm}$
- $\epsilon_{met} = -90 + i10$
- $\epsilon_{l,core} = 3.46^2 + i10^{-4}$ ,  $n_2 = 10^{-17} \text{ m}^2/\text{W}$

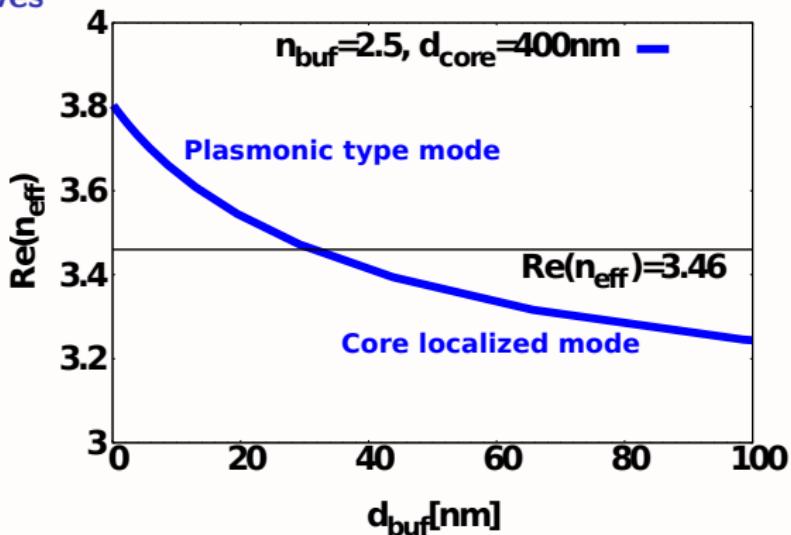


C. Lacava et al.

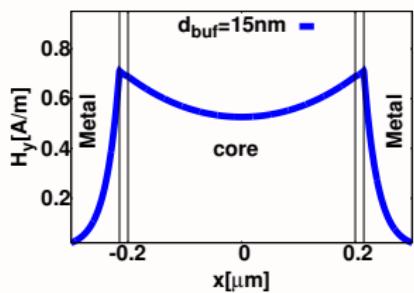
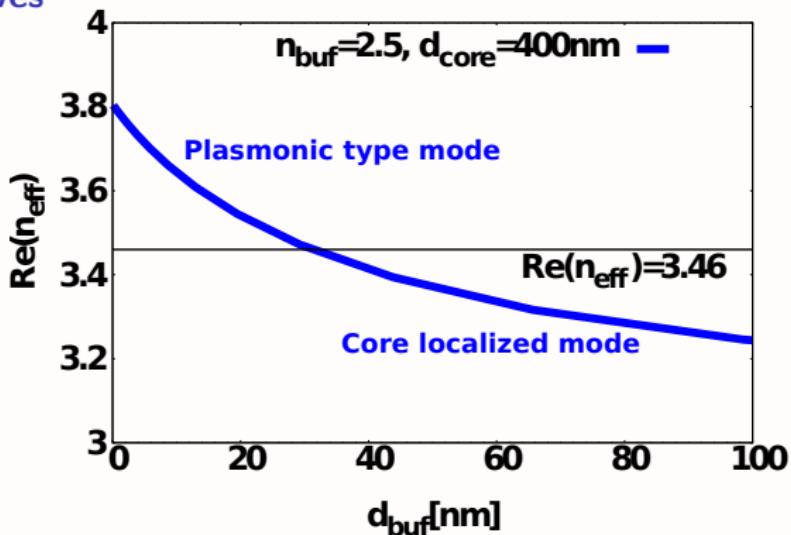
Nonlinear characterization of hydrogenated amorphous silicon waveguides and analysis of carrier dynamics.

Appl. Phys. Lett, 103: 141103 (2013)

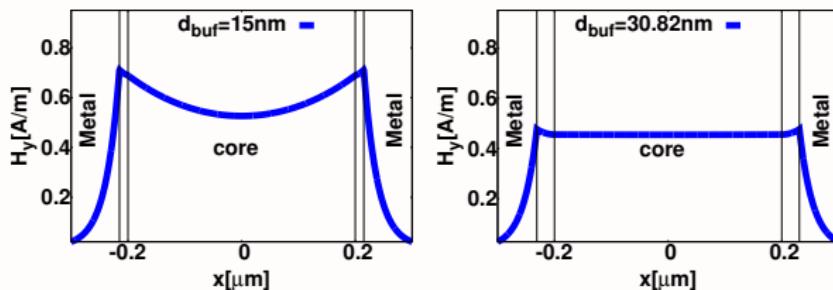
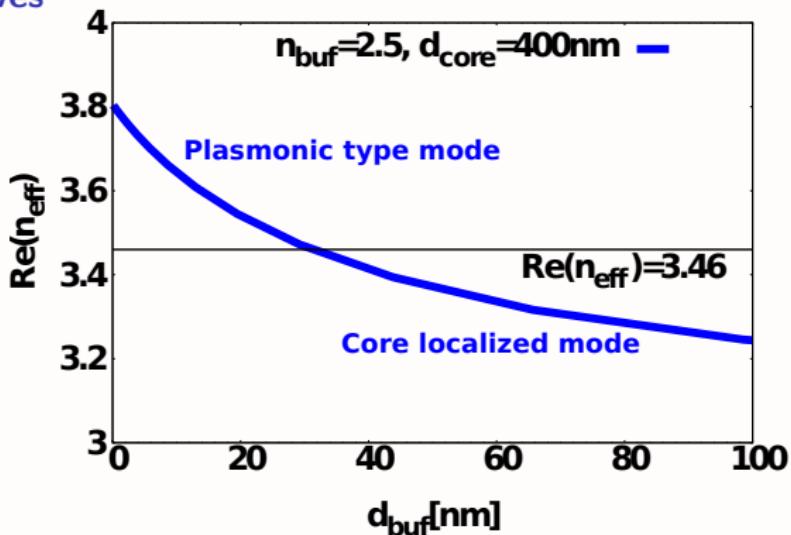
## Linear TM waves



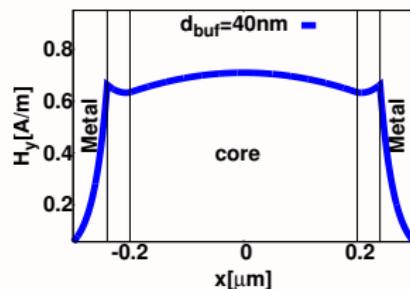
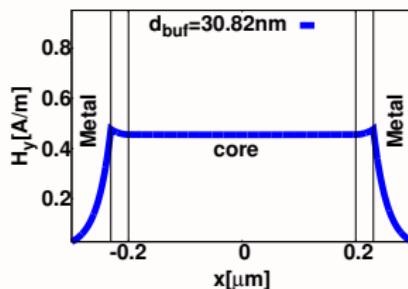
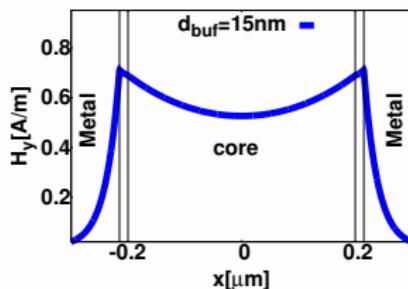
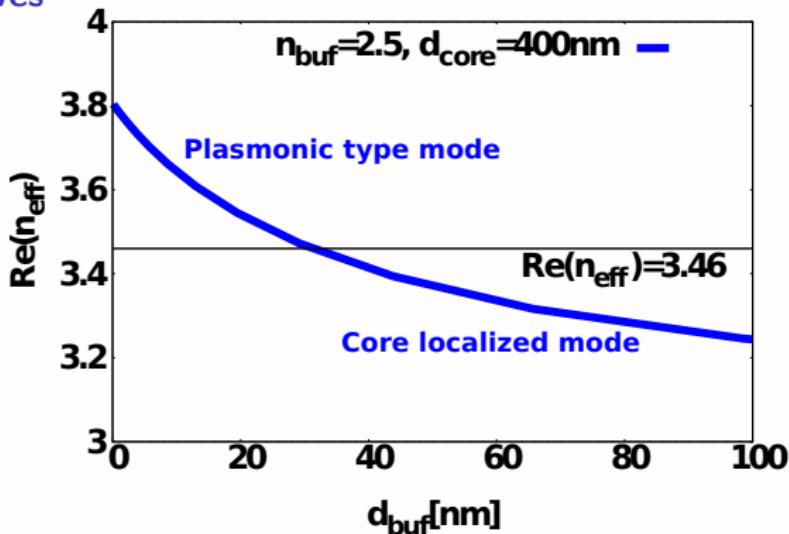
## Linear TM waves



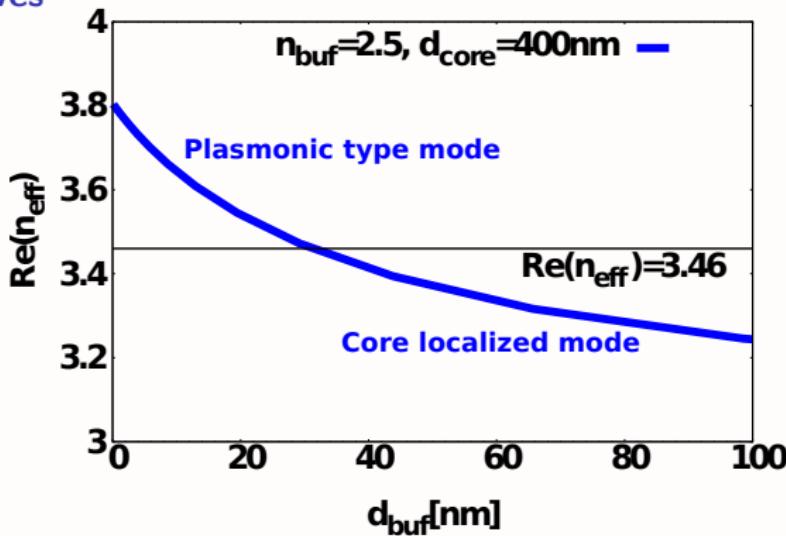
## Linear TM waves



## Linear TM waves



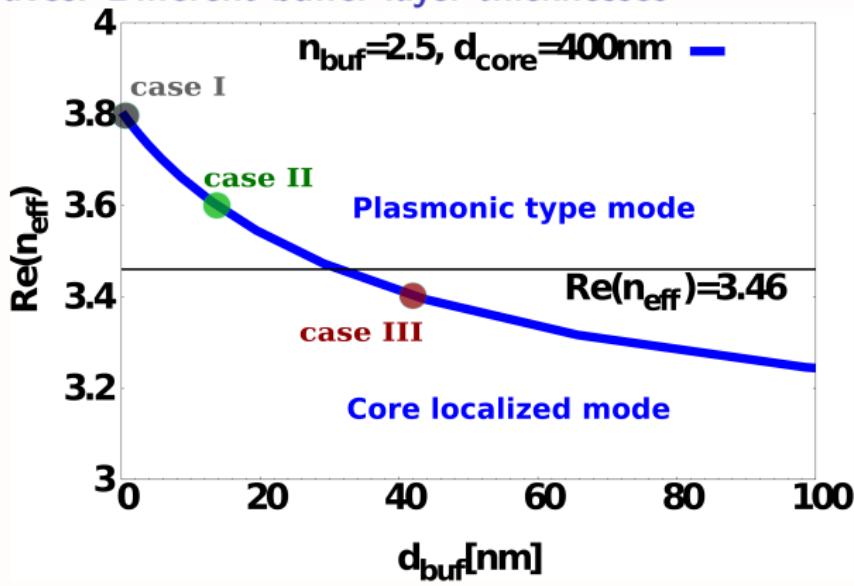
## Linear TM waves



Analytical evaluation of  $d_{\text{buf}}^{\text{up}}$

$$d_{\text{buf}}^{\text{up}} \approx \frac{\lambda}{4\pi \sqrt{\text{Re}(\epsilon_{I,\text{core}}) - \epsilon_{\text{buf}}}} \ln \left[ \frac{\text{Re}(\epsilon_{\text{met}}) \sqrt{\text{Re}(\epsilon_{I,\text{core}}) - \epsilon_{\text{buf}}} - \epsilon_{\text{buf}} \sqrt{\text{Re}(\epsilon_{I,\text{core}}) - \text{Re}(\epsilon_{\text{met}})}}{\text{Re}(\epsilon_{\text{met}}) \sqrt{\text{Re}(\epsilon_{I,\text{core}}) - \epsilon_{\text{buf}}} + \epsilon_{\text{buf}} \sqrt{\text{Re}(\epsilon_{I,\text{core}}) - \text{Re}(\epsilon_{\text{met}})}} \right]$$

## Linear TM waves: Different buffer layer thicknesses

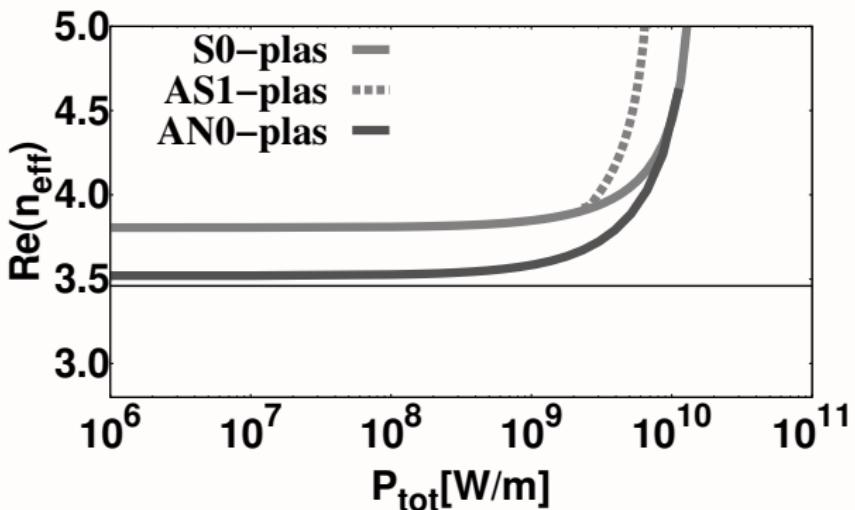


**case I (limit case):**  $d_{\text{buf}} = 0 \text{ nm} \rightarrow$  simple slot configuration

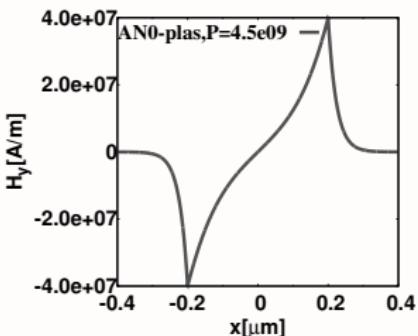
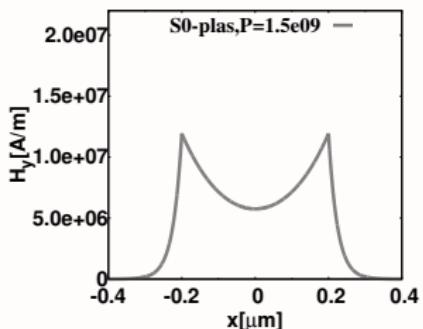
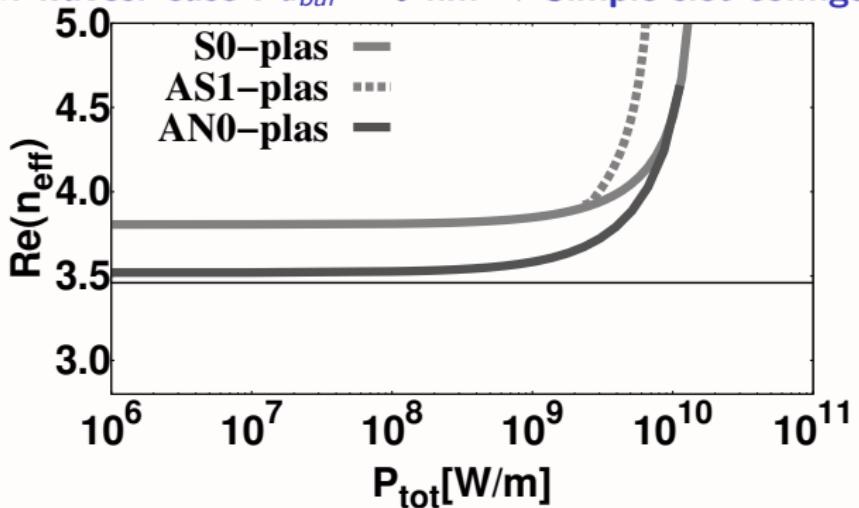
**case II:**  $d_{\text{buf}} < d_{\text{buf}}^{\text{up}}$  (The main linear symmetric mode is of plasmonic type)

**case III:**  $d_{\text{buf}} > d_{\text{buf}}^{\text{up}}$  (The main linear symmetric mode is mostly core localized)

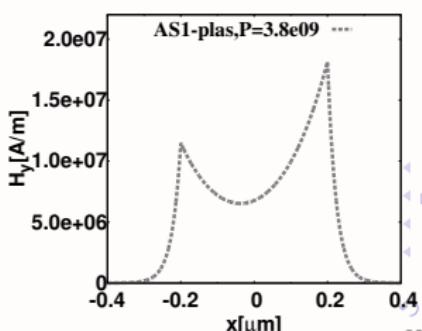
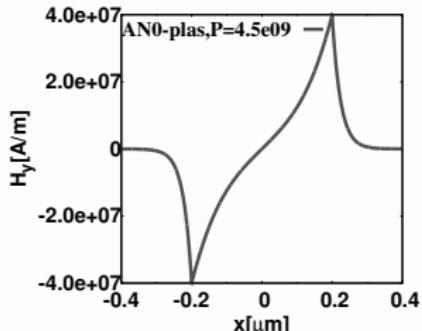
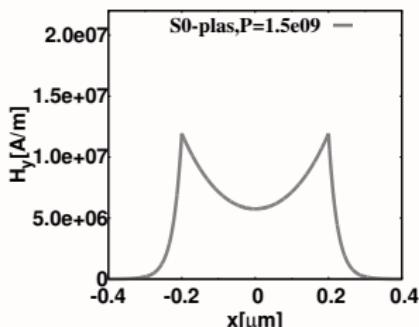
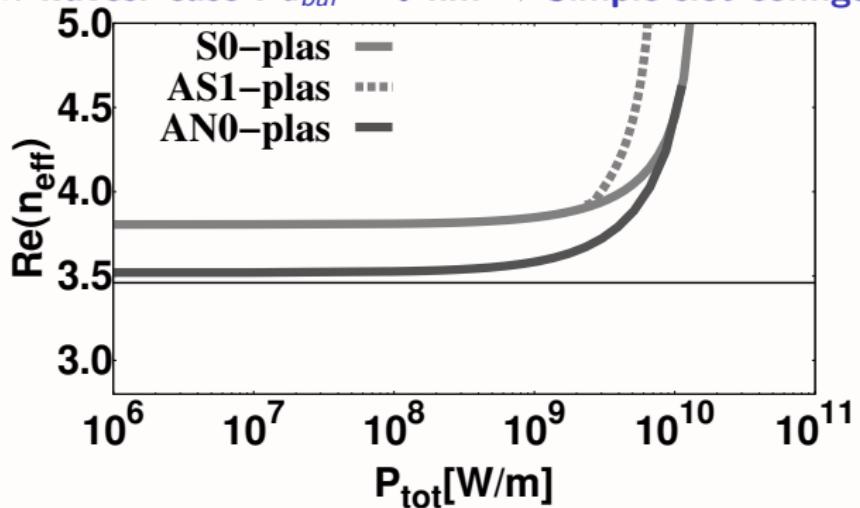
## Nonlinear TM waves: case I $d_{buf} = 0$ nm → Simple slot configuration



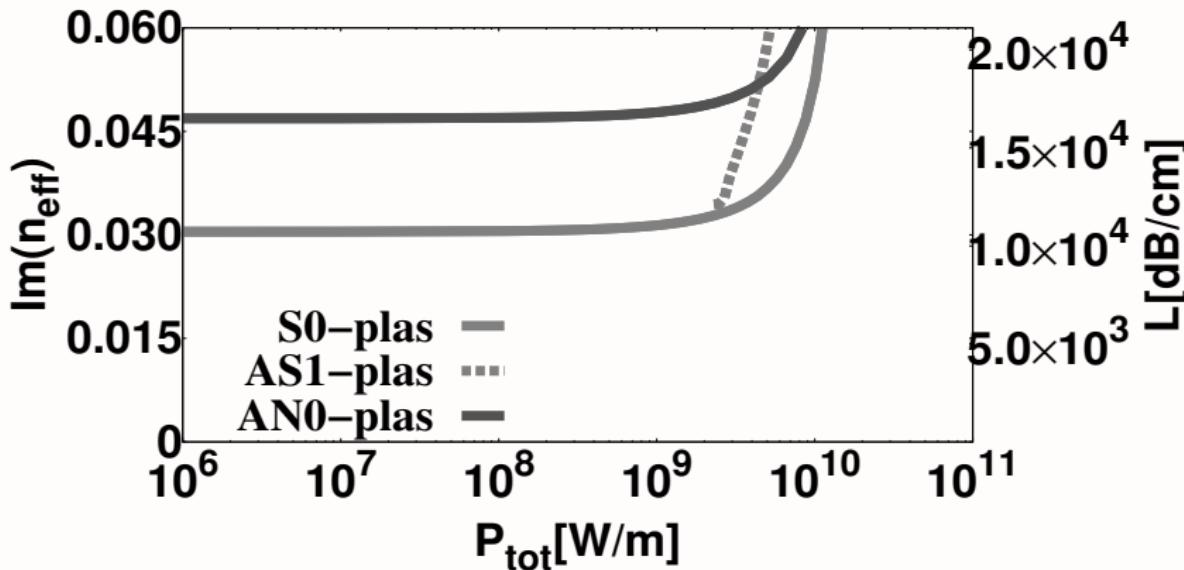
## Nonlinear TM waves: case I $d_{buf} = 0$ nm → Simple slot configuration



## Nonlinear TM waves: case I $d_{buf} = 0$ nm → Simple slot configuration

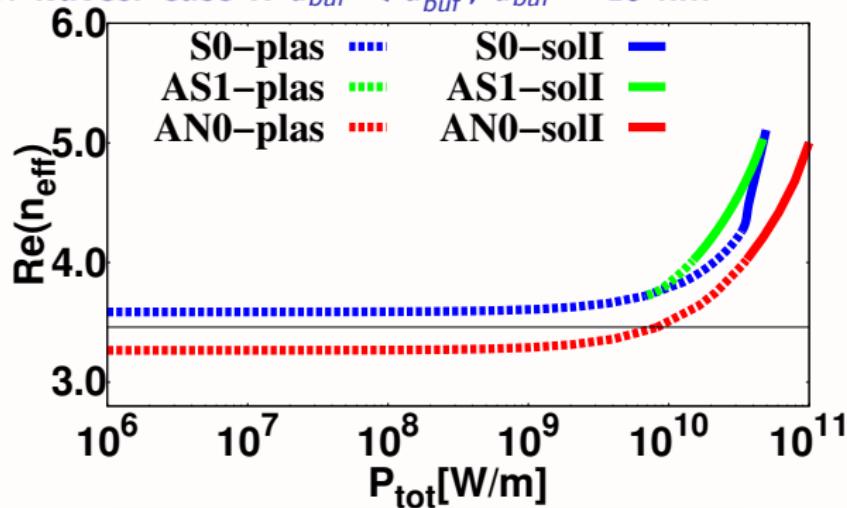


## Nonlinear TM waves. case I $d_{buf} = 0$ nm → Simple slot configuration (Losses)



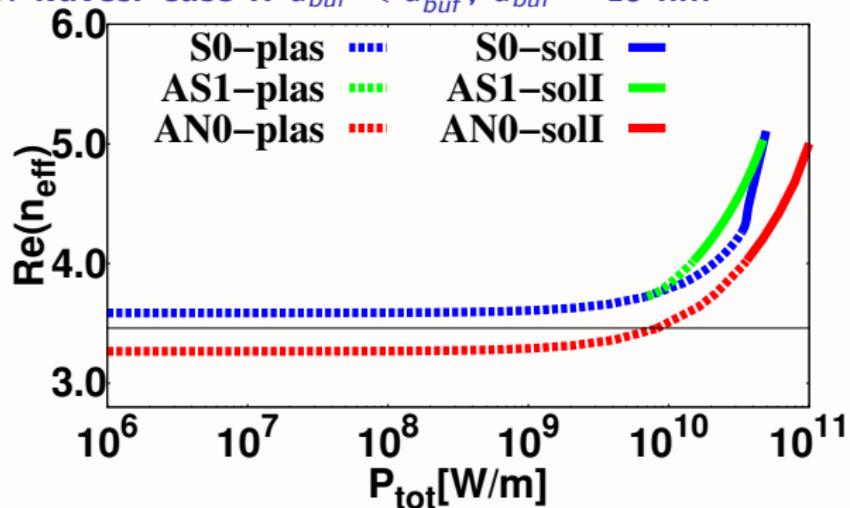
Losses increase with the increase of the power

## Nonlinear TM waves: case II $d_{buf} < d_{buf}^{up}$ , $d_{buf} = 15 \text{ nm}$

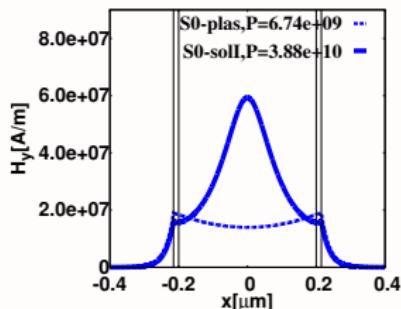


Nonlinear spatial modal transition

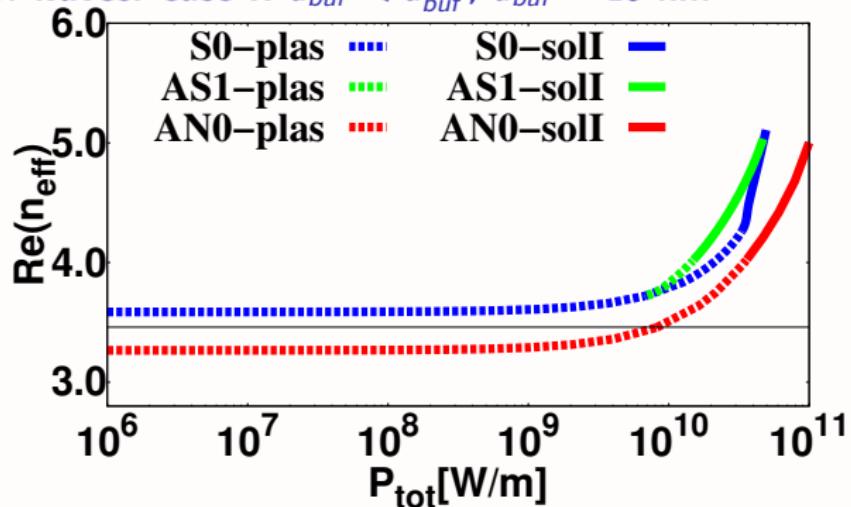
## Nonlinear TM waves: case II $d_{buf} < d_{buf}^{up}$ , $d_{buf} = 15 \text{ nm}$



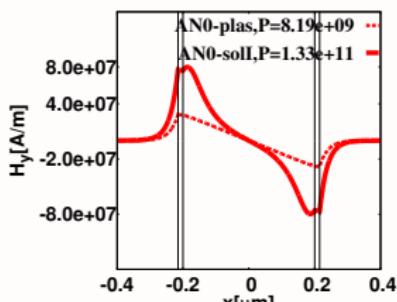
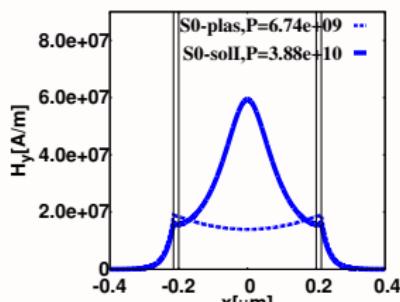
Nonlinear spatial modal transition



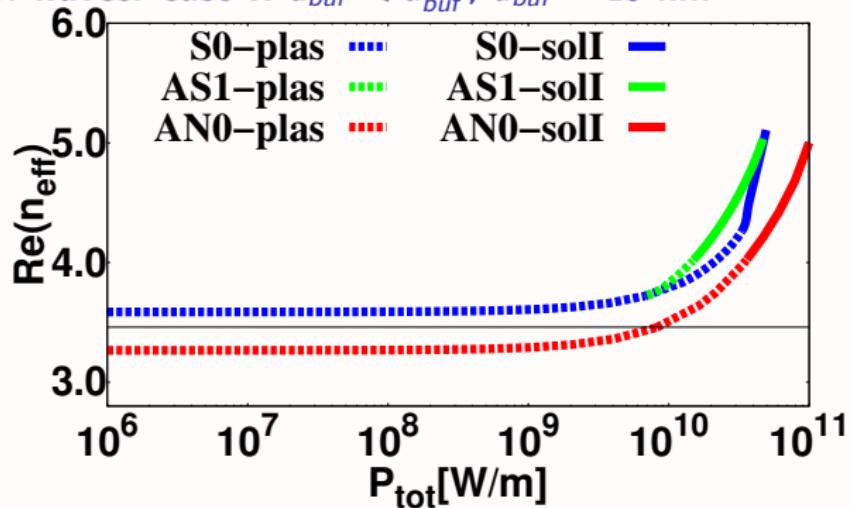
## Nonlinear TM waves: case II $d_{buf} < d_{buf}^{up}$ , $d_{buf} = 15 \text{ nm}$



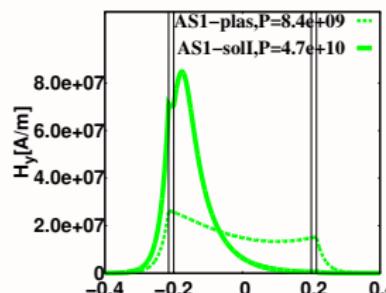
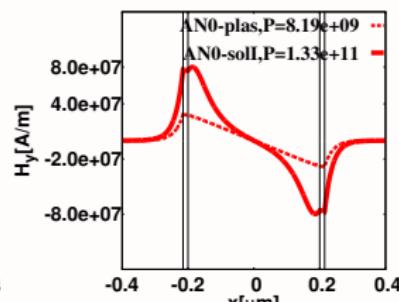
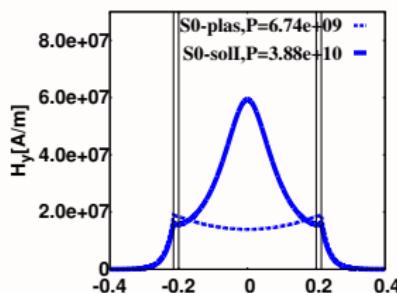
Nonlinear spatial modal transition



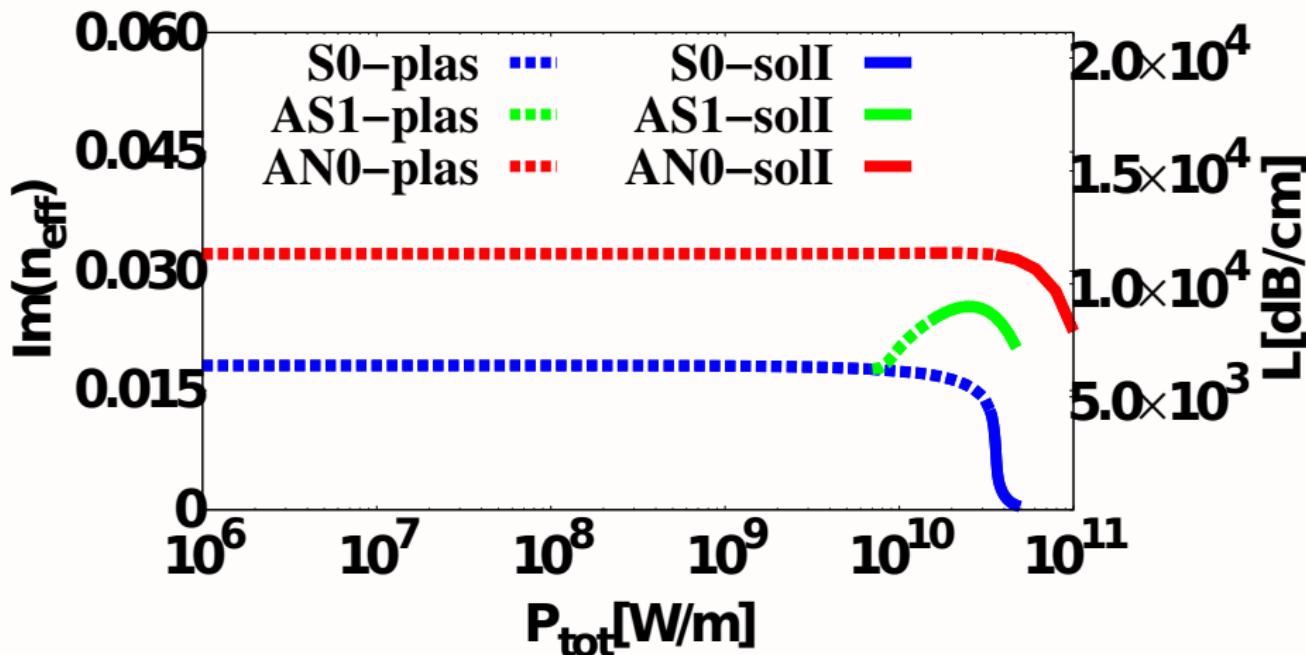
## Nonlinear TM waves: case II $d_{buf} < d_{buf}^{up}$ , $d_{buf} = 15 \text{ nm}$



Nonlinear spatial modal transition

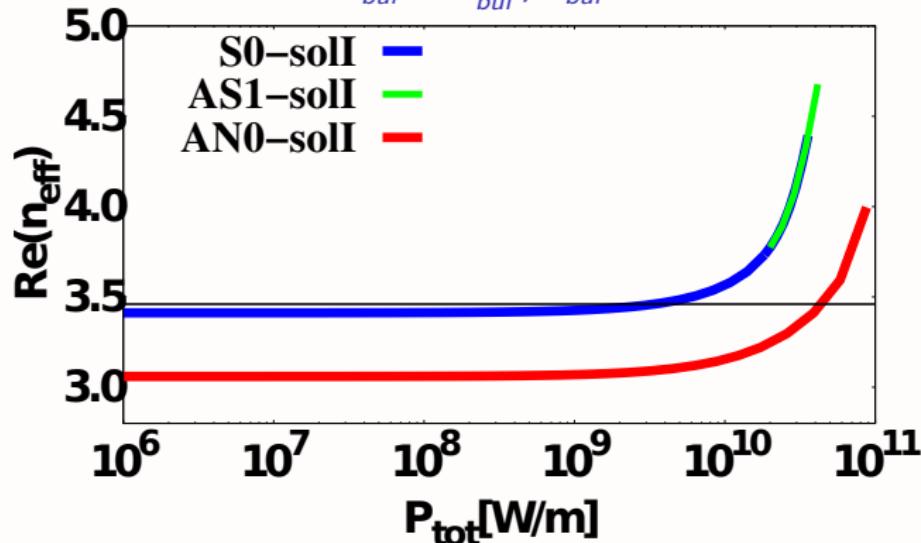


## Nonlinear TM waves: case II $d_{buf} < d_{buf}^{up}$ , $d_{buf} = 15 \text{ nm}$ (Losses)

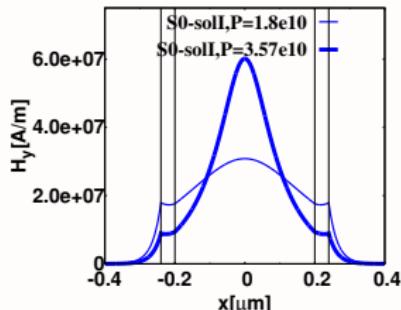
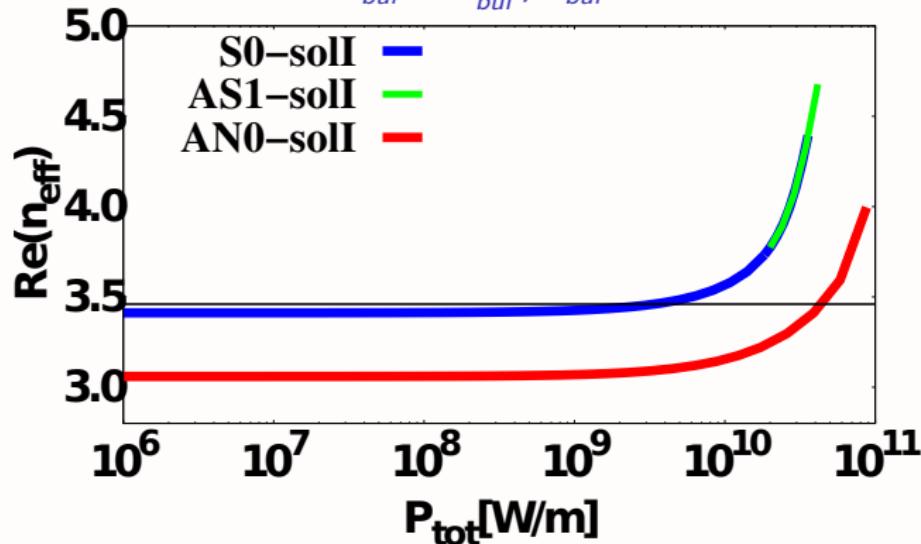


Losses decrease with the increase of the power

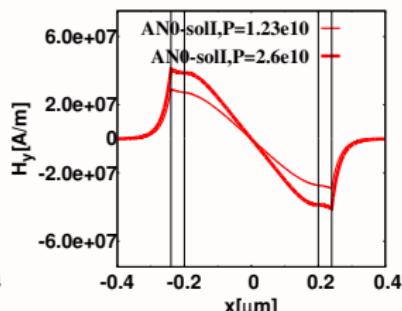
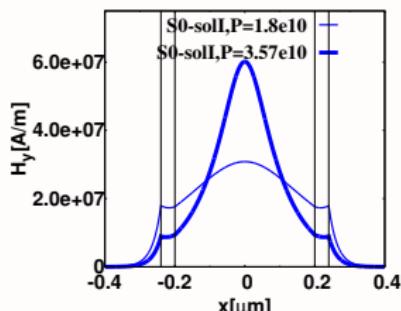
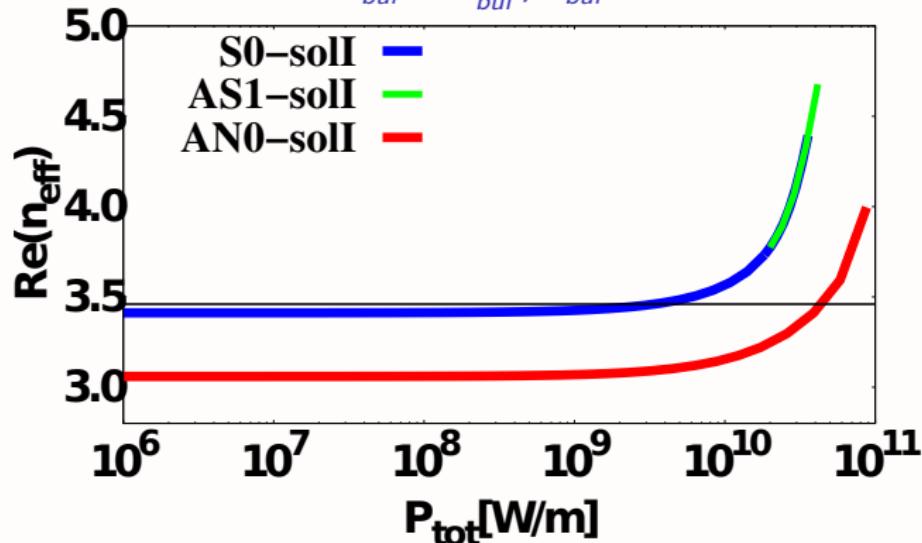
## Nonlinear TM waves: case III $d_{buf} > d_{buf}^{up}$ , $d_{buf} = 40$ nm



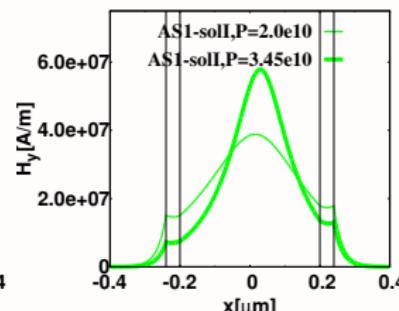
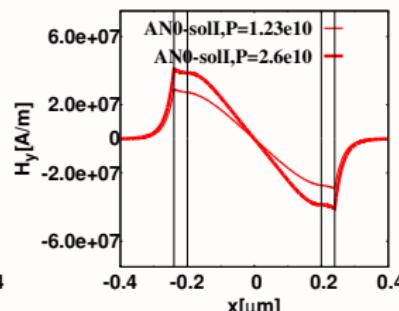
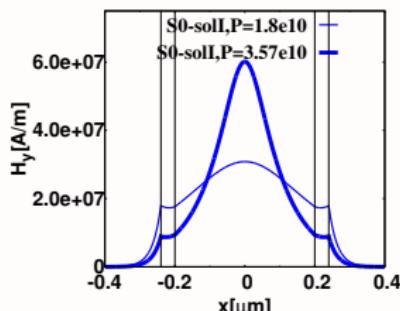
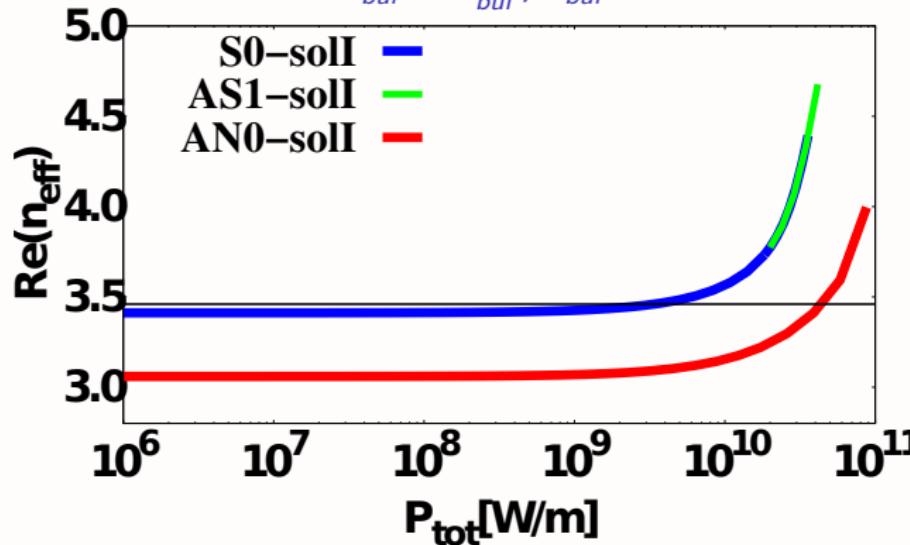
## Nonlinear TM waves: case III $d_{buf} > d_{buf}^{up}$ , $d_{buf} = 40 \text{ nm}$



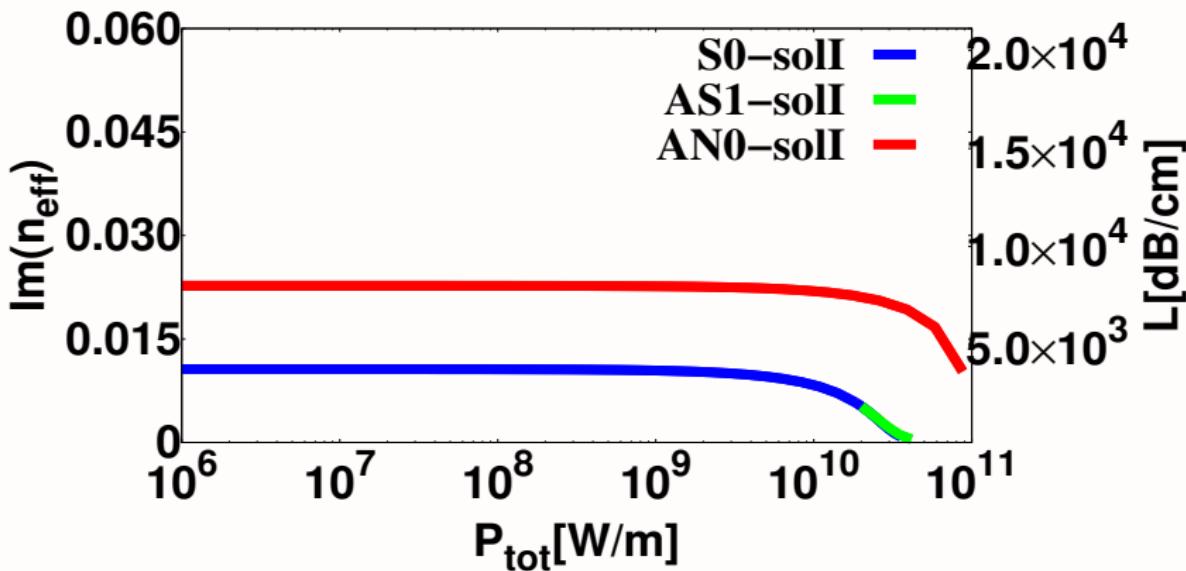
## Nonlinear TM waves: case III $d_{buf} > d_{buf}^{up}$ , $d_{buf} = 40 \text{ nm}$



## Nonlinear TM waves: case III $d_{buf} > d_{buf}^{up}$ , $d_{buf} = 40 \text{ nm}$

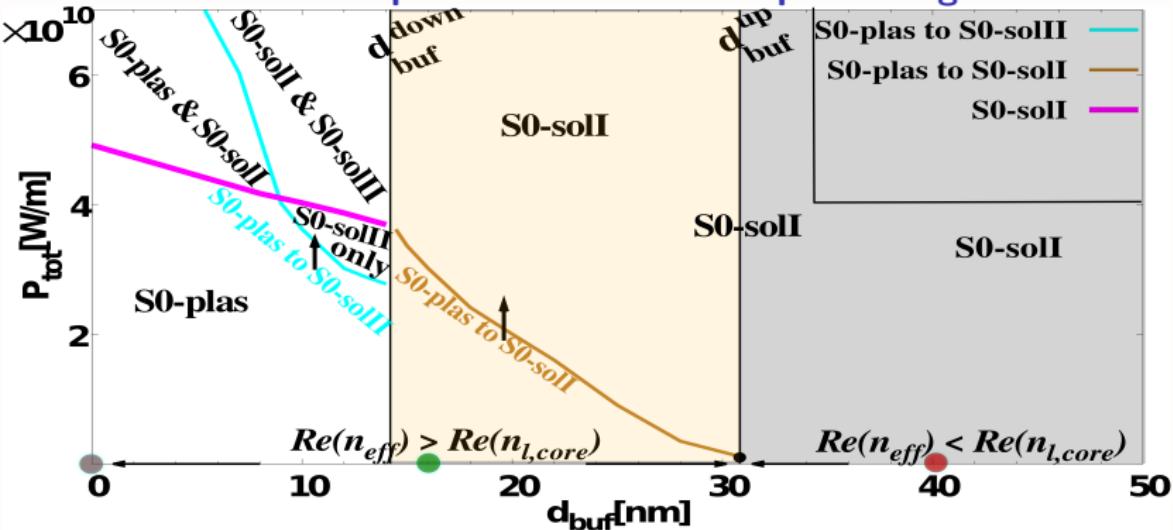


## Nonlinear TM waves: case III $d_{buf} > d_{buf}^{up}$ , $d_{buf} = 40$ nm (Losses)

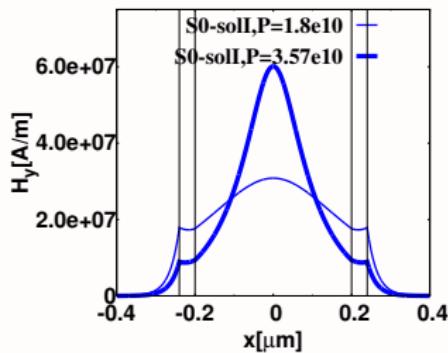
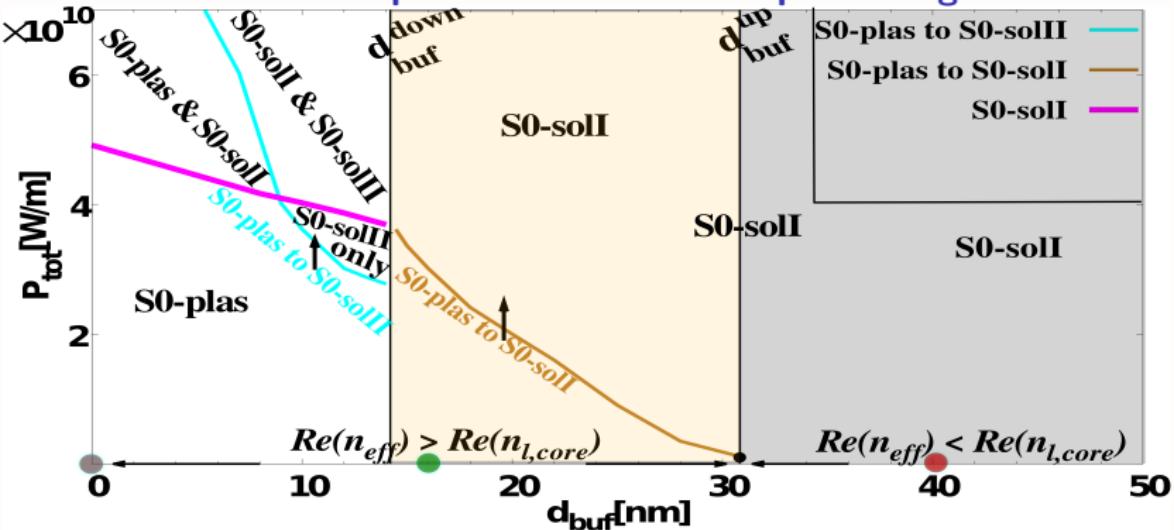


Losses decrease with the increase of the power

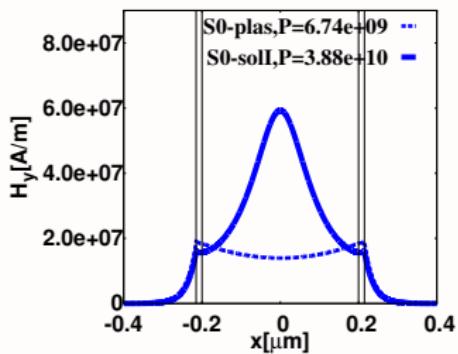
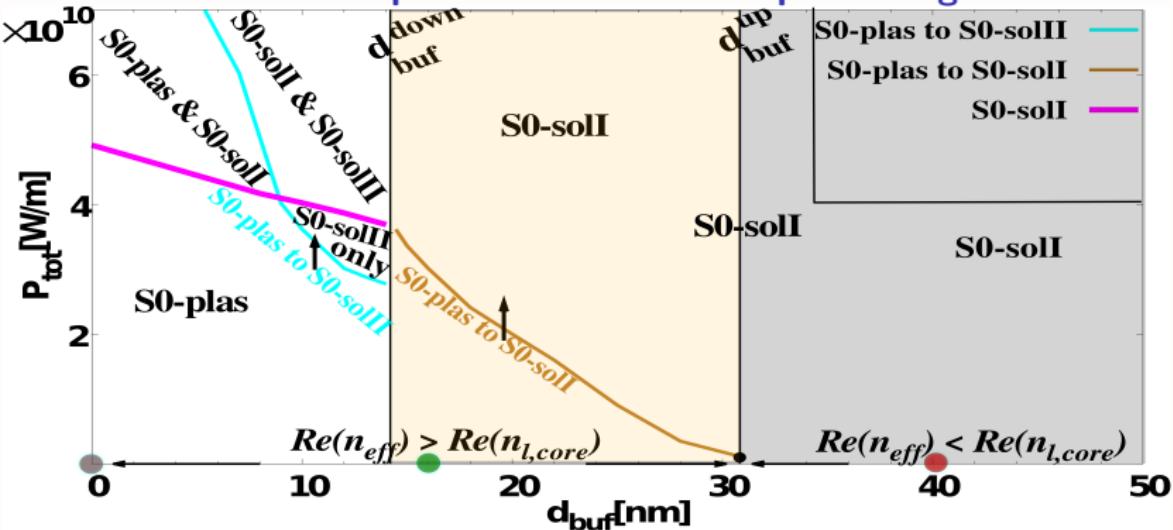
## Nonlinear TM waves in improved slot: nonlinear phase diagram



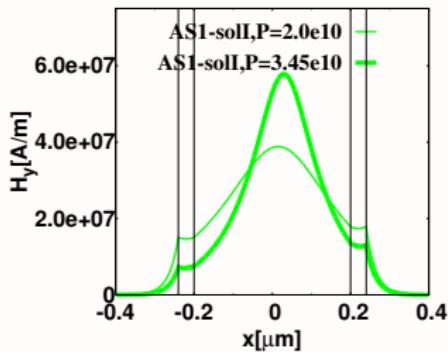
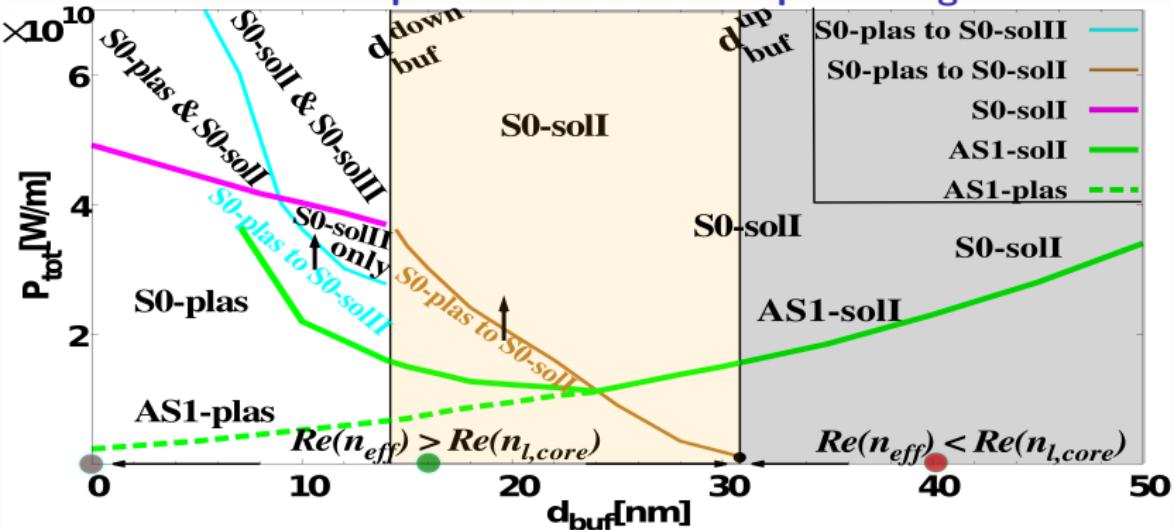
## Nonlinear TM waves in improved slot: nonlinear phase diagram



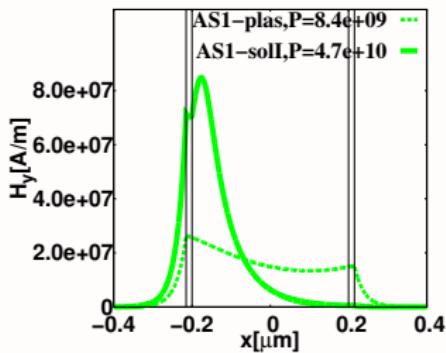
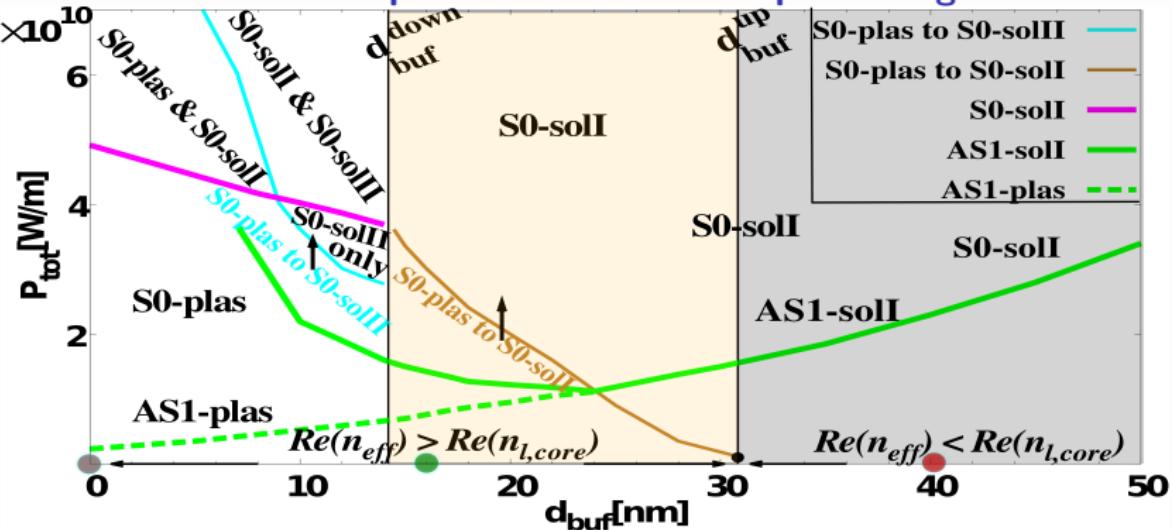
## Nonlinear TM waves in improved slot: nonlinear phase diagram



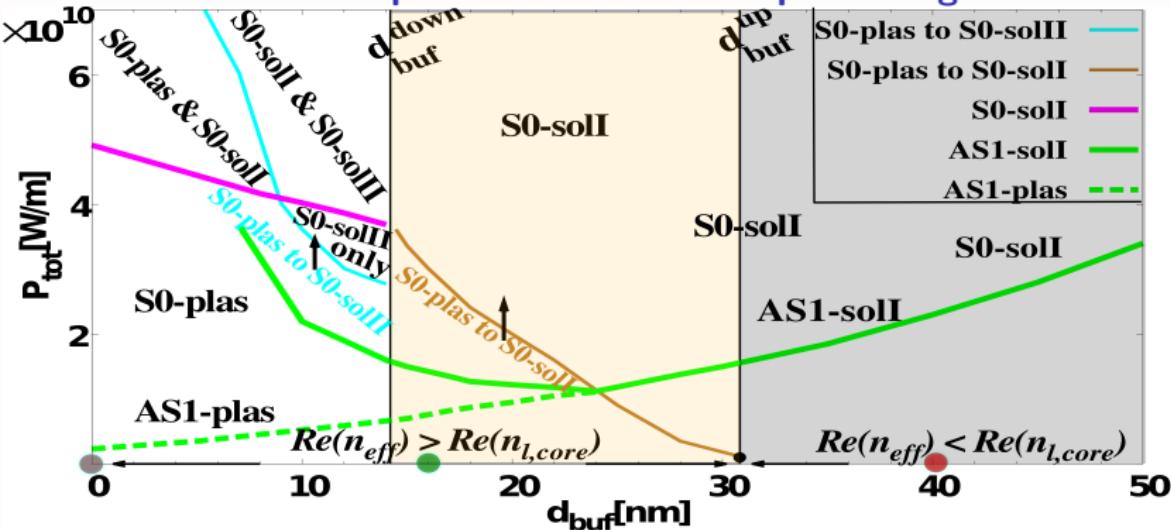
## Nonlinear TM waves in improved slot: nonlinear phase diagram



## Nonlinear TM waves in improved slot: nonlinear phase diagram



## Nonlinear TM waves in improved slot: nonlinear phase diagram



-M. M. R. Elsawy and G. Renversez, *Improved nonlinear slot waveguides using dielectric buffer layers: properties of TM waves.*, Opt. Lett., **41**, 1542-1545, (2016)

-M. M. R. Elsawy, V. Nazabal, M. Chauvet, G. Renversez, *Improved nonlinear plasmonic slot waveguide: a full study.*, Proc. SPIE 9884, Nanophotonics VI, 98840J, (2016)

## Slot with a metamaterial nonlinear core — Introduction

The idea to use metamaterial and/or epsilon-near-zero (ENZ) materials to enhance nonlinear effects was already proposed several times:



[A. Husakou and J. Hermann](#)

Steplike transmission of light through a Metal-Dielectric Mutilayer Structure due to an Intensity-Dependent Sign of the Effective Dielectric Constant

*Phys. Rev. Lett.*, 99, 127402, (2007)



[A. Ciattoni et al.](#)

Extreme nonlinear electrodynamics in metamaterials with very small linear permittivity,

*Phys. Rev. A.*, 81, 043839, (2011)



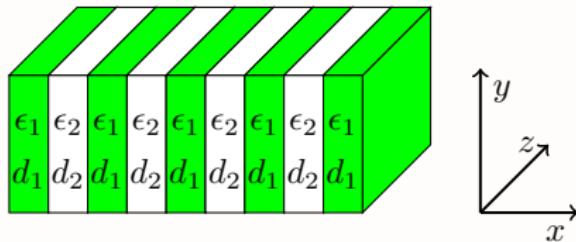
[A. D. Neira et al.](#)

Eliminating material constraints for nonlinearity with plasmonic metamaterials

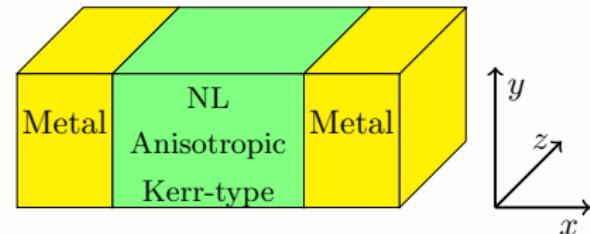
*Nature Comm.*, 6, 7757, (2015)

Nevertheless, **nonlinear ENZ waveguide problems and the key role of anisotropy seem to have been partially overlooked.**

## Slot with a metamaterial nonlinear core



Metamaterial based nonlinear core



Full slot with its metamaterial nonlinear core

$$\varepsilon_{core} \longrightarrow \bar{\varepsilon}_{core} = \begin{pmatrix} \varepsilon_x = \varepsilon_{\perp} & 0 & 0 \\ 0 & \varepsilon_y = \varepsilon_{//} & 0 \\ 0 & 0 & \varepsilon_z = \varepsilon_{//} \end{pmatrix} \quad (1)$$

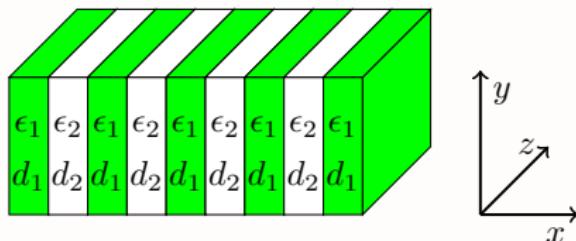
- Effective Medium Theory (EMT)  $\rightarrow \bar{\varepsilon}_{core}$  tensor for uniaxial anisotropic medium as:

$$\varepsilon_y = \varepsilon_z = r\varepsilon_2 + (1 - r)\varepsilon_1 = \varepsilon_{//}$$

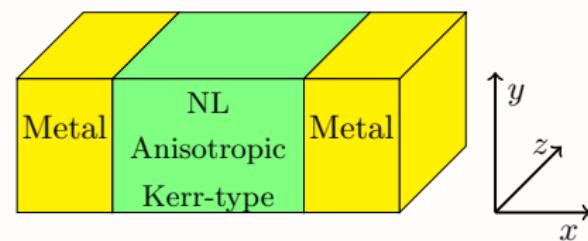
$$\varepsilon_x = \frac{\varepsilon_1\varepsilon_2}{r\varepsilon_1 + (1 - r)\varepsilon_2} = \varepsilon_{\perp}$$

$$r = \frac{d_2}{d_1 + d_2} \text{ is the ratio of the 2nd material in the layered structure}$$

## Slot with a metamaterial nonlinear core



Metamaterial based nonlinear core



Full slot with its metamaterial nonlinear core

$$\varepsilon_{core} \longrightarrow \bar{\varepsilon}_{core} = \begin{pmatrix} \varepsilon_x = \varepsilon_{\perp} & 0 & 0 \\ 0 & \varepsilon_y = \varepsilon_{//} & 0 \\ 0 & 0 & \varepsilon_z = \varepsilon_{//} \end{pmatrix} \quad (1)$$

- As first order approximation, the nonlinear part of the permittivity is isotropic. This hypothesis can be overcome in the nonlinear FEM method if needed.

## Slot with a metamaterial nonlinear core — Effective nonlinearity

- In the frame of our semi-analytical 1D model (Maxwell's equations & stationary TM waves), we obtain for the **effective nonlinearity**  $\alpha$  using Eq.(1):

$$\alpha_{ANISOTROPIC} \propto \frac{-1}{\varepsilon_{\perp}^2} \left( n_{eff}^2 (\varepsilon_{\perp} - \varepsilon_{//}) - \varepsilon_{\perp}^2 \right) \alpha_{ISOTROPIC} \quad (2)$$

- $\alpha_{ANISOTROPIC} \rightarrow \alpha_{ISOTROPIC}$  when  $\varepsilon_{\perp} \rightarrow \varepsilon_{//}$
- Metamaterial properties  $\Rightarrow \varepsilon_{\perp} = f(\varepsilon_1, d_1, \varepsilon_2, d_2)$  and  $\varepsilon_{//} = g(\varepsilon_1, d_1, \varepsilon_2, d_2)$
- $n_{eff}$  depends on the total power  $P_{tot}$  and on the opto-geometric parameters of the slot

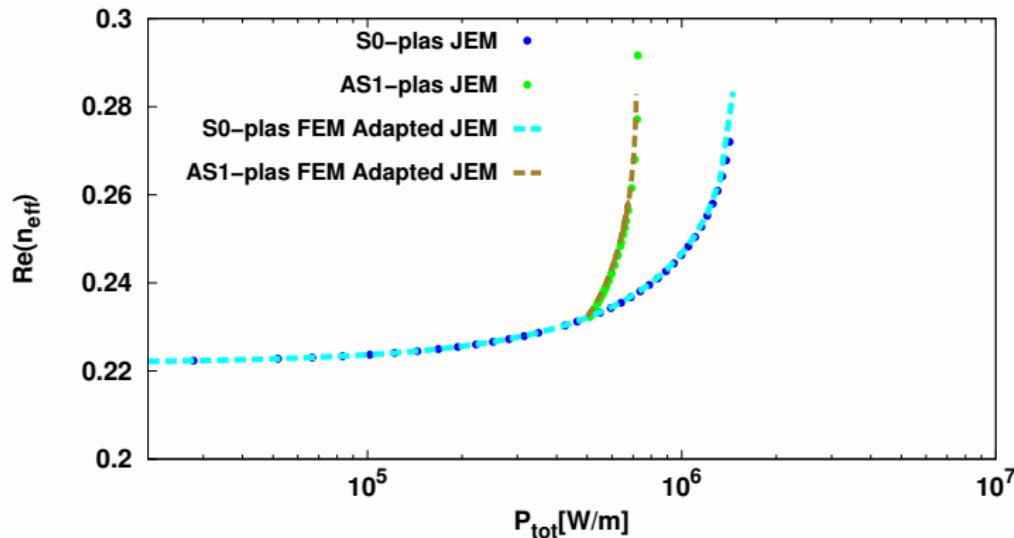
### Consequences in the elliptical case

- Lowering the bifurcation threshold for symmetry breaking :  
 $1 \text{ GW/cm}^2 \rightarrow 1 \text{ MW/cm}^2$

Elliptical and hyperbolic cases: M. M. R. Elsayy and G. Renversez, *Spatial nonlinearity at low power in metamaterial plasmonic slot waveguides, submitted, (2016)*

## Slot with a metamaterial nonlinear core — Numerical results

with our 2 models: semi-analytical Jacobi Elliptic function based Model (JEM) and nonlinear FEM (adapted)

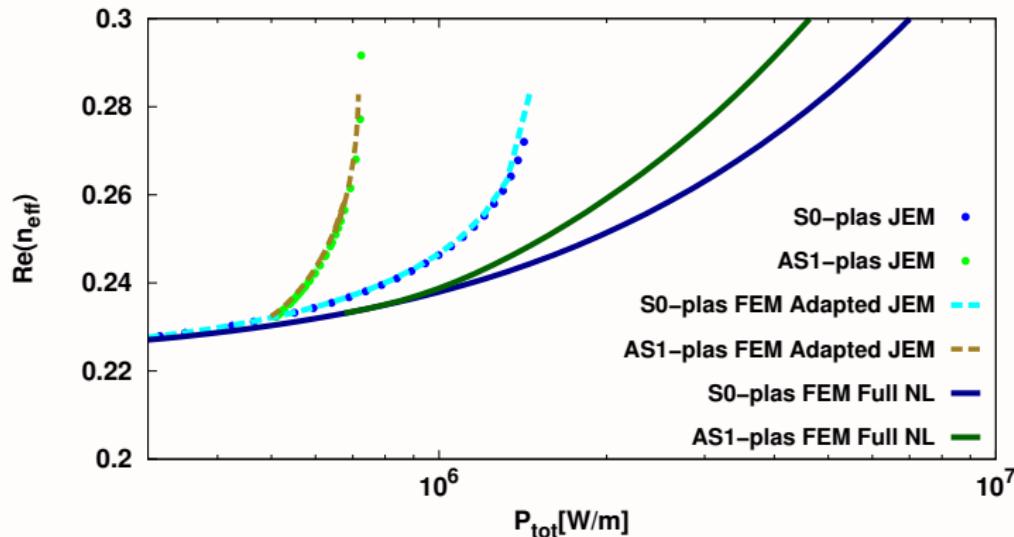


Full nonlinear dispersion relation for the symmetric and asymmetric modes as a function of total power  $P_{tot}$ .  $\lambda = 1.55 \mu\text{m}$ ,  $\varepsilon_{\perp} = 0.042$ ,  $\varepsilon_{//} = 9.07$ ,  $d_{core} = 400 \text{ nm}$  and  $n_2 = 2.10^{-17} \text{ m}^2/\text{W}$

- Here, in the *FEM Adapted JEM* only  $E_x$  is in the nonlinear term so as to correspond with JEM

## Slot with a metamaterial nonlinear core — Numerical results

with our 2 models : JEM and nonlinear FEM (adapted and full)



Zoom of the nonlinear dispersion relation for the symmetric and asymmetric modes as a function of total power  $P_{tot}$ , around the bifurcation.

- Here, our 2 FEM models: full nonlinearity and  $E_x$  only in the nonlinear term

## Conclusions

- **Stability of the asymmetric mode**
- **Loss reduction** and new nonlinear spatial modal transitions for the buffer improved isotropic structure
- **2 or 3 orders of magnitude reduction of the bifurcation threshold using realistic metamaterial based nonlinear core**  
→ **Important nonlinear effects in plasmonic waveguides at low power**

## Perspectives

- **Fabrication** of the designed waveguides by technological facilities
  - Use of recent ENZ materials like ITO in Alam, De Leon and R. Boyd's article in Science April 2016 or like Al-doped ZnO in Caspani *et al.* PRL article June 2016
  - Use of enhanced manufacturing capabilities (like CEA-LETI ones, or see E. Shkondin and A. V. Lavrinenko works shown yesterday in 2-A24 session)
- **Experimental observations** of the predicted waves in collaboration with experimental groups

**Job opening in my group:** Post-doc in computational photonics (semi-analytical approach, FEM, FDTD, GNLSE) to work on *integrated nonlinear plasmonics with metamaterials*