Review on spatial nonlinearity in plasmonic waveguides: single interface and slot configurations

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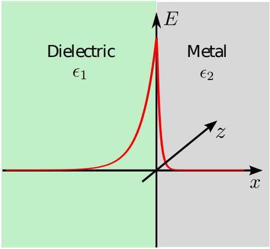
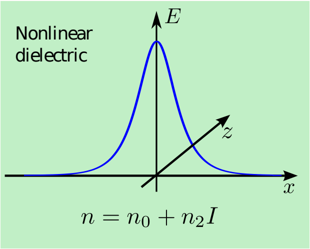
Outline

Plasmon–soliton wave building blocks

Surface plasmon polariton	Spatial optical soliton
Solution of a linear wave equation	Solution of a nonlinear wave equation





Propagation constant

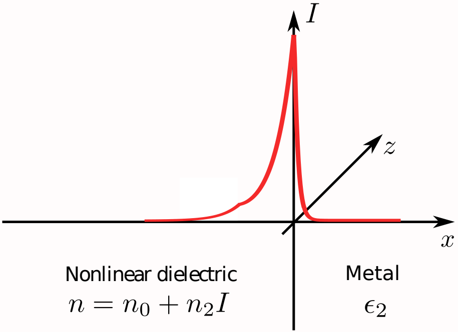
$$\beta_p = k_0 \sqrt{\frac{\epsilon_1 \epsilon_2}{\epsilon_1 + \epsilon_2}}$$

Propagation constant

$$\beta_s = k_0 n_0 \sqrt{1 + \frac{n_2 I}{n_0}}$$

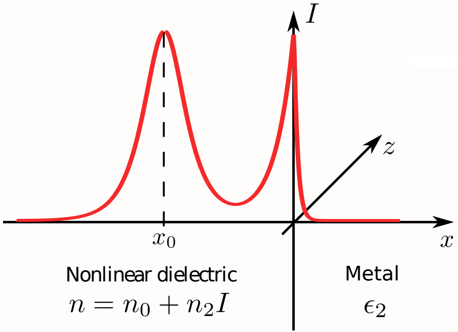
What is a plasmon–soliton wave?

A nonlinear optical wave combining a spatial soliton and a plasmon field with a single propagation constant

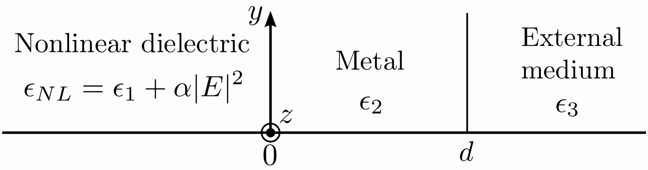


What is a plasmon–soliton wave?

A nonlinear optical wave combining a spatial soliton and a plasmon field with a single propagation constant

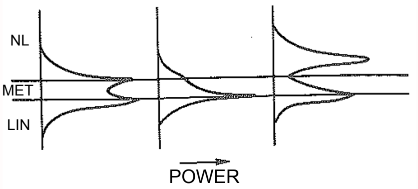


Motivation — Plasmon-soliton coupling in the semi-infinite NL region case



Geometry of the 3-layer nonlinear model

- Seminal articles:
  - V. M. Agranovich *et al.* Nonlinear surface polaritons. *Sov. Phys. JETP*, 32(8):512, 1980.
  - J. Ariyasu *et al.* Nonlinear surface polaritons guided by metal films. *J. Appl. Phys.*, 58(7):2460, 1985.



Motivation — Plasmon-soliton coupling in the semi-infinite NL region case

More recent articles:

- Using the 'interaction picture' approach:

K. Y. Bliokh, Y. P. Bliokh, and A. Ferrando.
Resonant plasmon-soliton interaction.
Phys. Rev. A, 79:41803, 2009.

C. Milián, D. E. Ceballos-Herrera, D. V. Skryabin, and A. Ferrando.
Soliton-plasmon resonances as Maxwell nonlinear bound states.
Opt. Lett., 37(20):4221–4223, 2012.

- Starting from nonlinear Schrödinger's equation:

A. Baron, T. B. Hoang, C. Fang, M. H. Mikkelsen, and D. R. Smith.
Ultrafast self-action of surface-plasmon polaritons at an air/metal interface.
Phys. Rev. B, **91**, 195412, 2015

Motivation — Plasmon-soliton coupling in the semi-infinite NL region case

More recent articles:

- Starting from Maxwell's equations:

A. R. Davoyan, I. V. Shadrivov, and Y. S. Kivshar.
Self-focusing and spatial plasmon-polariton solitons.
Opt. Express, 16(24):21732–21737, 2009.

-W. Walasik, V. Nazabal, M. Chauvet, Y. Kartashov, and G. Renversez,
Low-power plasmon-soliton in realistic nonlinear planar structures, Opt. Lett., 37(22): 4579, (2012)

-W. Walasik, G. Renversez, and Y. Kartashov,
Stationary plasmon-soliton waves in nonlinear planar structures: modeling and properties. Phys. Rev. A, 89: 023816, (2014)

Single interface configuration

Choice of a proper structure

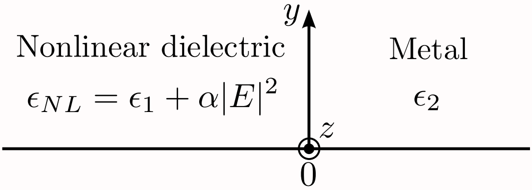
Can we design a **feasible simple** structure supporting **low power plasmon-soliton** that can be **excited directly and recorded?**

Single interface configuration

Choice of a proper structure

Can we design a **feasible simple** structure supporting **low power plasmon-soliton** that can be **excited directly and recorded?**

2-layer model (*W. J. Tomlinson, Opt. Lett. 5(7), 323 (1980)*):



Results

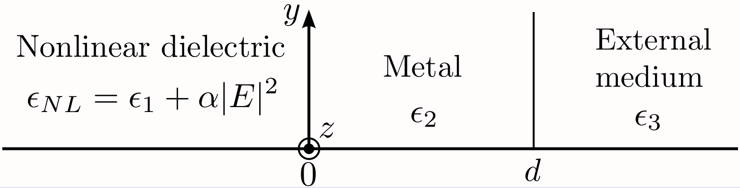
- no low power plasmon-soliton
- plasmon part cannot be recorded using NFO experiments

Single interface configuration

Choice of a proper structure

Can we design a **feasible simple** structure supporting **low power plasmon-soliton** that can be **excited directly and recorded?**

3-layer model (*J. Ariyasu et al., Appl. Phys. 58(7), 2460 (1985)*):



Our results

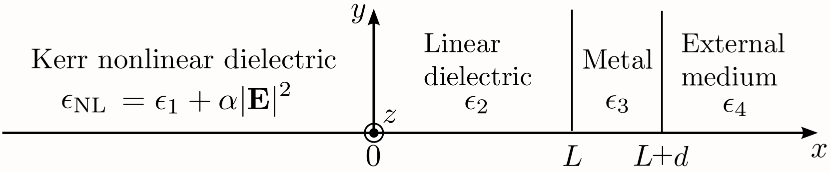
- **No low power plasmon-soliton when air or water is chosen as external medium**

Single interface configuration

Choice of a proper structure

Can we design a **feasible simple** structure supporting **low power plasmon-soliton** that can be **excited directly and recorded?**

a **4**-layer model must be developed



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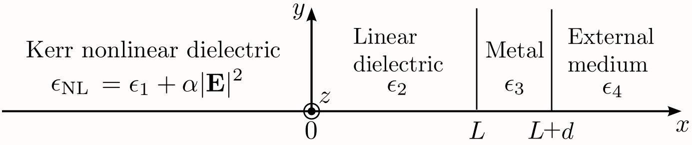
Formulation of our 1D model : TM vector case assumptions

Stationary waves and nonlinear eigenvalue problem



Geometry of the 4-layer nonlinear model used to study 3-layer structures.

- Metal layer $\longrightarrow$ **TM waves**:
  $\mathbf{E} = [E_x, 0, \imath E_z]$ and $\mathbf{H} = [0, H_y, 0]$
- **Kerr nonlinearity**
- Transverse field prevails $\longrightarrow |E_z| \ll |E_x|$ (verified *a posteriori*)
- We look for **stationary solutions**:
  $\mathbf{E}(x, z, t) = \mathbf{E}_{NL}(x) \exp[\imath(\beta_{NL} k_0 z - \omega t)]$
  $k_0 = 2\pi/\lambda$, and β_{NL} is the effective index of this nonlinear wave

Nonlinear wave equation in the frame of the nonlinear eigenvalue problem

Maxwell's equations $\longrightarrow$ **nonlinear wave equation** for $E_{\mathrm{NL},x}$

$$\frac{\mathrm{d}^2 E_{\mathrm{NL},x}}{\mathrm{d}x^2} - k_0^2 q^2(x) E_{\mathrm{NL},x} + k_0^2 \alpha(x) E_{\mathrm{NL},x}^3 = 0, \qquad (1)$$

with $q^2(x) = \beta_{\mathrm{NL}}^2 - \epsilon(x)$,
and $\alpha(x) = \mathcal{H}(-x)\epsilon_0 c \epsilon(x) n_2$

$\mathcal{H}(x)$ — Heaviside step function

Analytical solutions in the whole structure

- In nonlinear region: the well-known solitonic-type solution:

$$E_{\mathrm{NL},x}(x) = E_{0,x}(\beta_{\mathrm{NL}})\,\mathrm{sech}[k_0(\beta_{\mathrm{NL}}^2 - \epsilon_1)^{\frac{1}{2}}(x - x_0)]$$

  where $\mathbf{x_0}$ **denotes the center of the soliton**

- In the linear regions: decreasing and/or increasing exponentials

Nonlinear dispersion relation (NDR) for the single interface configuration

Boundary conditions ⟶ closed form for the **NDR of the 4-layer model**

$$\Phi_+\left(\widetilde{q}_4+\widetilde{q}_3\right)\exp(2k_0\widetilde{q}_3\epsilon_3 d)+\Phi_-\left(\widetilde{q}_4-\widetilde{q}_3\right)=0, \tag{2}$$

$$\Phi_\pm=\left(1\pm\frac{\widetilde{q_{1\mathrm{NL}}}}{\widetilde{q}_3}\right)+\left(\frac{\widetilde{q_{1\mathrm{NL}}}}{\widetilde{q}_2}\pm\frac{\widetilde{q}_2}{\widetilde{q}_3}\right)\tanh(k_0\widetilde{q}_2\epsilon_2 L), \tag{3}$$

witch $\widetilde{q}_j=q(x)/\epsilon(x)$ in the j-th layer,
and $\widetilde{q_{1\mathrm{NL}}}=\widetilde{q}_1\tanh(k_0\widetilde{q}_1\epsilon_1 x_0)$

All the plasmon-soliton characteristics can be evaluated

1. Eq. (2) ⟶ allowed β_{NL} of 1D nonlinear problem
2. β_{NL} and $E_{\mathrm{NL},x}(x)$ ⟶ other field components ($E_{\mathrm{NL},z}(x)$ and $H_{\mathrm{NL},y}(x)$)
3. β_{NL} and $E_{\mathrm{NL},x}(x)$ ⟶ power P
4. limiting cases : 3-layer and 2-layer model results

First example of low power plasmon-soliton waves

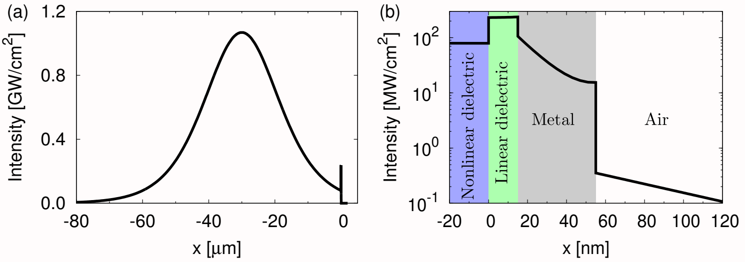
Realistic opto-geometric structure parameters

- Chalcogenide glasses ($\epsilon_1 = 2.4707^2$, $n_2 = 10^{-17}\mathrm{m}^2/\mathrm{W}$) $\longrightarrow$ high nonlinear coefficient
  - Coated planar chalcogenide waveguides already fabricated (*V. Nazabal et al., Int. J. Appl. Ceram Technol.*, **8**, *2011*)
  - Spatial solitons already observed in planar chalcogenide waveguides (*M. Chauvet et al., Opt. Lett.*, **34**, *2009*)
- Silica ($\epsilon_2 = 1.443^2$, $L = 15\mathrm{nm}$) $\longrightarrow$ well known, good compatibility
- Gold ($\epsilon_3 = -96$, $d = 40\mathrm{nm}$) $\longrightarrow$ low loss, good compatibility
- Air as external medium ($\epsilon_4 = 1$) $\longrightarrow$ Near field optics to record the plasmon part of the field

First example of low power plasmon-soliton waves

Realistic soliton parameters $\longrightarrow$ feasible excitation of the plasmon-soliton **peak power P $\simeq$ 1.07GW/cm^2**

($P \simeq 2GW/cm^2$ reported by *M. Chauvet et al., Opt. Lett.*, **34**, *2009*)

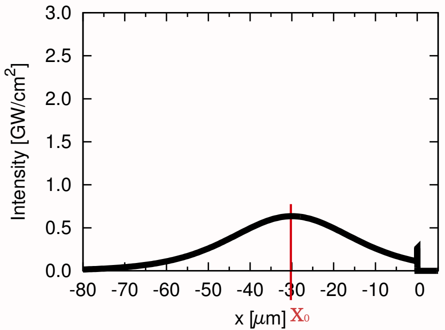


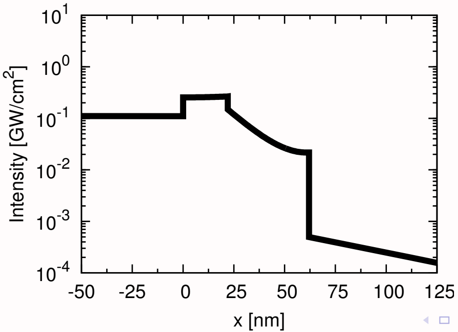
Recordable plasmon field ($E \simeq 4.5\,\mathbf{MV/m}$)

Soliton center position influence

- $x_0 = 20\lambda$
- $\beta = 2.4707317$
- total power $= 11.28\ kW$
- FWHM $= 34\ \mu$m

- soliton peak intensity $= 0.63\ GW/cm^2$
- metal/air interface intensity $= 0.49\ MW/cm^2$
- metal/air interface electric field $= 3.04\ MV/m$



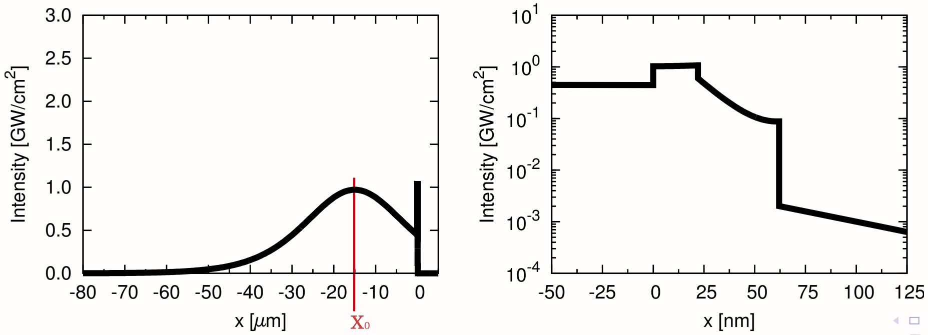


$\lambda = 1.5\ \mu$m $n_{NL} = 2.4707$ $n = 1.624$ $\epsilon_m = -96$ L $= 22$ nm d $= 40$ nm

Soliton center position influence

- $x_0 = 10\lambda$
- $\beta = 2.4707486$
- total power $= 10.01\ kW$
- FWHM $= 27\ \mu$m

- soliton peak intensity $= 0.97\ GW/cm^2$
- metal/air interface intensity $= 2.01\ MW/cm^2$
- metal/air interface electric field $= 6.11\ MV/m$

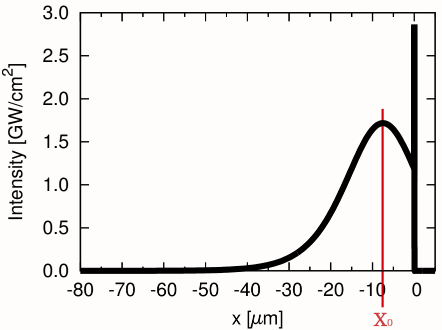


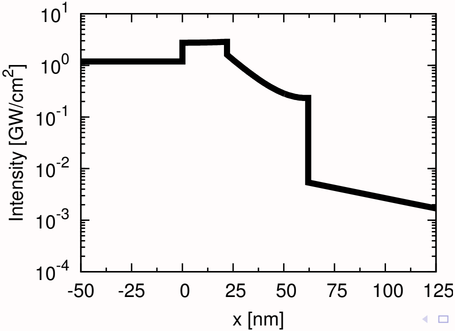
$\lambda = 1.5\ \mu$m $n_{NL} = 2.4707$ $n = 1.624$ $\epsilon_m = -96$ L $= 22$ nm d $= 40$ nm

Soliton center position influence

- $x_0 = 5\lambda$
- $\beta = 2.4707859$
- total power $= 8.88\ kW$
- FWHM $= 20\ \mu$m

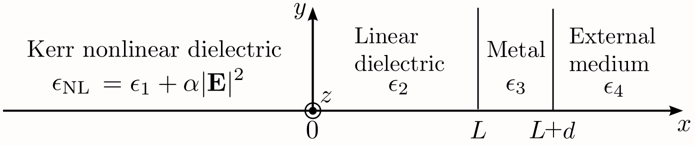
- soliton peak intensity $= 1.71\ GW/cm^2$
- metal/air interface intensity $= 5.36\ MW/cm^2$
- metal/air interface electric field $= 9.99\ MV/m$





$\lambda = 1.5\ \mu$m $n_{NL} = 2.4707$ $n = 1.624$ $\epsilon_m = -96$ L = 22 nm d = 40 nm

Model parameters for the experimental design: L and d, with $P_{peak} \leq 3\mathrm{GW/cm^2}$



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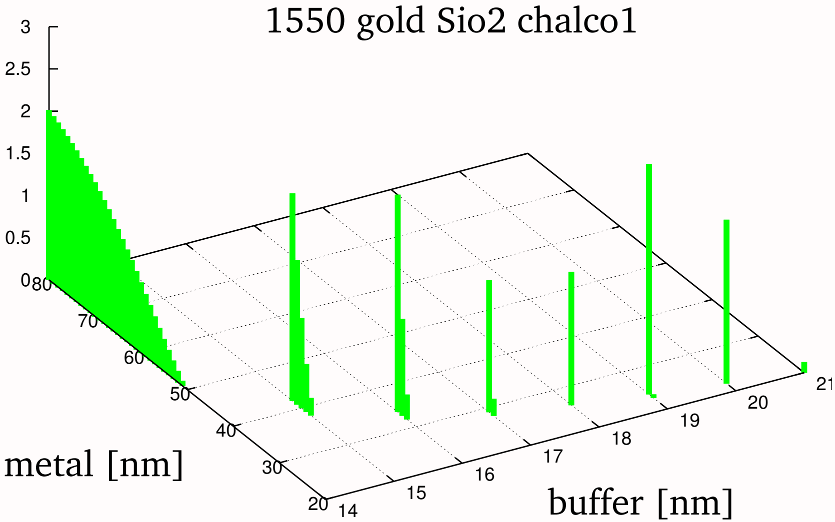

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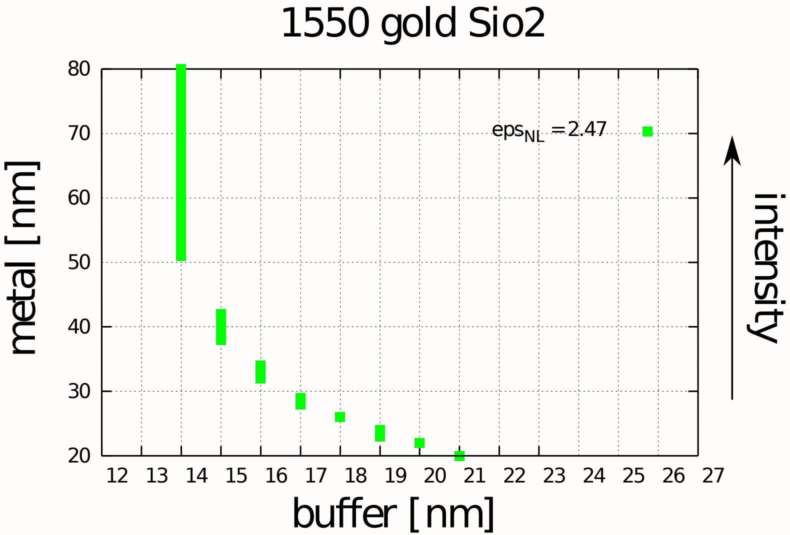
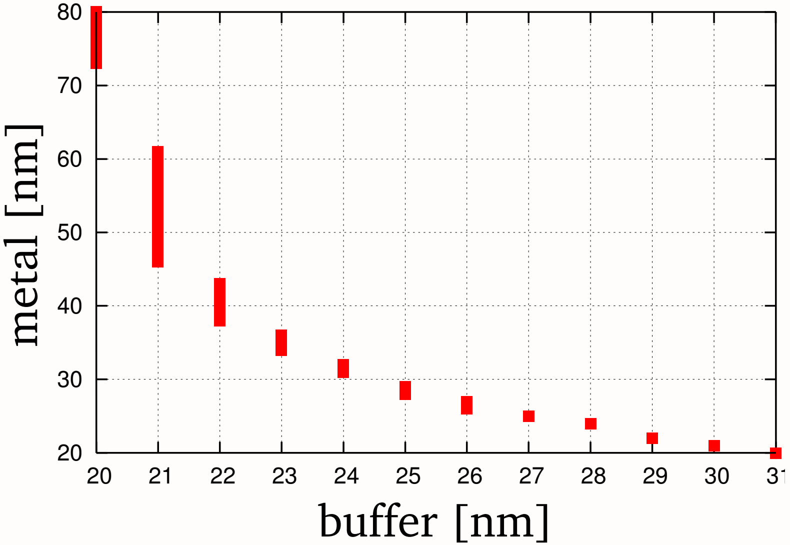


Model parameters for the experimental design: L and d, with $P_{peak} \leq 3\mathrm{GW/cm^2}$

Other composition for the buffer : silica $\longrightarrow$ mulit ($n = 1.624$)



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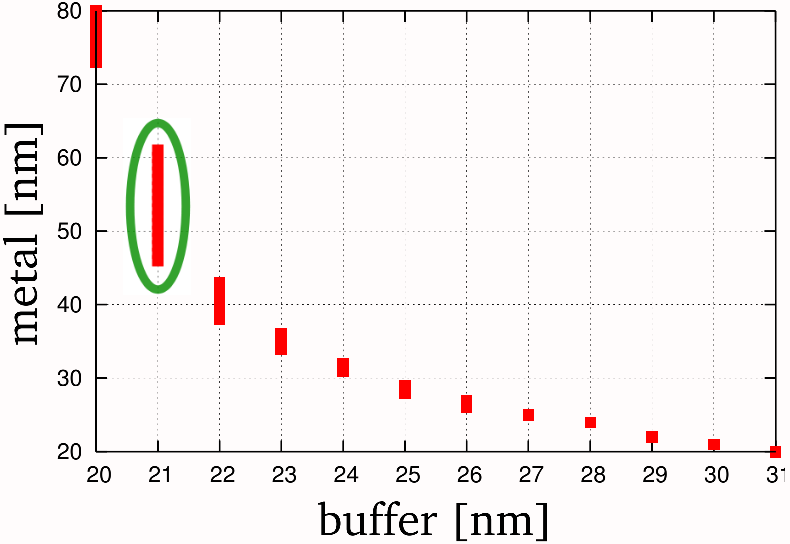
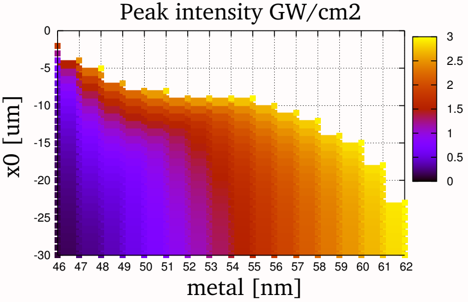
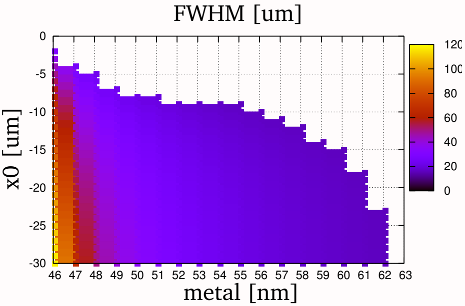
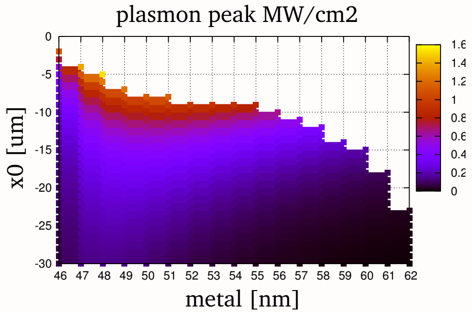


Metal thickness & soliton center position scan (Max 3GW/cm^2)







- $\lambda = 1.5\ \mu\text{m}$
- chalcogenide glass $n_{NL} = 2.4707$
- buffer: $n = 1.624$
- buffer thickness $= 21$ nm
- metal: gold $\epsilon = -96$

*

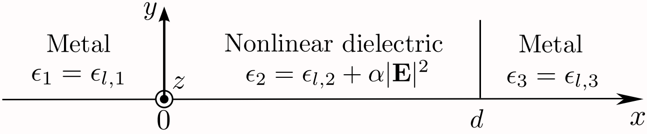
First fabrications using chalcogenide glass

with the chalcogenide layer above a silica film on a BK7 glass substrate

with the chalcogenide layer above a silica film on a silicon substrate

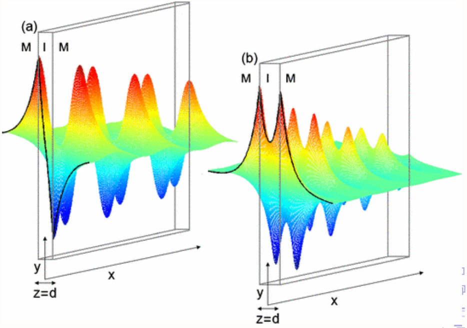
Structures made at the University of Rennes I by EVC-ISCR (courtesy of V. Nazabal)

Nonlinear slot waveguide — Introduction



Linear case:

- V. R. Almeida *et al.*
  Guiding and confining light in void nanostructure,
  Opt. Lett, 29:1209-1211, (2004)
- J. A. Dionne *et al.*
  Plasmon slot waveguides: Towards chip-scale propagation with subwavelength-scale localization,
  Phys. Rev. B, 73(3):035407, (2006)



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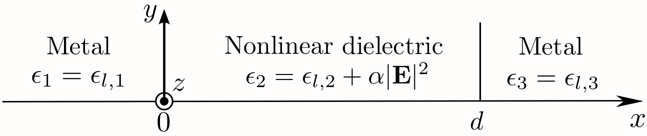
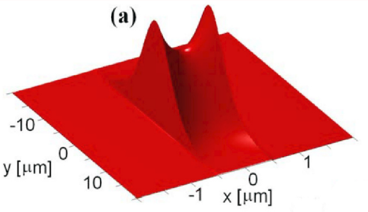


Nonlinear case:

E. Feigenbaum and M. Orenstein.
Plasmon–soliton.
Opt. Lett., 32(6):674, (2007)

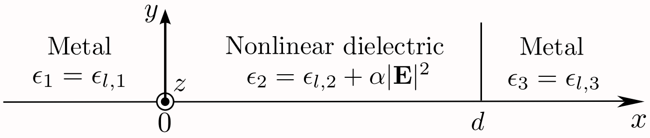
A. Davoyan, I. Shadrivov, and Y. Kivshar,
Nonlinear plasmonic slot waveguides,
Opt. Express, 16(26), (2008)



No experimental results on plasmon–soliton in nonlinear slot

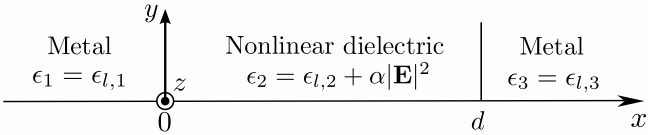
Too high nonlinear index change $\Delta n = n_2 I$

Nonlinear slot waveguide — Introduction



- **Waveguide configuration**
- **Subwavelength focusing**
- **Control the solutions with the power**
- **Peculiar nonlinear effects**

Nonlinear slot waveguide — Models: eigenvalue problem with nonlinearity



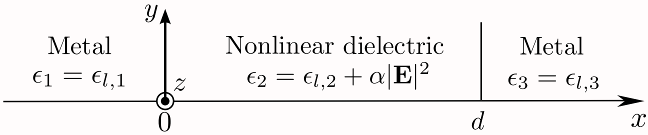
Focusing Kerr effect

Approach: the spatial dependency of transverse field components is kept

"Modal" nonlinear solutions of Maxwell's equations for TM stationary waves using field continuity conditions in 1D structure

Both n_{eff} and field profiles that depend on total power P_{tot} are computed

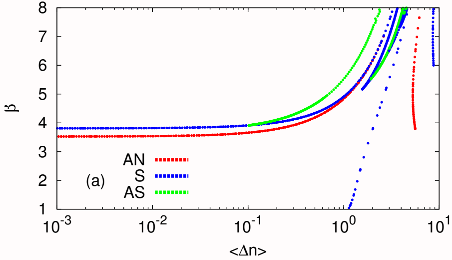
Nonlinear slot waveguide — Models: eigenvalue problem with nonlinearity

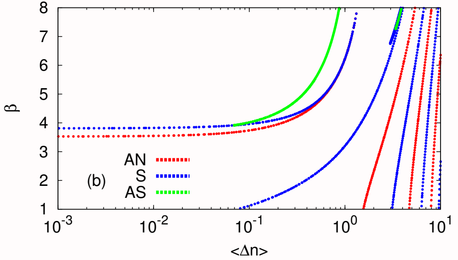


Common hypotheses to our two models

- **Stationary solutions of Maxwell's equations**:
  $$\begin{Bmatrix} \mathbf{E}(x,z,t) \\ \mathbf{H}(x,z,t) \end{Bmatrix} = \begin{Bmatrix} \mathbf{E}_{\mathrm{NL}}(x) \\ \mathbf{H}_{\mathrm{NL}}(x) \end{Bmatrix} \exp[\imath(\beta_{\mathrm{NL}} k_0 z - \omega t)]$$
  $k_0 = 2\pi/\lambda$, and β_{NL} is the effective index n_{eff} of this nonlinear wave
- **TM waves**:
  $\mathbf{E} = [E_x, 0, \imath E_z]$ and $\mathbf{H} = [0, H_y, 0]$
- **Kerr nonlinearity**
- Maxwell's equations + boundary conditions
  $\longrightarrow$ **Nonlinear dispersion relation**

Nonlinear slot waveguide — Models: eigenvalue problem with nonlinearity





Jacobi Elliptic function based Model (JEM)

Extension to slot configuration of
W. Chen and A. A. Maradudin
J. Opt. Soc. Am. B, 5, 529 (1988)

Finite Element Method (FEM)

Adaptation to slot configuration of:
F. Drouart, G. Renversez, *et al.*
J. Opt. A: Pure Appl. Opt., 10, 125101J (2008)

– Low nonlinearity depending only on the transverse electric field

+ **Analytical formulas for field shapes and nonlinear dispersion relation**

+ **Exact treatment of Kerr-type nonlinearity**

– Field shapes and dispersion curves obtained numerically

Nonlinear slot waveguide — Models: eigenvalue problem with nonlinearity

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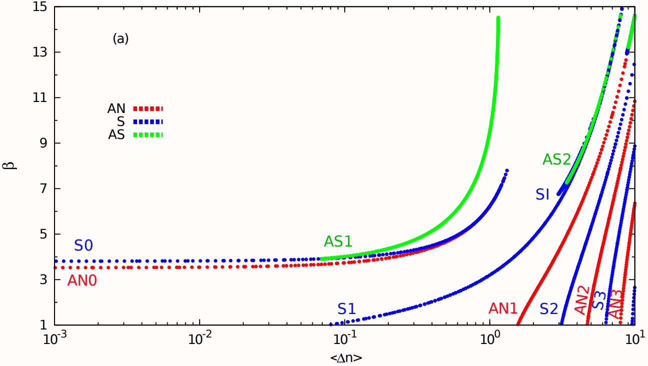
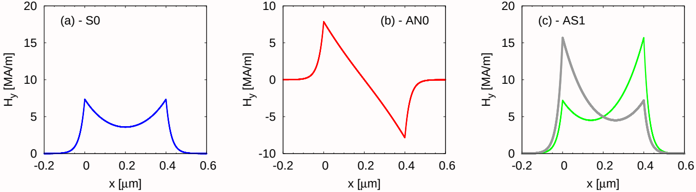
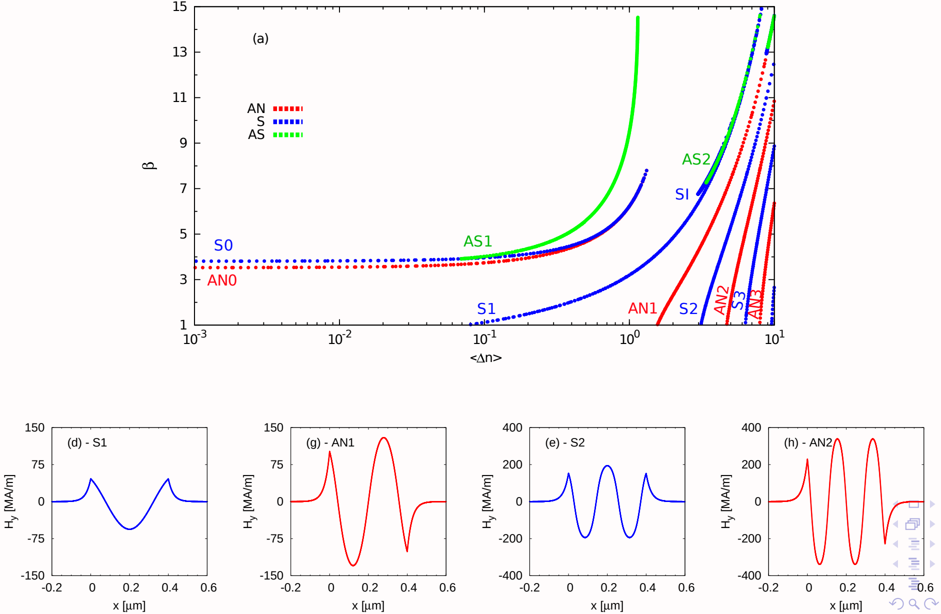
Nonlinear slot waveguide — Dispersion relations



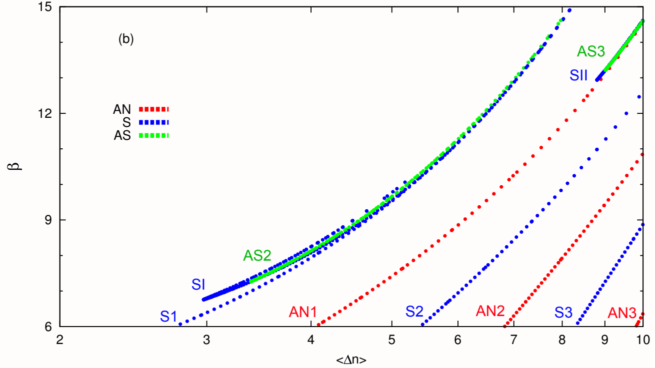


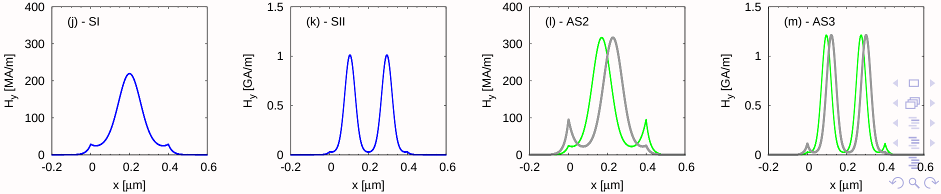
A. Davoyan, I. Shadrivov, and Y. Kivshar, Nonlinear plasmonic slot waveguides, *Opt. Express*, 16(26), 2008

Nonlinear slot waveguide — Dispersion relations

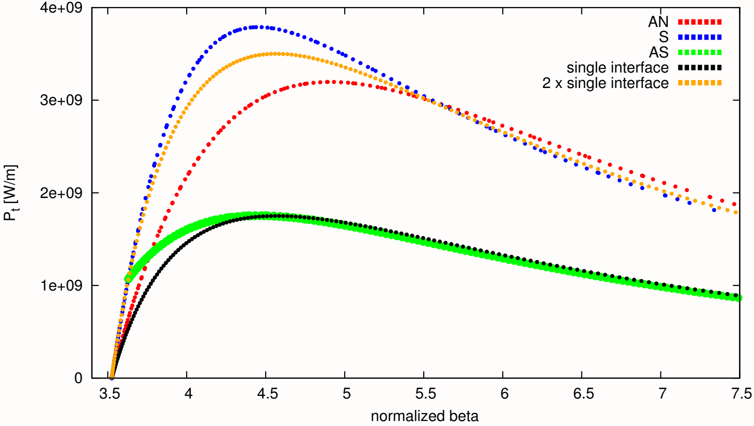


Nonlinear slot waveguide — Dispersion relations





Energy interpretation for the emergence of the asymmetric mode
from the symmetric mode in a symmetric nonlinear simple slot



Power as a function of normalized propagation constant β for the first three modes (S: symmetric, AN: anti-symmetric, AS: asymmetric), single interface is the structure with a semi-infinite nonlinear dielectric medium and a semi-infinite metal region. First: global view, Second: zoom on the bifurcation region where the asymmetric mode emerges.

Energy interpretation for the emergence of the asymmetric mode

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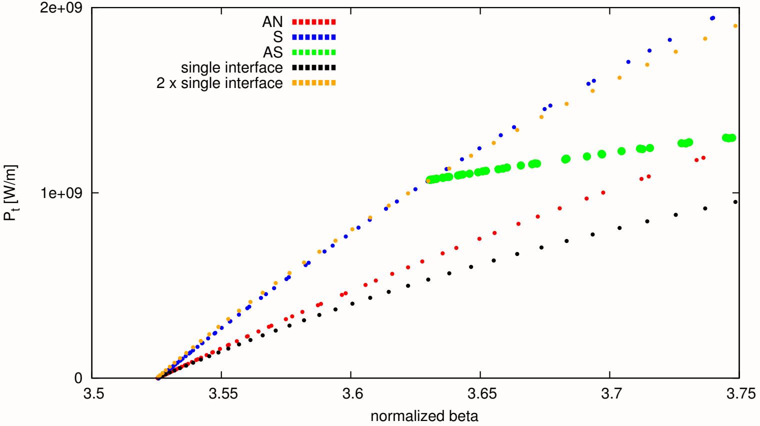
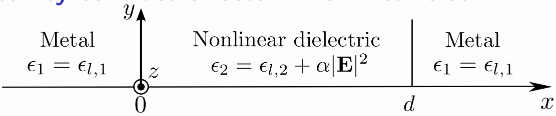
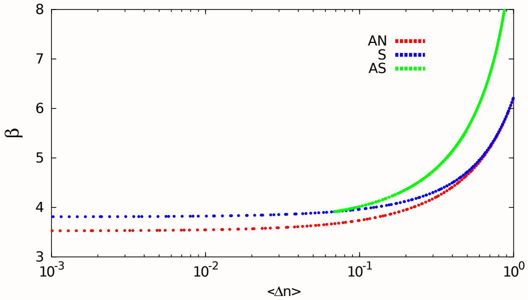
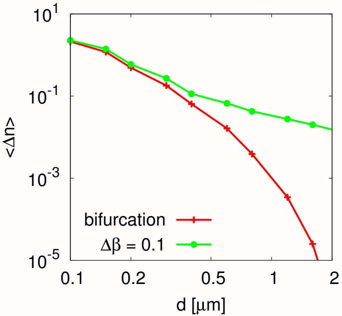


Power as a function of normalized propagation constant β for the first three modes (S: symmetric, AN: anti-symmetric, AS: asymmetric), single interface is the structure with a semi-infinite nonlinear dielectric medium and a semi-infinite metal region. First: global view, Second: zoom on the bifurcation region where the asymmetric mode emerges.

Size and permittivity contrast effects in nonlinear slot

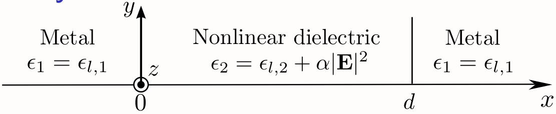


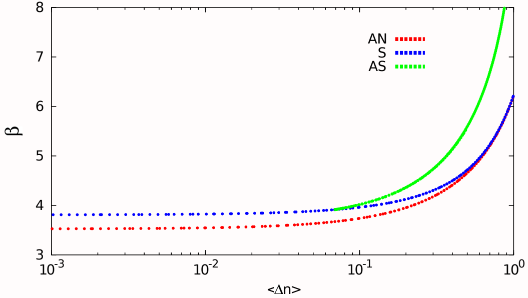


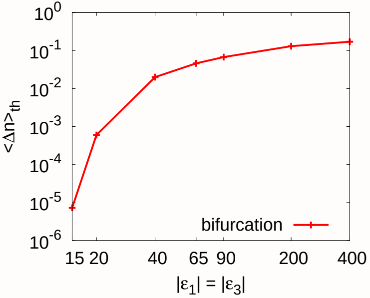


- Bifurcation — spontaneous symmetry breaking
- Asymmetric modes in symmetric structures
- Parameter rules to lower power needed for nonlinear effects

Size and permittivity contrast effects in nonlinear slot







- Bifurcation — spontaneous symmetry breaking
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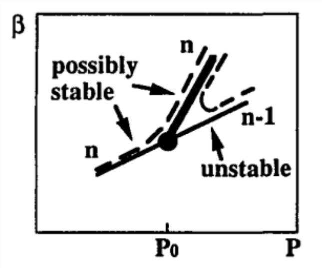
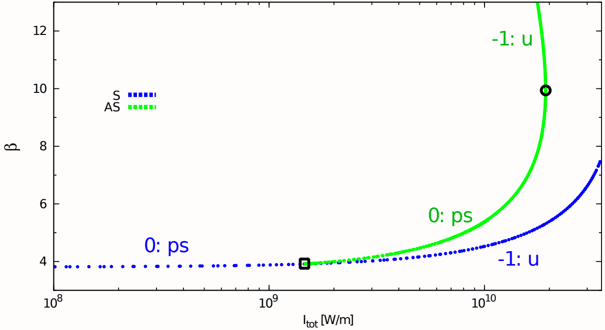
Stability analysis: introduction



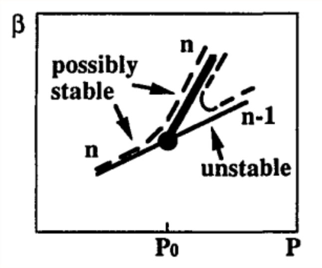


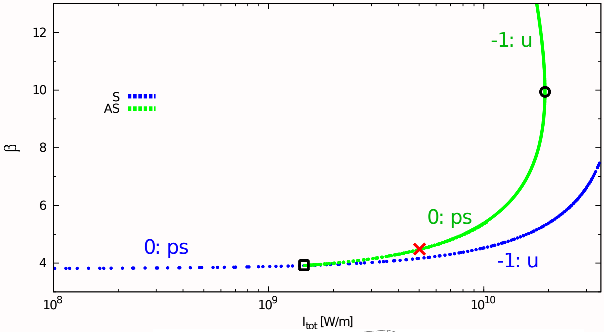
D. J. Mitchell and A. W. Snyder
Stability of fundamental nonlinear guided waves
J. Opt. Am. B, 10(9):1572, 1993

Vakhitov–Kolokolov criterion / linear stability analysis } ⟶ geometrical approach

- possible stability of a fundamental mode
- weak guiding approximation
- first use to nonlinear slot waveguides

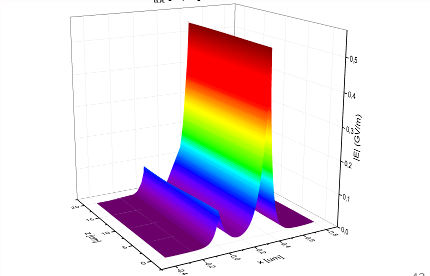
Stability analysis: introduction





Temporal propagation simulation over 15λ for the asymmetric mode $|\mathbf{E}|$ (comsol based)

Numerical results confirm the conclusions drawn from the geometrical method



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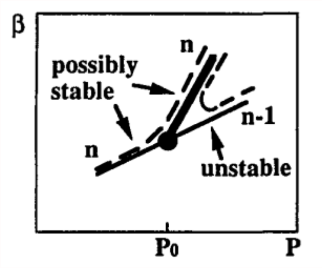
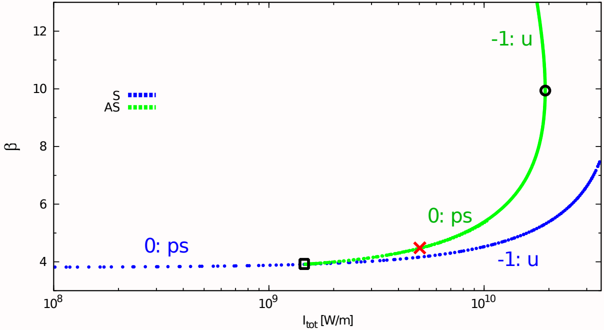
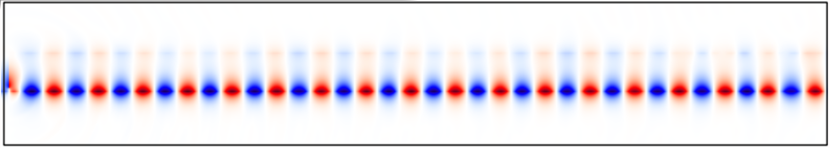
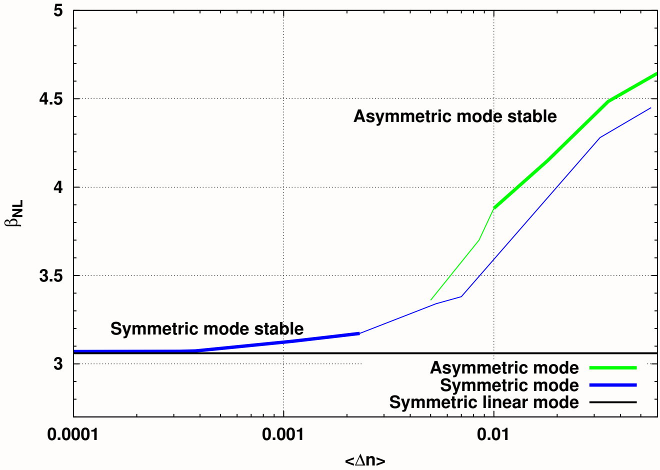




Temporal propagation simulation over 15λ
MEEP full vector nonlinear FDTD simulation for the asymmetric mode H_y



Stability analysis: full vector numerical results from nonlinear FDTD simulations



Dispersion curves with stability results for the symmetric and asymmetric modes from FDTD simulations, $d = 500$ nm amorphous Si core surrounded by gold

Intermediate conclusions

- Two semi-analytical models for **nonlinear slot waveguide** configuration with a finite size nonlinear core
- Prediction of the existence of **higher order modes** in nonlinear slot waveguides
- Study of size and permittivity contrast effects on **bifurcation threshold**
  → ways to reduce it
- **Stability study** of plasmon–solitons using two numerical methods
  → stable asymmetric mode

-W. Walasik, A. Rodriguez, G. Renversez, Symmetric Plasmonic Slot Waveguides with a Nonlinear Dielectric Core: Bifurcations, Size Effects, and Higher Order Modes, Plasmonics, **10***, 33, (2015)*

-W. Walasik, G. Renversez, Plasmon–soliton waves in planar slot waveguides: I. Modeling. Phys. Rev. A, **93***: 013825, (2016)*

-W. Walasik, G. Renversez, F. Ye, Plasmon–soliton waves in planar slot waveguides: II. Results for stationary waves and stability analysis. Phys. Rev. A, **93***: 013825, (2016)*

Intermediate conclusions

- Two semi-analytical models for **nonlinear slot waveguide** configuration with a finite size nonlinear core
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- Study of size and permittivity contrast effects on **bifurcation threshold**
  $\rightarrow$ ways to reduce it
- **Stability study** of plasmon–solitons using two numerical methods
  $\rightarrow$ stable asymmetric mode
- **But losses and bifurcation threshold are still high for realistic and useful parameters.**

Slot with a metamaterial nonlinear core — Introduction

The idea to use metamaterial and/or epsilon-near-zero (ENZ) materials to enhance nonlinear effects was already proposed several times:

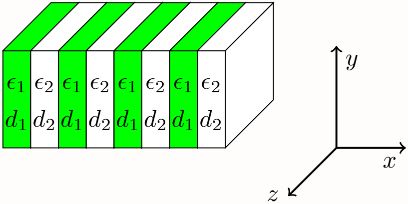
A. Husakou and J. Hermann
Steplike transmission of light through a Metal-Dielectric Mutilayer Structure due to an Intensity-Dependent Sign of the Effective Dielectric Constant
Phys. Rev. Lett., 99, 127402, (2007)

A. Ciattoni *et al.*
Extreme nonlinear electrodynamics in metamaterials with very small linear permittivity,
Phys. Rev. A., 81, 043839, (2011)

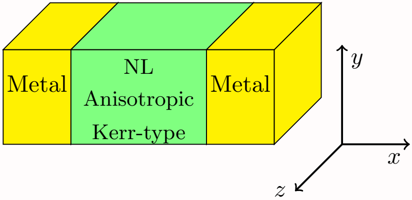
A. D. Neira *et al.*
Eliminating material constraints for nonlinearity with plasmonic metamaterials
Nature Comm., 6, 7757, (2015)

Nevertheless, **nonlinear ENZ waveguide problems and the key role of anisotropy seem to have been partially overlooked.**

Slot with a metamaterial nonlinear core



Metamaterial based nonlinear core



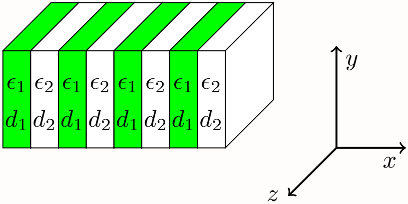
Full slot with its metamaterial nonlinear core

$$\varepsilon_{core} \longrightarrow \bar{\bar{\varepsilon}}_{core} = \begin{pmatrix} \varepsilon_x = \varepsilon_\perp & 0 & 0 \\ 0 & \varepsilon_y = \varepsilon_{//} & 0 \\ 0 & 0 & \varepsilon_z = \varepsilon_{//} \end{pmatrix} \quad (4)$$

- Effective Medium Theory (EMT) $\rightarrow$ $\bar{\bar{\varepsilon}}_{core}$ tensor for uniaxial anisotropic medium as:

$$\begin{aligned} \varepsilon_y &= \varepsilon_z = r\varepsilon_2 + (1-r)\varepsilon_1 = \varepsilon_{//} \\ \varepsilon_x &= \frac{\varepsilon_1 \varepsilon_2}{r\varepsilon_1 + (1-r)\varepsilon_2} = \varepsilon_\perp \\ r &= \frac{d_2}{d_1 + d_2} \text{ is the ratio of the 2nd material in the layered structure} \end{aligned}$$

Slot with a metamaterial nonlinear core



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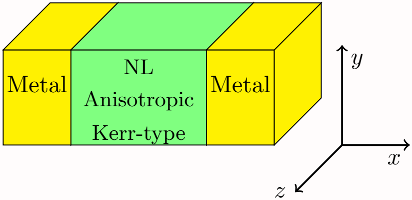

Full slot with its metamaterial nonlinear core

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- As first order approximation, the nonlinear part of the permittivity is **isotropic**. We have recently exented our methods to deal with **anisotropic nonlinearity**. In this case, only the nonlinear FEM method can be used.

Equations for the TM waves with linear anisotropy and Kerr type isotropic nonlinearity

$$E_x(x) = \frac{\Re e(n_{eff}) H_y(x)}{\varepsilon_0 \varepsilon_x(x) x} \tag{5}$$

$$E_z(x) = \frac{1}{\varepsilon_0 \varepsilon_z(x) \omega} \frac{dH_y(x)}{dx} \tag{6}$$

$$k_0 \Re e(n_{eff}) E_x(x) - \frac{dE_z(x)}{dx} = \omega \mu_0 H_y(x) \tag{7}$$

$$\varepsilon_j = \varepsilon_{jj} + \alpha |E_x^2 + E_z^2|, \forall j \in \{x, y, z\} \text{ (full nonlinearity)} \tag{8}$$

$$\text{or} \tag{9}$$

$$\varepsilon_j = \varepsilon_{jj} + \alpha |E_x^2|, \forall j \in \{x, y, z\} \text{ (full adapted to EJEM nonlinearity)} \tag{10}$$

For the FEM:
We use the two continuous components across the interfaces, H_y and E_z, to write the weak formulation.
$\longrightarrow$ **A nonlinear eigenvalue problem made of 2 coupled equations with nodal elements only.**

Slot with a metamaterial nonlinear core — Effective nonlinearity

- In the frame of our semi-analytical 1D model (Maxwell's equations & stationnary TM waves), we obtain for the **effective nonlinearity** a_{nl} using Eq.(4):

$$a_{nl}^{\text{EJEM}} = -\tilde{\alpha} n_{eff}^2 \left(n_{eff}^2 \left(\varepsilon_{xx} - \varepsilon_{zz} \right) - \varepsilon_{xx}^2 \right) / \left(\varepsilon_{xx}^4 c^2 \varepsilon_0^2 \right) \tag{11}$$

with $\tilde{\alpha} = \varepsilon_0 c \Re e(\varepsilon_1)(1 - r) n_{2,1}$.

- $a_{nl}^{\text{EJEM}} \longrightarrow a_{nl,ISOTROPIC}^{\text{EJEM}}$ when $\varepsilon_{xx} \longrightarrow \varepsilon_{zz}$
- Metamaterial properties $\Rightarrow \varepsilon_{xx} = \varepsilon_{\perp} = f(\varepsilon_1, d_1, \varepsilon_2, d_2)$ and $\varepsilon_{zz} = \varepsilon_{//} = g(\varepsilon_1, d_1, \varepsilon_2, d_2)$
- n_{eff} depends on the total power P_{tot} and on the opto-geometric parameters of the slot

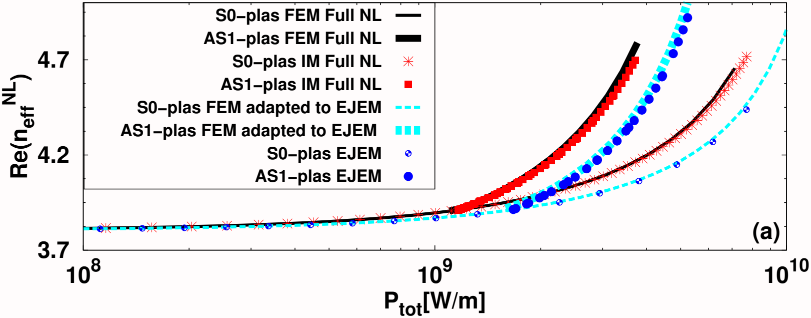
Consequences in the elliptical case: $\varepsilon_{\perp} \gg \varepsilon_{//} > 0$

- Lowering the bifurcation threshold for symmetry breaking : 1 GW/cm^2 $\longrightarrow$ 10 MW/cm^2

Elliptical and hyperbolic cases: M. M. R. Elsawy and G. Renversez, Spatial nonlinearity in anisotropic metamaterial plasmonic slot waveguides, *submitted*, (2016)

Slot with a metamaterial nonlinear core — Numerical results

with our 4 models for the isotropic case: semi-analytical Jacobi Elliptic function based Model (JEM), nonlinear FEMs (full or adapted), and the Interface Model (IM)

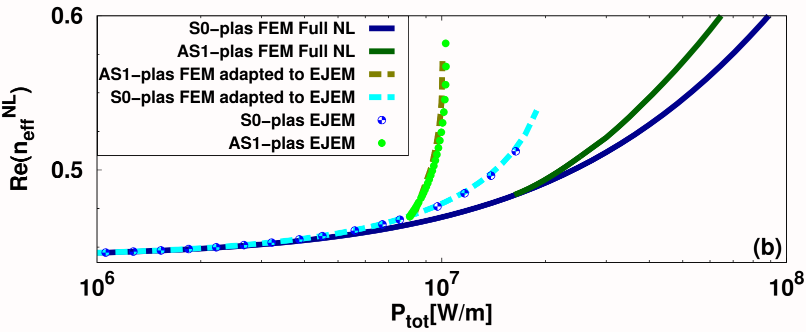


Isotropic case: nonlinear dispersion relation for the symmetric and asymmetric modes as a function of total power P_{tot}. $\lambda = 1.55\,\mu$m, $d_{core} = 400$ nm, and $n_2 = 2.10^{-17} m^2/W$, and for $\varepsilon_\perp = 0.042$, $\varepsilon_{//} = 9.07$.

- Here, in the *FEM adapted to JEM* only E_x is in the nonlinear term in order to correspond with JEM

Slot with a metamaterial nonlinear core — Numerical results

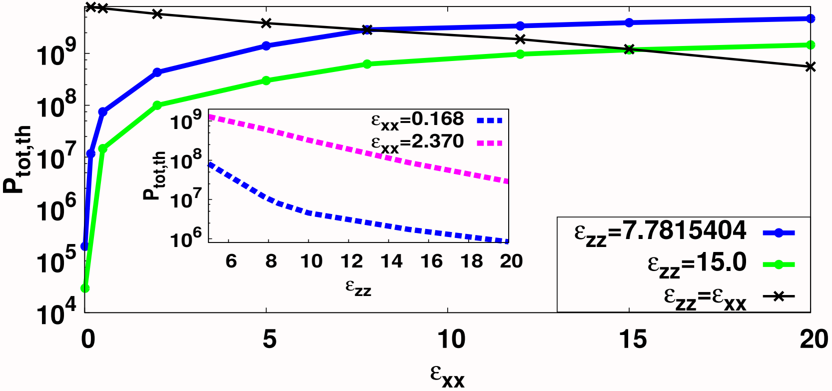
with our 3 models for the anisotropic case: semi-analytical Extended Jacobi Elliptic function based Model (EJEM), nonlinear FEMs (full or adapted)



Anisotropic case: zoom of nonlinear dispersion relation for the symmetric and asymmetric modes as a function of total power P_{tot}, around the bifurcation for $\varepsilon_\perp = 0.042$, $\varepsilon_{//} = 9.07$,

Slot with a metamaterial nonlinear core — Numerical results

Influence of the linear anisotropy on the bifurcation threshold



Anisotropic case: Power threshold $P_{tot,th}$ as a function of linear transverse permittivity ε_{xx} in the elliptical case for two longitudinal permittivity ε_{zz} values. Isotropic case is shown by the black curve. Inset: $P_{tot,th}$ as a function of ε_{zz} for two ε_{xx} values.

Conclusions

- **Stability of the asymmetric mode** of the isotropic slot waveguide
- **Loss reduction** and new nonlinear spatial modal transitions for the buffer improved isotropic structure (not shown today, see I*mproved nonlinear slot waveguides using dielectric buffer layers: properties of TM waves*, Optics Lett., 41(7), pp. 1542-1545, 2016.
- **2 or 3 orders of magnitude reduction of the bifurcation threshold using realistic metamaterial based nonlinear core**
  $\longrightarrow$ **Important nonlinear effects in plasmonic waveguides at low power**

2D models:

- We also investigated the 2D model: a rib waveguide with a nonlinear layer using vector FEM with edge elements. . . for the next time.
- We computed the nonlinear figures of merit taking into account the losses for comparison with state of the art full dielectric nonlinear waveguides.