Second-harmonic-generation-induced optical bistability in prism or grating couplers

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We deal with second-harmonic generation in $\chi^{(2)}$ nonlinear-optical resonators such as prism or grating couplers. The theoretical study is performed within the framework of a recently developed coupled-mode analysis leading to a set of equations governing the amplitudes of pump and second-harmonic frequency fields. We predict analytically that second-harmonic generation in prism or grating couplers may lead to optical bistability. We believe this to be the first demonstration of such an effect in $\chi^{(2)}$ optical resonators.

The characteristic features of second-harmonic (SH) generation in the depleted-pump regime have been known since the early research of Armstrong *et al.*[1] In recent years this kind of SH generation has attracted a great deal of interest[2] because, in the presence of depletion of this fundamental frequency field together with a small phase mismatch, this kind of interaction can lead to a nonlinear phase shift of several π.[2] The devices studied so far belong to the class of nonlinear directional couplers for which there is no optical bistability (OB). But it is understood that, for example, prism or grating couplers that include a Kerr-type nonlinear medium may exhibit OB.[3] Since one of the origins of OB is the phase shift arising from a Kerr nonlinearity, the same kind of behavior is expected to occur based on $\chi^{(2)}$ cascading. The aim of this Letter is to show that this is indeed the case.

Let us consider SH generation in optical resonators such as prism or grating couplers (Fig. 1) in the presence of an electromagnetic resonance such as a surface plasmon or a guided wave. In the following we focus on sharp resonances for which the full width at half-maximum is small compared with the distance between two isolated electromagnetic resonances. Moreover, it is assumed that these couplers are used in the immediate vicinity of a given isolated resonance. These conditions correspond to the so-called high-finesse case, which is the most interesting one when one is dealing with optical resonators.

It was shown in Ref. 4, from which we have adopted definitions and notation, that SH generation in high-finesse optical resonators can be studied in the framework of the modal theory leading to a simple set of equations at the fundamental and SH frequencies:

$$\frac{\mathrm{d}f_{1,p}}{\mathrm{d}x} + j(\beta - \gamma_{1,p})f_{1,p} = -j\omega\xi_1(x)f_{2,m}f_{1,p}^{*} - j\tau_i\mathcal{A}_i(x), \tag{1a}$$

$$\frac{\mathrm{d}f_{2,m}}{\mathrm{d}x} + j(2\beta - \gamma_{2,m})f_{2,m} = -j2\omega\xi_2(x){f_{1,p}}^2. \tag{1b}$$

Throughout this Letter an asterisk indicates a complex conjugate. In Eqs. (1) the quantities $\gamma_{1,p}$ and $\gamma_{2,m}$ are the pth and mth complex poles associated with the leaky homogeneous modes (guided-wave or surface-plasmon) at the pump and SH circular frequencies ω and 2ω, respectively. The amplitudes $f_{1,p}(x)$ and $f_{2,m}(x)$ are related to the mode amplitudes $c_{1,p}(x)$ and $c_{2,m}(x)$ at frequencies ω and 2ω, respectively, by

$$c_{1,p}(x) = f_{1,p}(x)\exp[j(\beta - \gamma_{1,p})x], \tag{2a}$$

$$c_{2,m}(x) = f_{2,m}(x)\exp[j(2\beta - \gamma_{2,m})x]. \tag{2b}$$

The quantity $\mathcal{A}_i(x)$ in Eq. (1a) represents the x dependence of the finite-width pump beam incident at the angle θ_i. When one is dealing with a prism coupler, $\beta = k(\omega)\sin\theta_i$; whereas for a grating coupler, $\beta = k(\omega)\sin\theta_i + r_0(2\pi/d)$, where r_0 is the integer that labels the evanescent diffracted order (at the pump frequency) leading to the resonant excitation of mode p. In Eq. (1a), τ_i is the in-coupling coefficient associated with the resonantly excited diffracted order r_0 at the pump frequency ω. The quantities $\xi_1(x)$ and $\xi_2(x)$ are nonlinear coefficients related to the overlap integral.[5] The presence of a modulated region for the case of a grating coupler explains the (x, y) dependence of the elements of the nonlinear susceptibility tensor $[\chi]$ and thus the x dependence in $\xi_1(x)$ and $\xi_2(x)$.

As assumed above, the operating point remains close to electromagnetic resonances at ω and 2ω. Thus the transverse field maps (along y) at the pump and at the SH frequencies are known[4] and correspond to the linear regime (i.e., when no nonlinear interaction takes place between the pump and the SH fields). Therefore in the case of grating couplers the in-coupling coefficient τ_i and the nonlinear coefficients $\xi_1(x)$ and $\xi_2(x)$ are calculated by use of the linear diffraction theory developed in Ref. 6. The use of reduced units leads to a simple form for Eqs. (1).

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In the plane-wave stationary regime these equations can be written as

$$(\Delta_1 - j)F_1 + \exp(j\psi_1)F_2F_1^* + A_i = 0\,, \tag{3a}$$

$$(\Delta_2 - j)F_2 + \exp(j\psi_2)F_1^2 = 0\,, \tag{3b}$$

where

$$f_{1,p} = \frac{1}{\omega}\left(\frac{\gamma_{1,p}''\gamma_{2,m}''}{2|\xi_1\xi_2|}\right)^{1/2} F_1\,, \tag{4a}$$

$$f_{2,m} = \frac{\gamma_{1,p}''}{\omega|\xi_1|}F_2\,, \tag{4b}$$

$$\mathcal{A}_i = \frac{\gamma_{1,p}''}{\omega\tau_i}\left(\frac{\gamma_{1,p}''\gamma_{2,m}''}{2|\xi_1\xi_2|}\right)^{1/2} A_i\,, \tag{4c}$$

$$\Delta_1 = \frac{\beta - \gamma_{1,p}'}{\gamma_{1,p}''}\,, \tag{4d}$$

$$\Delta_2 = \frac{2\beta - \gamma_{2,m}'}{\gamma_{2,m}''}\,, \tag{4e}$$

$$\xi_\nu = |\xi_\nu|\exp(j\psi_\nu) \qquad (\nu = 1, 2)\,, \tag{4f}$$

$$\gamma_\nu = \gamma_\nu' + j\gamma_\nu'' \qquad (\nu = 1, 2)\,. \tag{4g}$$

The existence of phases ψ_1 and ψ_2 is due to dielectric losses and to the leaky character of modes p and m. The detunings Δ_2 and Δ_1 with respect to the resonances at ω and 2ω are not independent. They are related by

$$\Delta_2 = \frac{2\gamma_{1,p}' - \gamma_{2,m}'}{\gamma_{2,m}''} + \frac{2\gamma_{1,p}''}{\gamma_{2,m}''}\Delta_1\,. \tag{5}$$

Now let us consider Eqs. (3), which yield

$$\left[-(j - \Delta_1) + \frac{\exp(j\psi)}{j - \Delta_2}|F_1|^2\right]F_1 = -A_i\,, \tag{6a}$$

where

$$\psi = \psi_1 + \psi_2\,. \tag{6b}$$

A careful consideration of the zeros of $\mathrm{d}|A_i|^2/\mathrm{d}|F_1|^2$ [from Eq. (6a)] shows that bistable behavior occurs, provided that the following condition is fulfilled:

$$-2[a \sin\psi + b\cos\psi] > \sqrt{3(a^2 + b^2)}\,, \tag{7a}$$

where $a = \Delta_1 + \Delta_2$ and $b = 1 - \Delta_1\Delta_2$. Inequality (7a) implies that

$$a\sin\psi + b\cos\psi < 0\,. \tag{7b}$$

The case of $\psi \simeq 0$ is of special interest since it corresponds to low-loss (dielectric losses and electromagnetic leakage) $\chi^{(2)}$ resonators. When $\psi \simeq 0$, inequality (7a) becomes

$$\Delta_1\Delta_2 - 1 > |\Delta_1 + \Delta_2|\sqrt{3}\,. \tag{8}$$

From this inequality it is apparent that Δ_1 and Δ_2 must be different from zero to yield OB, i.e., OB requires phase mismatch. According to Eq. (6a), OB

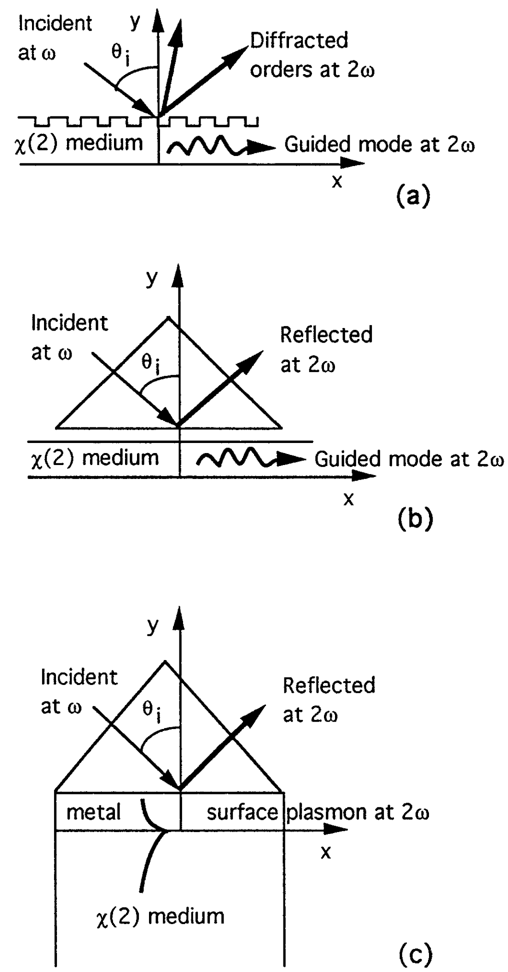

Fig. 1. $\chi^{(2)}$ optical resonators considered in this Letter: (a) grating coupler of periodicity d, permitting guided-mode excitation, (b) prism coupler permitting guided-mode excitation, (c) prism coupler in the Kretschmannn geometry, permitting surface-plasmon excitation.

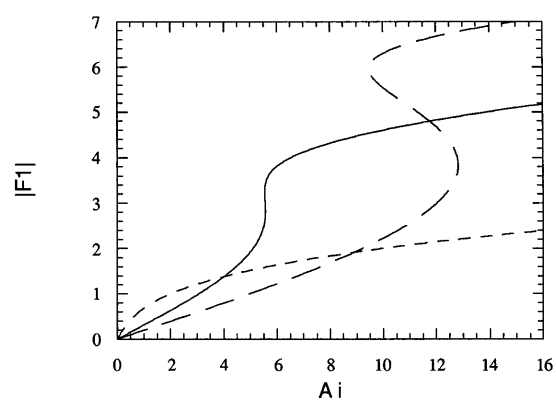

Fig. 2. Response of a $\chi^{(2)}$ optical resonator for detuning values $\Delta_1 = 5$ (long-dashed curve), $\Delta_1 = 3.08$ (solid curve; threshold values deduced from inequality (8), and $\Delta_1 = 0$ (short-dashed curve). $n_{\mathrm{eff},1} = 1.9100 + j1.4677 \times 10^{-3}$; $n_{\mathrm{eff},2} = 1.9099 + j9.6401 \times 10^{-4}$, where $n_{\mathrm{eff},\nu}(\nu = 1, 2)$ is the effective index of mode p at $\omega(\nu = 1)$ and that of mode m at $2\omega(\nu = 2)$; $\psi = -0.808496°$.

leads to an effective cascaded nonlinearity, $\chi_{\mathrm{eff}}^{(3)}$, given by (in unreduced units for the sake of convenience)

$$\chi_{\mathrm{eff}}^{(3)} = \frac{2\omega^2 \xi_1 \xi_2}{2\beta - \gamma_2}, \tag{9}$$

which includes both dispersive and absorptive contributions. Figure 2 is a plot of $|F_1|$ as a function of A_i for three different detunings below, at, and above the OB threshold.

In conclusion, the formalism that we have developed is based on a coupled-mode approach for SH generation in $\chi^{(2)}$ optical resonators. This modal framework is highly convenient since (i) it leads to analytical results and (ii) it remains simple, despite the fact that it applies to a wide class of $\chi^{(2)}$ resonators, such as prism or grating couplers. We have shown that prism and grating couplers may exhibit optical bistability when the depletion of the fundamental frequency field is taken into account. This new result points to the richness of phenomena provided by $\chi^{(2)}$ optical resonators.

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