# Phase-matched guided-wave optical bistability in $\chi^{(2)}$ optical resonators

**R. Reinisch**

*Laboratoire d'Electromagnétisme, Microondes et Optoélectronique, Unité de Recherche Associée au Centre National de la Recherche Scientifique, 833, ENSERG, 23 avenue des Martyrs, B.P. 257, 38016 Grenoble Cedex, France*

**M. Nevière and E. Popov**

*Laboratoire d'Optique Electromagnétique, Unité de Recherche Associée au Centre National de la Recherche Scientifique, 843, Faculté des Sciences et Techniques de St Jérome, 13397 Marseille Cedex 20, France*

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We treat $\chi^{(2)}$ optical resonators, such as prism or grating couplers, illuminated not only by the field at the fundamental frequency but also by the one at the second harmonic. We show that optical bistability may exist even when phase matching is fulfilled. This kind of optical bistability involves a new nonlinear resonance.


Cascaded second-order nonlinear effects attract great interest because of their potential use in devices that require nonlinear (NL) phase shifts.[1–9] It is only recently that the case of $\chi^{(2)}$ distributed couplers in the cascading regime has been considered.[10] The nonlinear effect studied in Ref. 10 is second-harmonic generation, for which only the fundamental frequency field (at $\omega$) is incident upon $\chi^{(2)}$ prism or grating couplers. But the situation is different whether one deals with second-harmonic generation, i.e., $(\omega, \omega) \rightarrow 2\omega$, or with subharmonic generation i.e., $2\omega \rightarrow (\omega, \omega)$, of light, as one may infer by considering the NL polarizations at frequencies $2\omega$ and $\omega$:

$$P_i^{\mathrm{NL}}(2\omega) = \varepsilon_0 \chi_{ijh}(2\omega) E_j(\omega) E_h(\omega)\,, \tag{1a}$$

$$P_h^{\mathrm{NL}}(\omega) = \varepsilon_0 \chi_{hij}(\omega) E_i(2\omega) E_j^{\,*}(\omega)\,. \tag{1b}$$

In Eqs. (1) the $\mathbf{E}$ represent the electric fields at $\omega$ and $2\omega$ in the NL medium and the $*$ indicates the complex conjugate.

An incident field at frequency $\omega$ gives rise to an electromagnetic field at the same frequency in the NL medium and, according to Eq. (1a), to the NL polarization at $2\omega$. The other situation, when only a $2\omega$ field is incident upon the coupler, is different: the $\omega$ field entering the expression of the NL polarization [Eq. (1b)] does not originate from an incident field at that frequency but can be generated only by the $\chi^{(2)}$ interaction itself. If the $\chi^{(2)}$ medium is the guiding layer of a prism or grating coupler, we anticipate that illuminating these $\chi^{(2)}$ optical resonators not only with the $\omega$ beam, as in Ref. 10, but also with the second harmonic may lead to different results. One of the goals of this Letter is to investigate the conditions under which this $2\omega \rightarrow (\omega, \omega)$ interaction is possible in $\chi^{(2)}$ distributed couplers. More generally, we study what happens when the two fields at $\omega$ and $2\omega$ are incident upon such devices. It is shown that the existence of a pump beam at $2\omega$ gives rise to new operating regimes.

As in Ref. 10, we are interested in sharp resonances, and we assume that the operating point remains in the immediate proximity of a given isolated resonance. We consider the situation in which resonance occurs at $\omega$ and at $2\omega$ under phase matching. We investigate the plane-wave regime and, for the sake of simplicity, we consider low-loss (dielectric losses and electromagnetic leakage) resonators. The definitions and notation are those of Ref. 10. Under these conditions, the reduced amplitudes $F_1$ and $F_2$ of the guided waves, at $\omega$ and $2\omega$, respectively, obey the following steady-state equations[10]:

$$-jF_1 + F_2 F_1^{\,*} = -A_1\,, \tag{2a}$$

$$-jF_2 + F_1^{\,2} = -A_2\,, \tag{2b}$$

In Eqs. (2), $A_1$ and $A_2$ denote the reduced amplitude of the incident beams at $\omega$ and $2\omega$, respectively. Equations (2) yield

$$[1 + |A_2|\exp(-j\Delta\varphi) + |F_1|^2]F_1 = -jA_1\,, \tag{3a}$$

with $F_1 = |F_1|\exp(j\phi_1)$, $A_2 = |A_2|\exp(j\varphi_2)$, and $\Delta\varphi = 2\varphi_1 - \varphi_2$. Equation (3a) leads to

$$I_{1,g}^{\,3} + 2aI_{1,g}^{\,2} + bI_{1,g} = |A_1|^2, \tag{3b}$$

with $I_{1,g} = |F_1|^2$, $a = 1 + |A_2|\cos\Delta\varphi$, and $b = a^2 + |A_2|^2\sin^2\Delta\varphi$.

Equation (3b) shows that the guided-wave response at $\omega|F_1(|A_1|)|$ exhibits optical bistability (OB), provided that the following conditions hold:

$$(5\pi/6) < \Delta\varphi < (7\pi/6)\,, \tag{4a}$$

$$|A_2| > \frac{1}{|\cos\Delta\varphi| - \sqrt{3}\,|\sin\Delta\varphi|}\,. \tag{4b}$$

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Thus, as shown in Fig. 1, prism or grating couplers illuminated by both $\omega$ and $2\omega$ fields may exhibit phase-matched OB. Two points are worth noting: for $\Delta\varphi = 180°$ nonzero values of $|F_1|$ exist even with zero input at $\omega$; the OB loops in Fig. 1 are obtained assuming no phase mismatch. Because retuning to resonance is one of the requirements for OB and the system is already tuned to the guided-wave resonance at $\omega$ and at $2\omega$, the question is: Which resonance and which retuning scheme are involved in this OB process? To answer this question it is convenient to rewrite Eq. (3b) to deal with the NL transmission $T^{\mathrm{NL}}$, defined as follows:

$$T^{\mathrm{NL}} = \frac{I_{1,g}}{|A_1|^2} = \frac{1}{U^2 + b - a^2}\,, \tag{5a}$$

$$U = I_{1,g} + a\,. \tag{5b}$$

$T^{\mathrm{NL}}$ is maximum for

$$I_{1,g} = I_{1,g,\mathrm{res}} = -a\,, \tag{6a}$$

which implies that

$$(\pi/2) < \Delta\varphi < (3\pi/2)\,, \tag{6b}$$

$$|A_2| > \frac{1}{|\cos\,\Delta\varphi|}\,. \tag{6c}$$

The peak value $T^{\mathrm{NL}}_{\max}$ is given by

$$T^{\mathrm{NL}}_{\max} = \frac{1}{|A_2|^2\,\sin^2\,\Delta\varphi}\,. \tag{6d}$$

Therefore, when inequalities (6b) and (6c) apply, Eq. (6d) shows that $T^{\mathrm{NL}}_{\max} \to \infty$ for $\Delta\varphi = \pi$. This indicates that downconversion $2\omega \to (\omega, \omega)$ is possible even with no incident beam at $\omega$. One can easily verify this by considering Eq. (3a), which shows indeed that a solution $F_1 \neq 0$ with $A_1 = 0$ exists, provided that the two following conditions are fulfilled:

$$\Delta\varphi = \pi\,, \tag{7a}$$

$$|F_1|^2 = |A_2| - 1\,. \tag{7b}$$

The latter equation corresponds to Eq. (6a) and requires that

$$|A_2| > 1\,. \tag{7c}$$

When Eqs. (7a) and (7b) are satisfied, Eq. (2a) leads to

$$|F_2| = 1\,. \tag{7d}$$

Expressions (7) are the conditions in which self-excited oscillations at $\omega$ are possible in prism or grating couplers. As long as the incident power at $2\omega$ is below the threshold value $|A_2| = 1$, there is no electromagnetic field at $\omega$, and the intensity $|F_2|^2$ of the $2\omega$ guided wave is proportional to $|A_2|^2$ [Eq. (2b)]. When inequality (7c) is fulfilled, $|F_2|^2$ remains clamped to the value given by Eq. (7d), independently of $|A_2|^2$, and the intensity $|F_1|^2$ of the $\omega$ guided wave grows linearly with $|A_2|$ [Eq. (7b)].

Considering Eq. (5a), it is seen that the curve $T^{\mathrm{NL}}(U)$ is a Lorentzian centered in $U = 0$ with half-width at half-maximum $\delta$ equal to $\delta = \sqrt{b - a^2}$. This Lorentzian is associated with a NL resonance with conditions of existence given by inequalities (6b) and (6c). Figure 2(a) is a plot of $T^{\mathrm{NL}}(U)$ with the values of $\Delta\varphi$ and $|A_2|$ corresponding to those in Fig. 1(b). Besides, one notices that the minimum value of $U$ is $U_{\min} = a$. Thus the curves in Fig. 2(a) begin at point S, in the region $U < 0$ (existence of a NL resonance) when inequalities (6b) and (6c) are fulfilled and in the region $U > 0$ (no NL resonance) when these inequalities do not apply. Equation (5a) and Fig. 2(a) suggest an alternative way to derive the threshold condition for phase-matched OB: the intersection, denoted $U_{\mathrm{int}}$ [Fig. 2(a)] (in the range $U < 0$) of the tangent to the curve $T^{\mathrm{NL}}(U)$ in its inflection point with the horizontal axis must satisfy

$$|U_{\min}| > |U_{\mathrm{int}}|\,. \tag{8a}$$

Inequality (8a) leads to inequalities (4). Defining $\Delta$ as the detuning for $A_1 = 0$, i.e., $|\Delta| = |a|$, we can cast inequality (8a) in the form

$$|\Delta| > |\Delta_{\mathrm{th}}|\,, \tag{8b}$$

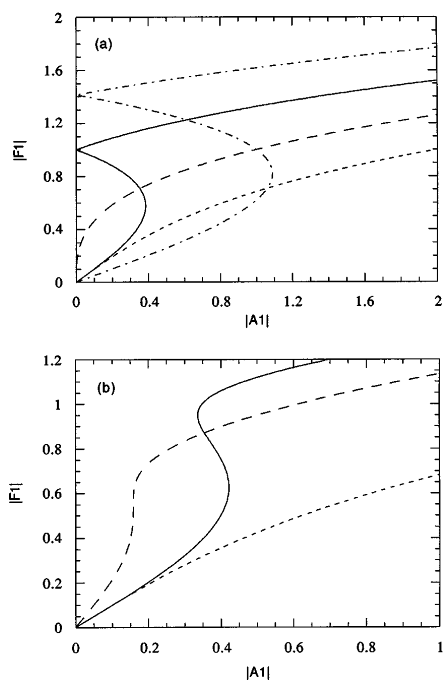


Fig. 1. Response of a $\chi^{(2)}$ optical resonator for $\Delta\varphi = 180°$ and $\Delta\varphi = 190°$. (a) $\Delta\varphi = 180°$: $A_2 = 0$ (short-dashed curve), $A_2 = 1$ [long-dashed curve, corresponding to the threshold value deduced from inequality (4b)], $A_2 = 2$ (solid curve), $A_2 = 3$ (dotted–dashed curve). (b) $\Delta\varphi = 190°$: $A_2 = 0$ (short-dashed curve), $A_2 = 1.4619022$ [long-dashed curve, corresponding to the threshold value deduced from inequality (4b)], $A_2 = 2$ (solid curve).

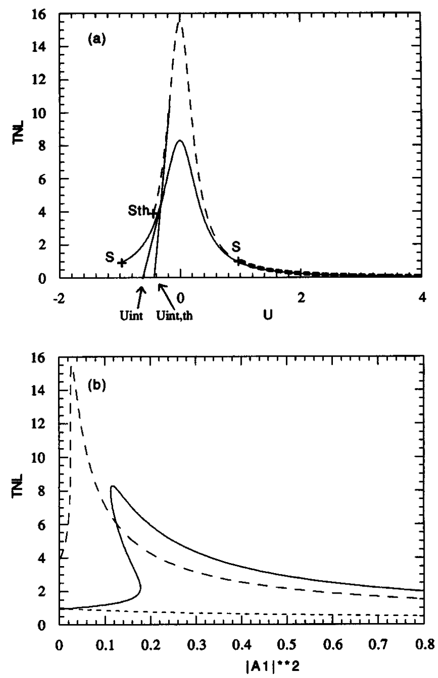


Fig. 2. $\Delta\varphi = 190°$. The values of $A_2$ and the captions are the same as in Fig. 1(b). (a) Nonlinear transmission $T^{NL}$ as a function of $U$. The $A_2 = 0$ curve is traced with thicker dashes to distinguish it from the two other curves. $S_{th}$ and $U_{int,th}$ are points S and $U_{int}$, respectively, for the threshold curve. The straight lines represent the tangent to the curves $T^{NL}(U)$ in their inflection point. (b) Nonlinear transmission $T^{NL}$ as a function of $|A_1|^2$.

where $\Delta_{th}$ is the threshold detuning: $|\Delta_{th}| = |U_{int,th}|$. Therefore, for OB, starting point S must be located at the left of point $S_{th}$ [Fig. 2(a)]. Figure 2(b), which is a plot of $T^{NL}(|A_1|^2)$, together with Fig. 2(a), demonstrates the link between the NL resonance and phase-matched OB.

We are now in a position to understand the origin of this kind of OB: there exists a nonlinear resonance that may be excited, provided that $\Delta\varphi$ fulfills inequality (6b) and $|A_2|$ is larger than a threshold value [inequality (6c)], which depends on $\Delta\varphi$. It follows that this nonlinear resonance does not exist when the system is pumped by the $\omega$ beam only [Fig. 2(a)]. On the other hand, OB requires that inequalities (4) rather than (6b) and (6c) be fulfilled. The OB process stems from the excitation of this NL resonance: at low $\omega$ incident power the NL resonance is detuned; the retuning takes place when, with increasing $A_1$, the value $I_{1,g} = I_{1,g,\mathrm{res}} = -a$ is reached. Inequalities (4) ensure the existence of the NL resonance together with a minimum detuning.

The phenomena investigated arise from the term $|A_2|F_1 \exp(-j\Delta\varphi)$ in Eq. (3a). Therefore they cannot be described with the concept of an effective cascaded nonlinearity $\chi_{\mathrm{eff}}^{(3)}$ as in Ref. 10. A useful value for the onset of these phenomena is $|A_2| = 1$, corresponding to incident intensities at $2\omega$ in the range $10^8$–$10^9$ W/cm$^2$ [we use the numerical values of Ref. 10 and $\chi^{(2)} = 70$ pm/V]. Despite the high field intensities required at $2\omega$, the transverse field map at either $2\omega$ or $\omega$ still corresponds to the linear ones,[11] implying that we do not operate in a strongly NL regime.

Optical bistability was observed recently in a triply resonant optical parametric oscillator.[9] The OB loops reported in Ref. 9 require some detuning. Thus that kind of bistability is different from the one studied in this Letter.

In conclusion, we have shown that illuminating $\chi^{(2)}$ distributed couplers with both $\omega$ and $2\omega$ beams may yield OB. It might seem surprising that this phenomenon of OB occurs with no detuning. But detuning is required, not with the usual linear electromagnetic resonance but with the new NL resonance.

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