

Mystery of the double limit in homogenization of finitely or perfectly conducting periodic structures

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We show that the difference in the effective medium properties of perfectly or finitely conducting short-period gratings is because for finitely conducting gratings it is possible to completely homogenize the electromagnetic field vector components, contrary to perfectly conducting gratings. As a consequence, for aluminum in the microwave domain, two possible effective media can be found, depending on whether the feature dimensions are much larger or shorter than the depth of field penetration inside the metal. © 2007 Optical Society of America

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The problem of homogenization of a given optogeometrical structure finds its foundations in the homogenization of the microscopic Maxwell equations to hide all the atomic nature of microcurrents and polarizations in a few macroscopic parameters. Starting with the fundamental work of Maxwell-Garnet [1], this domain presents many problems yet goes unsolved (see, for example [2,3], and references cited therein). Many attempts were made to obtain simple rules for homogenizing grating structures with a sub-wavelength period [4–7]. It has been shown that for nonmagnetic media the effective dielectric permittivity becomes anisotropic due to the geometrical anisotropy of the diffraction grating. However, numerical and theoretical works have shown that the values of this effective permittivity differ for finitely and perfectly conducting materials [5,6]. Due to these differences, the authors of [8] were obliged to consider that the effective properties of the homogenized system are characterized by both dielectric permittivity ε and magnetic permeability μ being anisotropic, even for a nonmagnetic system.

The aim of this Letter is to propose an explanation for this old mystery. If the thickness of the metallic features becomes much smaller than the wavelength of light, but still larger than the penetration (skin) depth of light, one obtains the perfectly conducting homogenization limit. However, this limit is not an absolute but an intermediate one, because when the feature dimensions are further reduced to become smaller than the skin depth, this limit is gradually transferred into the limit for a finitely conducting material.

Let us consider a lamellar metallic grating (the inset of Fig. 1) made of aluminum with relative permittivity ε_2 and illuminated in TM polarization under normal incidence. The groove is filled with air ($\varepsilon_1 = 1$). The period is d , the groove width is d_1 , the lamella width is d_2 , and the filling factor is $f_2 = d_2/d$. We consider two cases, one in the microwave (MW)

domain (wavelength of the order of 2 cm), where aluminum can be considered as almost perfectly conducting, with $\varepsilon_2 = -10^5 + i10^7$, the second case in the visible, where the conductivity is much smaller, with $\varepsilon_2 = -56.1 + i21$ at wavelength $\lambda = 0.6328 \mu\text{m}$. The filling ratio is equal to 0.5. When the groove period is small enough, no diffracted order can be supported by the grating. Its reflectivity is shown in Fig. 1 as a function of the grating period-to-wavelength ratio for three different cases, perfectly conducting (p.c.) or aluminum grating in the visible and in the MW region, for the case of a dielectric substrate ($\varepsilon_S = \varepsilon_1$). The lamellae height is equal to the quarter-

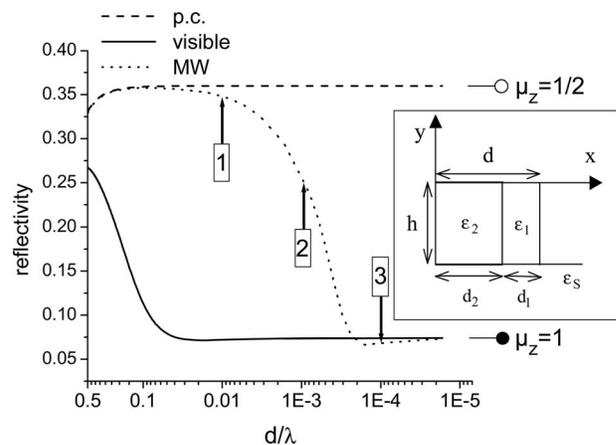


Fig. 1. Reflectivity as a function of the period for three different lamellae materials, p.c., aluminum in the MW, and in the visible. Substrate, cladding, and grooves are air, and the grating height is $h = \lambda/4$. Normal incidence is in TM polarization. Labeled points are analyzed in Fig. 2. Empty circle represents the reflectivity level of a homogeneous layer of the same thickness with anisotropic permittivity ($\bar{\varepsilon}_x = 2, \bar{\varepsilon}_y = \bar{\varepsilon}_z = \infty$) and anisotropic permeability ($\bar{\mu}_x = 1, \bar{\mu}_y = \bar{\mu}_z = 1/2$). The full circle represents the homogeneous layer with anisotropic permittivity and isotropic permeability ($\bar{\mu}_x = \bar{\mu}_y = \bar{\mu}_z = 1$). The inset represents schematically the diffraction grating.

wavelength, $h = \lambda/4$. The calculations for finitely conducting material are made using the rigorous coupled-wave (RCW) method [9,10]. Perfect conductivity is modeled by using the rigorous modal method [11,12].

For highly (or perfectly) conducting lamella material, a saturation level is reached quite rapidly for periods smaller than $d < \lambda/10$, while in the visible it is necessary to reduce the period to less than $\lambda/50$ to obtain saturation, and the limit values are quite different. The limit value in the visible maintains stable, whatever small the features are (although it is clear from a physical point of view that dimensions smaller than several angstroms cannot be treated using macroscopic permittivity values). More amazing is the behavior of the highly, but finitely, conducting grating. At first, one observes a saturation value equal to the value of perfectly conducting material. However, with the further reduction of the period, the reflectivity changes gradually to approximately approach the finitely conducting limits. The physical origin of this difference can be understood when analyzing in Fig. 2 the horizontal distribution of the z component of the magnetic field in the middle of the groove height for the aluminum grating in the microwave domain at three different characteristic points labeled 1, 2, and 3 in Fig. 1. In the first point, with $d/\lambda = 0.01$, there is no homogenization of H_z across the grooves. This is also valid for perfectly conducting lamellae, as whatever their width, the electromagnetic field vanished inside them. This fact explains why, for relatively larger grooves, the highly conducting grating behaves as if the conductivity is infinite. However, for finite conductivity, the field always penetrates inside some thin layer at a depth known as a skin depth. As the groove period is reduced further, the lamellae width becomes comparable with the skin depth t_s (of the order of $1 \mu\text{m}$, i.e., $\lambda/t_s \sim 10^{-4}$), and the field inside them cannot be neglected, as can be observed in Fig. 2 for the point labeled 2 in Fig. 1 with $d/\lambda = 0.001$. Thus, it is necessary to reduce further the groove period to obtain a homogenization of the field, as shown for point 3 with $d/\lambda = 10^{-4}$. As a consequence, when the conductivity is very high, one can observe two saturation values when the period is reduced. First, when the groove dimensions are much smaller ($\sim 1/20$) than the wavelength, but much

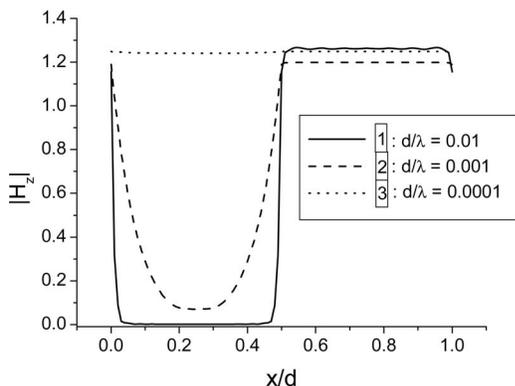


Fig. 2. X dependence of the z component of the magnetic field for three different groove periods, labeled in Fig. 1, calculated at the middle height of the lamellae ($y = -h/2$).

larger than the skin depth, there is an intermediate saturation value, characterized by an electromagnetic field, which is constant inside the grooves but vanishes inside the lamellae, a saturation value that corresponds to the homogenized limit for perfectly conducting materials. This structure is equivalent to an effective medium having both anisotropic electric permittivity and the magnetic permeability [8]. When the features become comparable in dimension with the skin depth, a second saturation level can be reached, corresponding to a perfectly homogenized field, and equivalent to a homogeneous layer with electrical anisotropy, but isotropic magnetically, as discussed in the next paragraph. In the case of aluminum in the visible, the skin depth is of the order of $\lambda/50$ (i.e., 13 nm), so that for $d \sim \lambda/20$ the half-lamellae width is larger than the skin depth, and the first saturation cannot be observed in Fig. 1.

The effective permittivity and permeability can be determined according to different models [4–8], which give the same results for the effective permittivity but differ for the permeability in the case of perfect conductivity. The boundary conditions for the electric and magnetic fields on the vertical lamellae walls impose the continuity of the tangential components of electric and magnetic field vectors. In particular, the continuity of E_y results in an effective permittivity in y direction given by the mean arithmetic value of permittivity $\bar{\epsilon}_y = f_1 \epsilon_1 + f_2 \epsilon_2$, tending in modulus toward ∞ when $|\epsilon_2| \rightarrow \infty$. In a similar manner, the continuity of the normal component of electric induction $D_x = \epsilon E_x$ requires that in the x direction the mean permittivity $\bar{\epsilon}_x$ is equal to the harmonic mean value:

$$\bar{\epsilon}_x = \frac{\epsilon_1 \epsilon_2}{f_1 \epsilon_2 + f_2 \epsilon_1} \rightarrow \frac{\epsilon_1}{f_1} \quad (1)$$

The continuity of H_z for finitely conducting materials results in the effective magnetic permeability in the z direction that is given by the mean arithmetic value $\bar{\mu}_z = (f_1 \mu_1 + f_2 \mu_2)$. For a magnetically homogeneous structure ($\mu_1 = \mu_2$), this gives simply that the mean permeability is equal to

$$\bar{\mu}_z = \mu_1 \quad (2)$$

However, this is not the case for perfectly conducting lamellae, where the magnetic field vanishes completely inside the lamellae, as observed in Fig. 2 for point 1 of Fig. 1. This means that H_z cannot be completely homogenized, however small the period may be, i.e., in the case of perfect conductivity, the electromagnetic field “sees” the structure whatever its fineness, so that one can expect homogenization of the field only inside the groove region. As the values of H_z vanish inside, the effective permeability is decreased by a factor equal to $f_1 (< 1)$, when compared to Eq. (2):

$$\bar{\mu}_{z,p.c.} = f_1 \mu_1 \quad (3)$$

The authors of [8] propose a more physical explanation of this reduction by using a modal presentation of the field inside the grooves having perfectly conducting walls, with a mode having a constant of

propagation in the y direction equal to $k_{y,p.c.} = k_0\sqrt{\varepsilon_1\mu_1} = k_0\sqrt{\bar{\varepsilon}_x\bar{\mu}_z,p.c.}$ [see Eqs. (1) and (3)]. When using Eqs. (1) and (2) instead of Eqs. (1) and (3), the y -propagation constant of the cavity mode is increased by a factor of $1/\sqrt{f_1}$, a difference due to the penetration of an electromagnetic field inside the lamellae.

Similar considerations can be applied in TE polarization for the other field components. As a result, the small-period perfectly conducting grating (or the highly conducting grating in the intermediate saturation region) is equivalent to a homogeneous layer having both electric and magnetic anisotropy ($\bar{\varepsilon}_x = \varepsilon_1/f_1$, $|\bar{\varepsilon}_y| = |\bar{\varepsilon}_z| = \infty$, $\bar{\mu}_x = \mu_1$, $\bar{\mu}_y = \bar{\mu}_z = \mu_1 f_1$). The reflectivity of such layers is presented with a hollow circle in Fig. 1. Finitely conducting gratings with features smaller than the skin depth is equivalent to a homogeneous layer only with electrical anisotropy, $\bar{\varepsilon}_x = \varepsilon_1\varepsilon_2/(f_1\varepsilon_2 + f_2\varepsilon_1)$, $\bar{\varepsilon}_y = \bar{\varepsilon}_z = f_1\varepsilon_1 + f_2\varepsilon_2$, $\bar{\mu}_x = \bar{\mu}_y = \bar{\mu}_z = \mu_1$. The reflectivity of such layers is represented by a black circle in Fig. 1.

Understanding this difference can play an important role in the study of metamaterials, where feature dimensions vary in several orders of magnitude, and their applications tend to extend from the MW to the visible. Let us consider the same grating as above, but with a metallic substrate, made of the same material as the lamellae. Figure 3 presents the dependence of the real part of the propagation constant k_p in the x direction of the spoof plasmons (for perfectly conducting lamellar grating) or of the real and imaginary parts of the canonical plasmon surface wave (for aluminum grating in the MW domain) as a function of the period-to-wavelength d/λ ratio. The groove depth is $h = \lambda/6$. As can be observed, the behavior is similar to Fig. 1. The saturation value for a perfectly conducting grating is reached for relatively larger periods ($\sim \lambda/10$), and the value can be determined for the surface wave guided by the equivalent layer having electrical and magnetic anisotropy [8]. The same is observed for the aluminum grating but for groove periods larger than $\lambda/500$. Further reduction of the dimensions causes the relatively stronger penetration of the field inside the lamellae (as shown in Fig. 2), and thus an increase of the losses. With the further decrease of the period, another limit is reached with the complete homogenization of the field, corresponding to the plane layer with only electrical anisotropy. There is an interesting difference between Figs. 1 and 3 in the characteristic value of the groove period corresponding to the

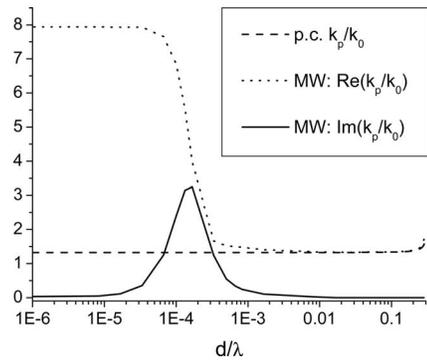


Fig. 3. Variation of the normalized plasmon surface wave propagation constant k_p/k_0 as a function of the period-to-wavelength ratio d/λ for a lamellar grating with metallic substrate, and groove depth $h = \lambda/6$. Dashed line, perfect conductivity; dotted curve, real part of k_p/k_0 for aluminum grating in the MW ($\varepsilon_2 = -10^5 + i10^7$); solid curve, its imaginary part.

transfer from the intermediate saturation value (equal to the perfectly conducting limit) and the saturation value characteristic to the complete homogenization case. In Fig. 1, the middle of the transition interval occurs at about $d/\lambda = 10^{-3}$, while in Fig. 3 it is close to 10^{-4} , a difference probably due to the different penetration of the electromagnetic field inside the lamellae in the two cases of an incident wave and plasmon propagation.

References

1. J. C. Maxwell-Garnet, *Philos. Trans. R. Soc. London, Ser. A* **203**, 385 (1904).
2. T. Mackay, *J. Nanophotonics* **1**, 019501 (2007).
3. T. Mackay and A. Lakhtakia, *Opt. Commun.* **234**, 35 (2004).
4. P. Yeh, A. Yariv, and C.-S. Hong, *J. Opt. Soc. Am.* **67**, 423–448 (1977).
5. P. Yeh, *Opt. Commun.* **26**, 289 (1978).
6. R. McPhedran, L. Botten, M. Craig, M. Nevière, and D. Maystre, *Opt. Acta* **29**, 289 (1982).
7. G. Bouchitte and R. Petit, *Electromagnetics* **5**, 17 (1985).
8. F. J. Garcia-Vidal, L. Martin-Moreno, and J. B. Pendry, *J. Opt. A, Pure Appl. Opt.* **7**, S97 (2005).
9. M. G. Moharam and T. K. Gaylord, *J. Opt. Soc. Am.* **72**, 1385 (1982).
10. P. Lalanne, *J. Opt. Soc. Am. A* **14**, 1583 (1997).
11. C. Botten, M. S. Craig, R. C. McPhedran, J. L. Adams, and J. R. Andrewartha, *Opt. Acta* **28**, 413 (1981).
12. C. Botten, M. S. Craig, R. C. McPhedran, J. L. Adams, and J. R. Andrewartha, *Opt. Acta* **28**, 1087 (1981).