Analytical model of the optical response of periodically structured metallic films: Comment

Evgeny Popov and Michel Nevière

Institut FRESNEL, Unité Mixte de Recherche associée au Centre National de la Recherche Scientifique n°6133, Faculté des sciences et techniques de Saint-Jérôme, case 161, Avenue Escadrille Normandie Niemen,13397 MARSEILLE CEDEX 20, France

e.popov@fresnel.fr

Abstract: A comparison is made between rigorous numerical results and a recently proposed analytical method for modeling light diffraction by lamellar diffraction gratings. The conclusion is that the analytical model is quite restrained in its applicability and can be misleading in determining the behavior of the grating efficiencies.

©2006 Optical Society of America

OCIS codes: ((050.0050) Diffraction and gratings; (2400240) Optics at surfaces; (240.6680) Surface plasmons; (240.7040) Tunneling

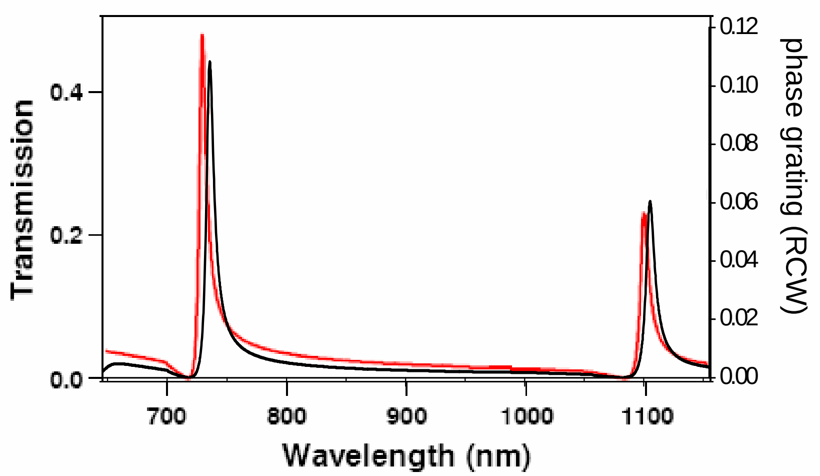
References and links

1. A. Benabbas, V. Halté, J.-Y. Bigot, "Analytical model of the optical response of periodically structured metallic films," Opt. Express **13**, 8730-8745 (2005)
2. Lord Rayleigh O. M., "On the dynamical theory of gratings," Proc. Phys. Soc. (London) A **79**, 399-416 (1907)
3. S. T. Peng, T. Tamir, and H. L. Bertoni, "Theory of periodic dielectric waveguides," IEEE Thans. Microwave Theory Techn. **MTT-23**, 123-133 (1975)
4. E. I. Krupitsky and B. C. Chernov, "Rigorous analysis of arbitrary slanted volume holographic gratings," Proc. IX All-Union School of Holography, Leningrad, 84-85 (1977), *in Russian*.
5. G. Moharam, and T. K. Gayrold, "Rigorous coupled-wave analysis of planar-grating diffraction," J. Opt. Soc. Am. **71**, 811- 818 (1981), "Rigorous coupled-wave analysis of dielectric surface-relief gratings," J. Opt. Soc. Am. **72**, 1385-1392 (1982)
6. M. Nevière and E. Popov, *Light Propagation in Periodic Media, Differential Theory and Design* (Marcel Dekker, New York, 2003)

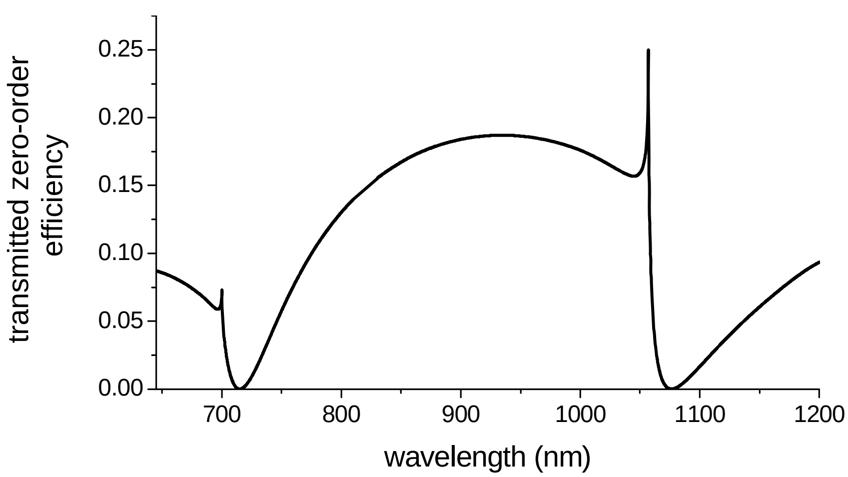
A recent paper by Benabbas *et al*. [1] has proposed an analytical model for treating the diffraction by periodic slits in metallic films. While such a task is not *a priori* impossible, the existence of such a reliable analytical method should throw a shadow on the already one-century-old attempts to build an electromagnetic theory of grating efficiencies, started with the early work of Lord Rayleigh [2] published in 1907.

The authors of ref.1 use the well-known rigorous coupled-wave (RCW) theory [3-5] to develop it in a first-order approximation with respect to the Fourier components of the permittivity ε, subjected to abrupt variations inside the grating modulated region. In fact, the initial lamellar diffraction grating is replaced by a phase grating with sinusoidal modulation of ε in a direction perpendicular to the grooves. The second approximation consists of severely truncating the number of diffraction orders taken into account in the calculations, preserving only the 0-th and the ± 1-st orders. While these approximations could apply for a low-modulated phase grating or a low-contrast lamellar dielectric grating, they are very inaccurate when considering metallic gratings. For example, in the near-IR, a silver grating with a filling factor equal to 2/7 would be equivalent to a phase grating having mean value of equal to $\varepsilon_{\text{mean}} \approx -17 + i0.56$ and modulated with $\Delta\varepsilon \approx 6 - i0.2$, the exact values varying with the wavelength and the groove aspect ratio.

Using two well-established rigorous numerical methods, namely the RCW method and the classical differential method [6], we made a comparison between our numerical results and some results presented in Ref. [1]. Figure1(a) presents a superposition of Fig. 3 of Ref. [1] and the rigorous results obtained for a phase grating with a sinusoidal modulation of the dielectric permittivity, as described in Eq. (30) of Ref [1]. In addition to the slight variation of the position of the resonant maxima (maxima due to the excitation of surface plasmons on the upper and lower grating interface), it is necessary to stress out the 4-fold discrepancy between the transmission values (note the different scales on the left and right). Thus the conclusion that the analytical results cannot be reliable even if one could assume the equivalence between a metallic lamellar grating and a phase one. Moreover, such an equivalence is not valid at all, as illustrated in Fig. 1(b), which presents the rigorous results for the true lamellar (slit) grating. The transmission behavior is completely different from the simple Fano-like resonances observed in Fig. 1(a).



(a)

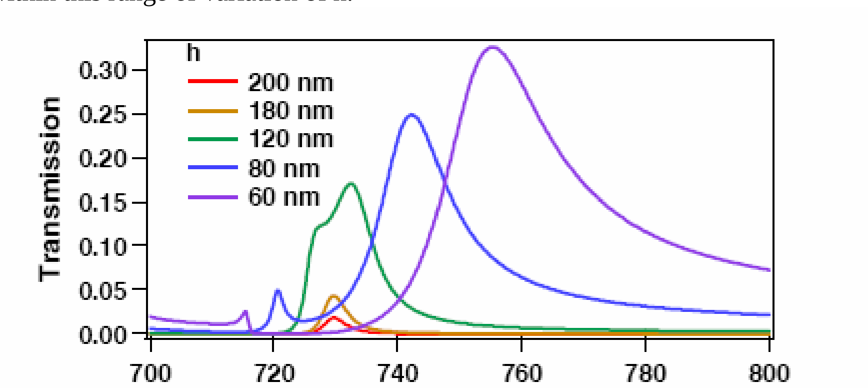


(b)

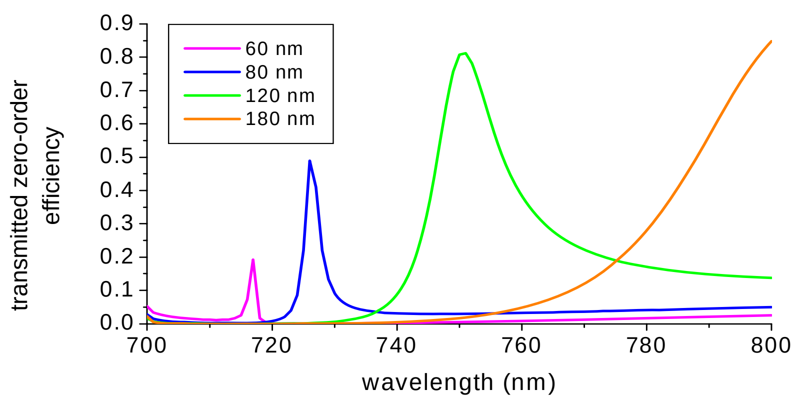
Fig. 1. . Zeroth transmission order efficiency of a 110 nm thick silver grating on a glass substrate at normal incidence in TM polarization: Groove period equal to 700 nm, groove width 200 nm. (a) Equivalent phase grating with a sinusoidal modulation of electric permittivity. Red curve and the left scale, results of the analytical method, ref.[1], black curve and the right scale, rigorous numerical results. (b) The rigorous numerical results for a lamellar silver grating.

Received 8 November 2005; accepted 6 December 2005
(C) 2006 OSA

A similar or even more pronounced difference between the analytical and the rigorous results exists when a symmetrical structure (substrate the same as the cladding) is analysed. The advantage of such a structure is that the plasmons at the two grating interfaces are identical and thus much strongly coupled than in the asymmetrical geometry. Figure 2 presents a comparison between the analytical results given in Fig. 6(a) of [1] and the rigorous results obtained for a true lamellar grating. The difference is not only in the position and the height of the resonant maximum, but also in the tendancy. While the analytical model predicts a blue shift and decrease of the maximum with the grating thickness h, the lamellar grating exhibits the opposite behavior, the transmission increasing and presenting a strong red shift, at least within this range of variation of h.



(a)



(b)

Fig. 2. The same as in fig.1 but for a symmetric structure, cladding, grooves and substrate having optical index of 1. Results for different groove depth (grating thickness), as indicated in the figures. (a) Analytical results reproduced from ref.[1]. (b) numerical results for a true lamellar profile.

Results that are not important in the context of the comparison between the two approaches show that further increase of the groove depth leads again to a red shift with decrease of maxima and then a quasiperiodical behavior is observed when the groove depth is varied within the range of several wavelengths. This peculiarity is due to the existence of the TEM hollow-waveguide mode inside the slits that propagates without cut-off and is leads to Fabry-Perot like resonances as function of the waveguide length (i.e., the groove depth h)

The conclusion is that the analytical model presented in [1] could present results which significantly differ from the behavior of the real grating under study and could be misleading in drawing physical conclusions. Thus the usefulness (and necessity) of rigorous grating theories.