ELECTRICALLY INDUCED STRIP WAVEGUIDE MODES

I. SAVATINOVA, V. TONCHEV, E. POPOV and L. MASHEV
Institute of Solid State Physics, Bulgarian Academy of Sciences, Blvd. Prof. Bl. Tsankov 3, Sofia 1784, Bulgaria

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Numerical results of the dispersion characteristics and field distribution of the transverse modes of an electrically induced strip waveguide with gaussian refractive index depth profile are presented. Consol.onditions are discussed in connection with the transverse energy localisation.

1. Introduction

Recently Yamamoto et al. [1] have demonstrated experimentally that light can be confined into a thin film only by a partial cladding with metal strips. Far-field patterns of several transverse modes have been demonstrated using Al$_2$O$_3$ layer on a SiO$_2$/Silicon substrate. In the same time, an electrically induced optical waveguide has been fabricated using a pair of electrodes deposited on electro-optic single crystals. The applied voltage (4 V/µm in the case of LiNbO$_3$ [2] and 3 V/µm in the case of KNbO$_3$ [3]) provides cut-off modulation.

Replacing the pure LiNbO$_3$ substrate with a Ti:LiNbO$_3$ waveguide, such electro-optic effect can be observed at much lower voltages [4], due to the lower localization in the guide depth. It is worth noting in this case both the amplitude and the phase of the guided light propagating in the electrode gap can be changed. An effective amplitude modulation and beam-splitting have been demonstrated in ref. [4].

The purpose of this paper is to investigate the characteristics of electrically induced strip waveguide transverse modes. The influence of the metal electrodes and the applied voltage on the effective refractive indices and the field distribution of the modes are discussed in detail.

2. Presentation of the physical problem

The structure under consideration is presented schematically in Fig. 1. The waveguide and electrode parameters at wavelength λ = 0.6328 µm are given in Table 1. We have considered a planar Y-cut Ti:LiNbO$_3$ waveguide with a gaussian refractive index profile

$$n(z, x) = n_0 + d_0 \exp(-x^2/D^2),$$

(1)

where $D$ is the waveguide effective thickness, $n_0$ and $d_0$ are the waveguide and electrode parameters, respectively.

![Fig. 1. An electrically induced channel in Ti:LiNbO$_3$ planar waveguide.](image)

Table 1: Structure parameters

<table>
<thead>
<tr>
<th>waveguide</th>
<th>electrodes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_0 = 2.203$</td>
<td>$d_0 = 30$ µm</td>
</tr>
<tr>
<td>$d_0 = 2.203$</td>
<td>$w = 50$ µm</td>
</tr>
<tr>
<td>$d_0 = 3.20$</td>
<td>$d = 10$ µm</td>
</tr>
<tr>
<td>$d_0 = 3.20$</td>
<td>$D = 5$ mm</td>
</tr>
<tr>
<td>$d_0 = 2.20$</td>
<td>$w' = 30$ µm</td>
</tr>
<tr>
<td>$d_0 = 2.20$</td>
<td>$w'' = 60$ µm</td>
</tr>
</tbody>
</table>
$n_0$ are the refractive indices at the surface and of the substrate respectively and $2n_0 = n_0 + n_m$. The set of waveguide parameters provides the existence of only one $T_{00}$ mode with an effective refractive index $n_{e0}$. An applied voltage $U$ between the two electrodes increases the waveguide refractive index according to

$$\Delta n(\tau, z) = \frac{U}{2} \Delta n_0^* \frac{\tau}{z}$$

where the electric field distribution is given by [5]

$$E_z = \frac{U}{2} K(\omega d) \text{Re} \left[ \left( \frac{1}{1 - (\omega d)^2} \right)^{1/2} \left( \frac{1}{1 - (\tau z)^2} \right)^{1/2} \right]$$

where $K$ is the elliptical integral of the first kind, and $\tau = \tau - \tau_0$.

In the electrically induced strip waveguide, light tends to propagate in the gap between the electrodes because of the $\Delta n$-channel formed by the additional influence of the metal cladding $\Delta n_0^*$ and the electrooptic effect $\Delta n_0^*$. Therefore, the external electric field can be used to enhance the optical strip line effect. The shape of the lateral refractive index profile is similar to that of the so-called W-fibers, where a high index core is surrounded by two layers with different indices. Two symmetric potential barriers round the central gap are formed under the metal-clad region. Such a system is a self-filtering waveguide [6] which has the higher-order modes in our configuration all modes are more or less leaky, the fundamental one being the best confined. Thus the channel contains a small amount of modes with effective mode indices $n_0^* > n_0$, where $n_0^* = n_0 + n_0^*$ is the depth mode effective index value under the metal layer.

3. Numerical procedure

Since in the presence of electrodes the refractive index distribution is a function of two variables, $y$ and $z$, the problem becomes two-dimensional (2-D). A set of 2-D partial differential equations can be solved rigorously by a 2-D numerical integration using the finite-difference method. In the case of small lateral refractive index fluctuations, however, the method of effective refractive indices [7] can be applied efficiently for a step-index distribution. In order to fit it to the case of gradient index profile, we have combined the effective index method with the matrix method, used for analysis of multi layered planar waveguides [8]. The calculation scheme is the following.

(i) The Gaussian refractive index profile of the planar waveguide is represented as a step-like $N$-layer structure. Its mode propagation constant is calculated by a matrix method [8]. Table 2 shows the convergence of the effective index to $n_{e0}$ of the initial gradient waveguide as a function of $N$. Furthermore in the calculations we have taken $N = 100$.

(ii) According to eqs. (2) and (3), the applied voltage changes the refractive indices of each of the $N$ layers in the $z$-direction. A depth mode index has been calculated for each value of $z$ in the same way as $\Delta n = 0$, using the corresponding modified values of $n_{e0}$.

The interval $z = [1, 50]$ has been divided into 1000 parts.

(iii) Representing the depth mode index distribution in $z$ as a stack of 1000 layers, the matrix method is applied for the calculation of the transverse mode indices and fields.

(iv) The procedures (iii) and (iii) are repeated, inserting another value of $U$.

Two perturbations have to be pointed out.

(i) Due to the complex refractive index of the aluminum, the mode effective indices are complex too. Thus the calculations of the zero of the characteristic matrix have to be performed in the complex $n_{e0}$ plane. The most powerful method for the search for zeros proves to be Newton iterative procedure, generalized for complex variables.

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(ii) In the investigated region of $U = 0 - 60$ V the depth mode propagation constant is nearly a linear function of $U$ and can be represented in the form $\beta(z, \lambda) = \beta_0(z) + \gamma U \partial \beta_0(z) / \partial U$.

This fact facilitates to a great extent the calculation, since only the values of $\partial \beta_0(z) / \partial U$ as a function of $z$ are sufficient, calculated only at a certain value of $U$.

4. Results

Fig. 1 shows the influence both of the metal coating and the electrostatic effect on the depth mode effective index. The great negative real part of the Al dielectric constant diminishes the depth mode effective index and leads to the partial localization of the optical field. On the other hand $10^4$ across $10 \mu m$ electrode distance provides an index variation $300 \times 10^{-8}$ which is comparable to $\beta_0(z)$, associated with the metal overlay. An important feature of the structure in fig. 2 is that only even modes with maximal field amplitude along the guide axis can be excited. Applying higher voltages, $\beta_0$ increases and some of the leaky modes can pass to the normal ones.

The behavior of $\beta_0(z)$ and $\gamma$ calculated as a function of $U$ is displayed in fig. 3. At low voltages the second even mode is leaky, but at $U \geq 30$ V its effective index exceeds the critical level $\beta_0(z)$.

More impressive is the picture of the mode fields, given in fig. 4. The fundamental mode remains well confined in the voltage interval $0 - 60$ V, but the evolation of the second even mode is drastic. Its field amplitude flows to the electrode gap center rather quickly, making the coincidence with the fundamental...
tilt amplitude more complete. It must be pointed out
that condition \( n_{2} > n_{0} \) is not a rigorous one;

\[ \text{as \: in \: fact \: truly \: bound \: between \: the \: electrodes \: the \: modes} \]

characterized with \( n_{2} > n_{0} \), the only modes that are
not radiated towards \( z \rightarrow -\infty \). Nevertheless, if
\( n_{2} > n_{0} \), the mode field is located near the elec-

trode gap and as its effective index grows up the en-
ergy flow towards \( z \rightarrow -\infty \) becomes more and more
noticeable. Moreover, since the depth of the sym-
metrical potential barriers under the electrodes in-
creases with \( U \), even modes with \( n_{2} = n_{0} \) are
oscillated. In fact the most important criterion is the
ratio between the “bounded” and “radiated” energy.

In fig. 5 the transverse distribution of the intensity of
\( \text{TE}_{0} \) mode is shown in the region \( 0 \leq z \leq 20 \). \( V \).

The following conclusions can be drawn:

1. For \( U = 0 \) the value of the intensity maxi-
mum is comparable with the intensity of the ra-
diated field (the ratio not exceeding a value of \( 4 \));
2. Above \( U > 10 \) that ratio exceeds 10 and the
radiated energy can be neglected, in comparison to
the bounded one between and below the electrodes,
though for \( U > 30 \) \( n_{2} > n_{0} \). Moreover, a peak in
the center region is formed and it can interfere with
the field of \( \text{TE}_{0} \) mode, depending on their phase
difference.

It is well known that the modes of an infinite-length
waveguide can interfere with each other because of
their orthogonality. The presence of a physical bound-
ary (at the waveguide edge), however, cancels this
requirement. The interference between the even
modes results in oscillations of the output intensity
with a high extinction ratio. Thus an amplitude
modulation can occur at the end of the electrode re-

dion in \( z \). If the length of the electrodes is \( L \), and if
we assume a constant and optimum excitation con-
dition for both \( \text{TE}_{0} \) and \( \text{TE}_{2} \) modes, the output
intensity would depend on the difference between
their phases after a path of \( L \) along the electrodes,

\[ \Delta \phi(U, L) = \frac{2 \pi}{L} \left[ n_{0}^{2}(U) - n_{2}^{2}(U) \right] L. \]  

(5)

Assuming an independence of the initial phase dif-
ference \( \Delta \phi(U, 0) \) on \( U \), the voltage change \( \Delta U \) nec-
sary to obtain a transition from a minimum to a
maximum is defined by

\[ \Delta \phi(U + \Delta U, L) - \Delta \phi(U, L) = \pi. \]  

(6)

The results presented in fig. 3 allow us to assume that
\( n_{2}^{2} \) is practically independent on \( U \) in comparison
with the change of \( n_{0}^{2} \). In that case condition (6) can
be represented as

\[ n_{0}^{2}(U + \Delta U) - n_{0}^{2}(U) = \frac{\pi}{2L}. \]  

(7)

Numerical results (fig. 3) enable us to interpolate
\( n_{0}^{2} \) as a linear function of \( U \)

\[ n_{0}^{2}(U) = n_{0}^{2}(0) + qU, \]  

(8)

with

\[ q = 1.1 \times 10^{-4} \text{V}^{-1}. \]  

(9)

Substituting of \( \lambda = 0.6328 \mu \text{m} \) and \( L = 5 \text{ mm} \) results in

\[ \Delta L = \lambda / 2qL = 5.35 \text{ V}. \]  

(10)

To illustrate that phenomenon a waveguide with the
same parameters as in table 1 was prepared. When
a DC voltage is applied to the electrodes, a sequence
of maxima and minima of the output intensity has
been observed (fig. 6). A saw-tooth generator was
used as a voltage source. The output intensity was
measured with a detector located in the center of the
optical fiber. A maximum modulation efficiency
up to 95% was achieved under optimum extinction
conditions. The half-wave voltage was estimated to
be about 7 V, in good agreement with the theoreti-
cal value (10).
Discussion

If a weak non-linearity of $n_e^2(U)$ and a slow variation of $n_e^2(U)$ are taken into account, a small change of the half-wave voltage $V$ with increase of $U$ can be expected. Higher modes excitation would have a weak influence on the process, too.

Despite the obtained agreement between the theoretical and experimental values $\Delta V$, we refrain from the conclusion that there is a quantitative, rather than qualitative, experimental confirmation of the theoretical considerations, because of the existence of one question that needs further investigations: In order to achieve a great value of the modulation depth in the entire voltage range (Fig. 6), the central peaks of $E_{h_n}$ and $E_{h_n}$ mode fields have to be almost identical, contrary to Fig. 4 and Fig. 5. There are two possible explanations:

(i) From an experimental point of view: if the excitation conditions are more optimal for $TE_{h_n}$ mode, it would compensate the greater value of $TE_{h_n}$ field at $z = 0$.

(ii) From a theoretical point of view: the waveguide parameters ($d = 0.0016$ and $D = 0.9 \mu m$) in the calculations may not correspond exactly to the experimental ones, although $n_e^2$ is the same. Furthermore, if the imaginary part of the refractive index $n_2$ of the metal electrodes has a higher value ($n_e^2$ depends on the deposition process), the electrodes can have a stronger influence on $n_e^2$, thus $TE_{h_n}$ can be more confined between them even without applied voltage.

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References


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