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# DIFFRACTION ANOMALIES OF COATED DIELECTRIC GRATINGS IN CONICAL DIFFRACTION MOUNTING

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A theoretical study of the anomaly in the zero-order efficiency caused by waveguide mode excitation is carried out for the case of conical diffraction mounting. The influence of the incident light polarization on the mode excitation is discussed. In particular it is shown that the coupling into the guided wave vanishes at a given polarization. A generalization of the phenomenological approach is presented for conical diffraction mounting.

## 1. Introduction

Near the point of excitation of a guided wave in a coated dielectric grating an abrupt change of the reflectivity of the system occurs [1,2]. The theoretical limit of 100% reflection proves to be a consequence only of the symmetry of the system [1]. Another interesting feature of this phenomenon is that the zeroth order acquires wavelength selectivity. A narrow band tunable optical filter in a reflection regime with a halfwidth of 3 Å has been demonstrated [2].

A very interesting bistable behaviour of the reflected light in the vicinity of guided wave resonance has been reported in refs. [3,4]. However, both the theoretical and the experimental investigations in refs. [1-4] have been carried out for the case when the incident wave vector is perpendicular to the grooves. In some cases it is important to know the behaviour of the system response in conical diffraction mounting. For example, some of the plane gratings in the XUV region, in the monochromators, as well as most of the concave gratings operate when the light propagation direction lies out of the plane perpendicular to the grooves. Very recently, we have discussed the application of the conical diffraction treatment to the mode coupling phenomena in planar corrugated waveguides [5].

The difficulties in the conical diffraction treatment follow from the fact that neither the Maxwell equa-

tions, nor the boundary conditions can be devided into two independent fundamental cases of polarization, thus rigorous electromagnetic theories have to be used.

An extension of rigorous electromagnetic methods based on the integral [6] and differential [7] formalisms have been reported. In this paper we have utilized the approach in ref. [8] to analyse the effect of guided wave excitation in off-plane light incidence.

#### 2. Numerical example

Let us consider a corrugated symmetrical waveguide with  $n_1 = n_3 = 1$ ,  $n_2 = 2.3$  and  $t = 0.1 \, \mu \text{m}$  (fig. 1). The grating is sinusoidal with a period  $d = 0.3 \, \mu \text{m}$  and  $h = 0.02 \, \mu \text{m}$  so that in the whole domain of the incident angles only the zeroth reflected and transmitted orders exist. The planar system can support one TE and one TM mode with propagation constants  $\gamma_g^{\text{TE}} = 1.76065$  asn  $\gamma_g^{\text{TM}} = 1.18012$ .

With TE (TM) type we denote the polarization of the incident wave when the electric (magnetic) field vector is perpendicular to the plane of incidence determined by  $k_1$  and the y-axis.

In the rectangular coordinate system Oxyz (fig. 1) the incident wave-vector  $k_i$  is given by  $k_i = k(\alpha_0, -h_0, \beta_0)$ , the reflection wave-vector by  $k(\alpha_0, h_0, \beta_0)$  and the transmission wave-vector  $k(\alpha_0, h_0, \beta_0) = k(\alpha_0, -h_0, \beta_0)$ 

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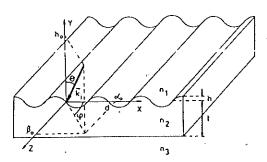


Fig. 1. Schematic representation of the corrugated waveguide in conical diffraction mounting.

 $\beta_0$ ), where  $k = 2\pi/\lambda$  and  $\lambda$  is a wavelength. If the phase matching condition is satisfied,

$$\gamma = \gamma_{\rm g}$$
, (1)

where

$$\gamma^2 = (\alpha_0 + m\lambda/d)^2 + \beta_0^2, \quad m = 0, \pm 1, \pm 2, \dots,$$
 (2)

then in the waveguide a mode is excited with angle of propagation  $\phi$  with respect to the grating vector  $K = 2\pi/d$  given by

$$\gamma_{\rm g} \sin \phi = \beta_0. \tag{3}$$

The reflectivity of the system in conical diffraction mounting in the vicinity of TE mode excitation is shown in fig. 2 for the two polarizations of the inci-

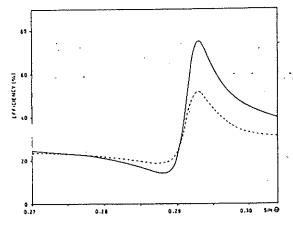


Fig. 2. Angular dependence of reflected zero-order efficiency for TE (with solid curve) and TM (with dashed curve) polarization of the incident wave.  $\lambda = 0.6 \ \mu m$ ,  $\varphi = 31.7^{\circ}$ .

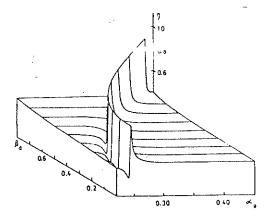


Fig. 3. General view of the zero-order efficiency in the vicinity of mode excitation in a non-polarized light.

dent wave. Contrary to the in-plane case, where TE or TM modes can be excited only by TE or TM polarized wave, in conical diffraction mounting the reflectivity in the both polarizations is influenced by the interaction.

The general behaviour of the anomaly for non-polarized light is displayed in fig. 3. The minimum and the maximum of the curves are located on two circles, each one with centre  $(\alpha_0 = \lambda/d, \beta_0 = 0)$  and radius approximately equal to  $\gamma_g$ .

# 3. Effect of non-excitation of guided waves

If the grating vector is collinear with the mode wave-vector the coupling of guided waves into radiation modes is accomplished with a polarization conservation. At oblique to the grating grooves incidence the radiation modes become, in general, elliptically polarized. The polarization of a vector plane wave is characterized by the parameters  $\delta$  and  $\tan \tau$  [9], defined respectively as a phase difference and a ratio of the amplitudes of the parallel and perpendicular to the plane of incidence components of the electric field vector.

The dependences of the radiation field components on  $\phi$  for TE and TM excited modes are shown in fig. 4 calculated by the numerical method described in ref. [5]. From the reciprocity theorem it follows that at each angle of propagation  $\phi$  the coupling is maximum for the parameters of the incident wave given in

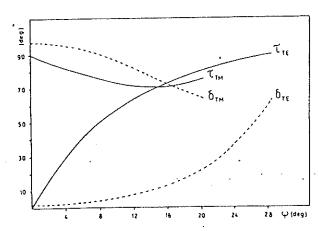


Fig. 4. Variation of the polarization parameters with the angle of propagation. The subscripts TE and TM correspond to the TE and TM waveguide modes.

fig. 4. An interesting consequence is that under the same conditions the wave with an orthogonal polarization can not excite a guided wave. Indeed, replacing  $\tau$  with  $\tau = 90^{\circ}$  no anomalies in the calculated zero-order efficiency curves have been observed.

## 4. Phenomenological approach

It is well known that the wave with an arbitrary polarization can be decomposed into two elliptically polarized components mutually orthogonal. The decomposition can be made so that only one of these components (with an amplitude  $C_1$ ) interacts with the waveguide mode, while for the second one (with an amplitude  $C_2$ ) no anomaly occurs. Mathematically the existence of guided waves in a planar waveguide means that the system has a pole  $\gamma^p = \gamma_g$ . Since in the reflectance from a plane layer no resonance anomaly is exhibited, a reflection zero  $\gamma_{r}^{z}$  =  $\gamma^{p}$  , compensating the pole must exist. As the corrugation is introduced, both  $\gamma^{p}$  and  $\gamma^{z}_{t}$  become complex, and in general  $\gamma^{p} \neq \gamma^{z}_{t}$ . Since the amplitude on the interacting component is proportional to  $(\gamma - \gamma_r^z)/(\gamma - \gamma^p)$ , the total zero-order diffraction efficienty of corrugated waveguide can be represented in the following "phenomenological" form:

$$\eta = \left| C_1 r_1 \frac{\gamma - \gamma_r^z}{\gamma - \gamma^p} + C_2 r_2 \right|^2. \tag{4}$$

The coefficients  $r_1$  and  $r_2$  are slowly varying functions of h and  $\gamma$  and for shallow gratings they coincide with the Airy reflection coefficients of plane layer.

It must be pointed out, however, that in general, all parameters in (4) are complex quantities.

The calculations of  $\gamma^p$  and  $\gamma_1^z$  have been performed by a Newton iterative procedure.

The coincidence between the calculated curves from (4) with phenomenological parameters  $|r_1|^2 = 0.2884$ ,  $|r_2|^2 = 0.2772$ ,  $\gamma_r^z = 1.761 - i 1.138 \times 10^3$ ,  $\gamma^p = 1.7585 - i 1.746 \times 10^{-3}$  and the curves in fig. 2 is better than 0.1%.

The pole and the reflection zero are equal to the corresponding values for the in-plane case because of the small value of  $\phi$  ( $\phi$  = 5°). Increasing the angle of propagation, the values of  $\gamma_r^z$  and  $\gamma^p$  are slightly changed, due to the angular anisotropy of the mode propagation constant. For example, at  $\phi$  = 25.25°,  $\gamma^p$  = 1.7575 - i1.304.10<sup>-3</sup> and  $\gamma_r^z$  = 1.762 - i468×10<sup>-3</sup>.

In our previous paper [1] we have shown that the reality of the reflection and/or transmission zero is determined only by the symmetry of the system. The same considerations can be applied in conical diffraction mounting for the interacting polarization. As a consequence, a system with a symmetry about Oyzplane (fig. 1) has a real transmission zero, thus a 100%

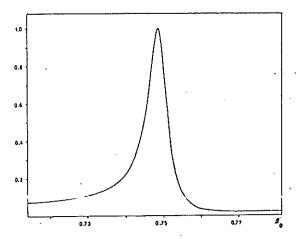


Fig. 5. Grating efficiency as a function of  $\beta$  at a fixed  $\alpha = 0.41$ .

reflectance can be achieved with a properly polarized incident wave. That is illustrated in fig. 5 for an elliptically polarized wave with parameters  $\tau = -87.25^{\circ}$  and  $\delta = 43.65^{\circ}$ .

### References

- [1] E. Popov, L. Mashev and D. Maystre, Optica Acta 33 (1986) 607.
- [2] L. Mashev and E. Popov, Optics Comm. 55 (1985) 377.

- [3] P. Vincent, N. Paraire, M. Neviere, H. Koster and R. Reinisch, J. Opt. Soc. Am. B2 (1985) 1106.
- [4] M. Neviere, P. Vincent, N. Paraire and R. Reinisch, Proc. SPIE 503 (1984) 216.
- [5] E. Popov and L. Mashev, J. Opt. Commun., in press.
- [6] D. Maystre and R. Petit, J. Spectr. Soc. Japan 23, Suppl. 1 (1974) 61.
- [7] P. Vincent, M. Neviere and D. Maystre, Nucl. Instr. Meth. 152 (1978) 123.
- [8] E. Popov and L. Mashev, J. Optics (Paris), in press.
- [9] M. Born and E. Wolf, eds., Principles of optics (Pergamon Press, 1968).