# DISPERSION CHARACTERISTICS OF MULTILAYER WAVEGUIDES

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Received 17 October 1984

The dispersion characteristics of planar multilayer optical waveguides have been calculated. It was found that splitting and degeneration of the modes occur, if the thickness of the intermediate layer between two waveguides is small.

## 1. Introduction

Multilayer waveguides have been a subject of continuous interest in integrated optics because of its application in modulators, heterostructures, directional couplers, etc.

Mode coupling characteristics of twin waveguides at a finite distance (both lossless and lossy) have been investigated by Kapany [1] and Marcuse [2]. The influence of a thin film on the waveguide has been studied for the propagation characteristics [3] and for the efficiency of mode excitation [4].

Recent works of multilayer structures consider the equal phase velocity [5,6] or the mismatch [7] of TE and TM modes.

In this paper the dispersion characteristics of a five-layer system, consisting of two parallel waveguides with step refractive indexes are calculated.

In section 2 a brief description of the calculation method for the mode dispersion characteristics is given. The method is applied to the case of two equivalent symmetrical waveguides in section 3. If the distance between two waveguides is small, TE–TE and TM–TM mode splitting and TE–TM degeneration occur. In section 4 the dispersion curves of a five-layer system, consisting of two different waveguides show well pronounced TE–TE quasi-degeneration.

## 2. Theoretical considerations

Let us consider a multilayer system of $N$ dielectric layers, bounded by semiinfinite dielectric media. As usually, the field in each layer is represented as a sum of exponentials with constant amplitudes. Wave equation and boundary conditions applied to the system, lead to a homogeneous system of linear equations. The matrix of coefficients can be factorized into a product of $N$ characteristics matrices, each of them having the form [8]

$$T_j = \begin{bmatrix} \cos(h_j d_j) & h_j^{-1}\sin(h_j d_j) \\ -h_j \sin(h_j d_j) & \cos(h_j d_j) \end{bmatrix}, \qquad (1a)$$

where $h_j = (kn_j^2 - \beta^2)^{1/2}$, $d_j$ and $n_j$ are the thickness and the refractive index of the $j$th layer respectively and $\beta$ is the phase velocity of the mode.

For layers, where $\beta/k > n_j$ the characteristic matrix has the form

$$T_j = \begin{bmatrix} \mathrm{ch}(h_j d_j) & h_j^{-1}\mathrm{sh}(h_j d_j) \\ h_j\,\mathrm{sh}(h_j d_j) & \mathrm{ch}(h_j d_j) \end{bmatrix}. \qquad (1b)$$

The relation between the field amplitude $A_0$ and amplitude of the field in $N+1$ media $A_{N+1}$ is given by

$$M_0 A_0 = T M_{N+1} A_{N+1}, \qquad (2)$$

where

$$A_j = \begin{bmatrix} a_j^+ \\ a_j^- \end{bmatrix}, \quad T = \prod_{j=1}^{N} T_j,$$

$$M_0 = \begin{bmatrix} 1 & 1 \\ -ih_0 & ih_0 \end{bmatrix}, \quad t_N = \sum_{j=1}^{N} d_j,$$

and

$$M_{N+1} = \begin{bmatrix} \exp(-\mathrm{i}h_{N+1}t_N) & \exp(\mathrm{i}h_{N+1}t_N) \\ -\mathrm{i}h_{N+1}\exp(-\mathrm{i}h_{N+1}t_N) & \mathrm{i}h_{N+1}\exp(\mathrm{i}h_{N+1}t_N) \end{bmatrix} \quad (3)$$

Here $a^+$ and $a^-$ denote the amplitudes of the wave travelling "upwards" and "downwards" respectively. For guided waves $a_0^+ = a_{N+1}^- = 0$. The dispersion relation is calculated stating the determinant of the matrix homogeneous linear equation equal to zero. It must be pointed out, that this treatment is equivalent to the method of poles [9].

In some simple cases the dispersion relation can be obtained in closed form. For example, for a five-layer system consisting of two identical waveguides, the dispersion relation takes the form

$$(h_1^2 - h_0h_2)^2\left[\tan(h_1d_1) - \frac{h_0 + h_2}{h_1 - h_0h_2/h_1}\right]^2$$

$$= [1 - \mathrm{th}(h_2d_2)]\,[\tan(h_1d_1)(h_1^4 + h_0^2h_2^2) + 2\tan(h_1d_1)h_1h_0(h_2^2 - h_1^2) + h_1^2(h_2^2 + h_0^2)]\,. \quad (4)$$

Another particular case of infinite number of periodical layers is described in ref. [6].

## 3. Splitting and degeneration of modes of two identical symmetrical waveguides

The dielectric system is shown schematically in fig. 1. Only when the separation of the waveguides is great, one can speak about two waveguides. In the case of small optical thickness of the central layer the whole system must be considered as one waveguide.

The dispersion curves for different thickness $d_3$ are shown in figs. 1 and 2.

In the case of infinite distance, the dispersion curves of two separated waveguides coincide (fig. 1, heavy lines). When the thickness of the central layer becomes finite, each of the doubly degenerate curves splits into two, both for TE and TM modes (fig. 1, light lines).

Further increasing of the coupling leads to larger splitting and to the intersection of TE and TM dispersion curves (fig. 2). It is worth noting, that there is no polarization conversion in the cross points between the TE and TM curves.

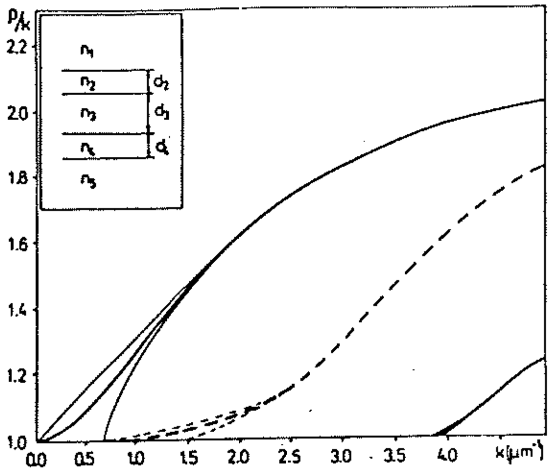


Fig. 1. Dispersion characteristics of a five-layer waveguide with parameters: $n_1 = n_3 = n_5 = 1$, $n_2 = n_4 = 2.3$, $d_2 = d_4 = 0.41$ μm. Heavy lines $d_3 = \infty$, light lines $d_3 = 2.83$ μm, solid lines TE case, dashed lines TM case.

For three-layer waveguides, TE and TM modes have different phase velocities and the orthogonality of the modes is achieved by the orthogonality of the propagating modes.

In the case of equal phase velocities the orthogonality of the modes with different polarization is achieved through the vector orthogonality of the fields themselves.

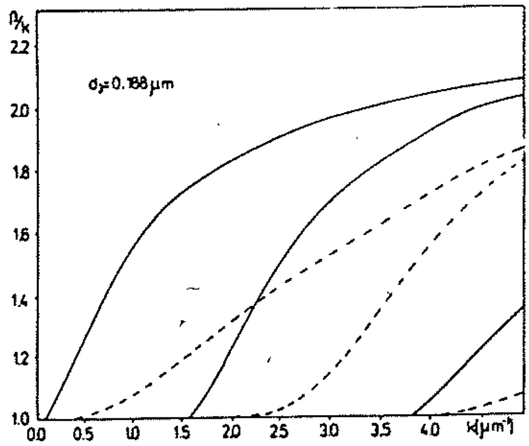


Fig. 2. Same as fig. 1, except for $d_3 = 0.188$ μm.

As can be expected, dispersion curves of the system coincide with these of symmetrical waveguide with $2d_2$ when $d_3 = 0$.

## 4. Quasi-degeneration of modes

The system with dispersion curves in fig. 3 can be considered as consisting of two waveguides, one of them symmetrical and the other asymmetrical. For the sake of simplicity, in fig. 3 only the TE dispersion curves are shown. When the thickness of the middle layer is infinite, the dispersion curves of two waveguides superimpose (fig. 3). The dispersion characteristics of the same system with a small thickness of the middle layer is shown in fig. 4. Some interesting features of the curves are the following:

(i) The dispersion curves in fig. 4 are shifted in comparison with fig. 3, due to the waveguide interaction and the shift is stronger for smaller $k$.

(ii) In the vicinity of the intersection points of the curves the modes are quasi-degenerate. In fig. 5 the vicinity of point 2 is shown with a magnification of 70 times and the splitting is visualized. It must be pointed out, that in the ideal case there is no coupling between the modes. However, any perturbation (periodical or aperiodical) can cause coupling between $TE_1-TE_2$ modes.

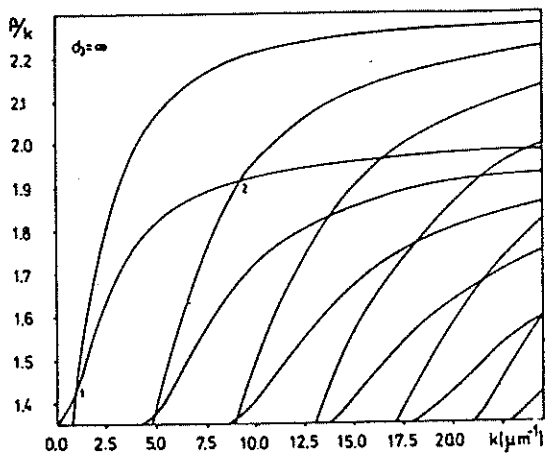

Fig. 3. Dispersion characteristics of a five-layer waveguide with parameters: $n_1 = 1$, $n_2 = 2.3$, $n_3 = n_5 = 1.35$, $n_4 = 2.0$, $d_2 = 0.41$ μm, $d_4 = 0.47$ μm, $d_3 = \infty$.

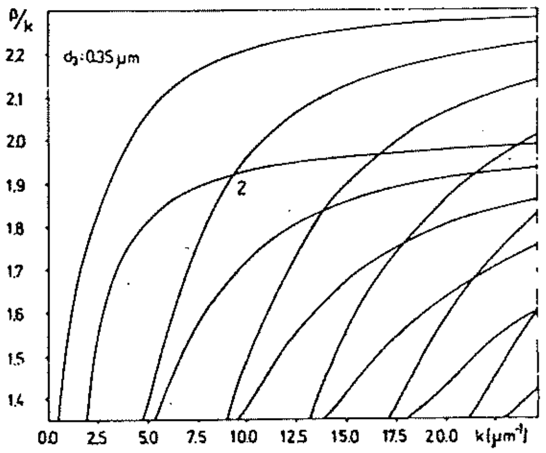

Fig. 4. Same as fig. 3, except for $d_3 = 0.35$ μm.

(iii) For the thickness $d_3 = 0.35$ μm of the middle layer the splitting of point 1 in fig. 3 is stronger and the separation of the curves is obvious without any magnification (fig. 4). The same features are intrinsic for the TM modes.

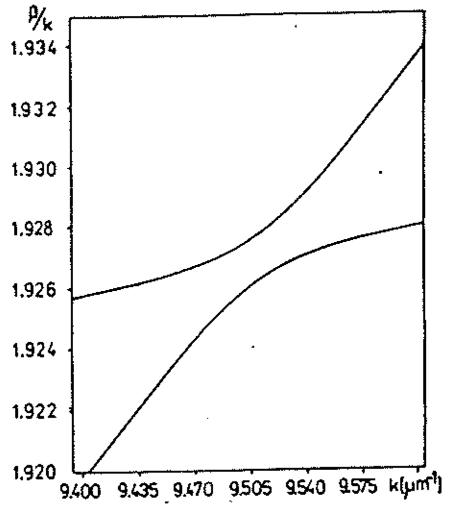

Fig. 5. 70 times magnification of the vicinity of cross point 2 of fig. 4.

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