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Rigorous Electromagnetic Treatment of Planar Corrugated Waveguides

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Summary

A generalization of the method for conical diffraction mounting based on the rigorous differential formalism of Chandezon et al. is presented for the case of planar corrugated waveguides. The coupled mode equations are derived from the phenomenological theory. A detailed comparison between the numerical results and the results of mode matching approach is given. It is shown that the Brewster's law analogy depends on the corrugation depth.

1 Introduction

In the last twenty years mode interactions in planar corrugated waveguides have been of continious interest for both theoreticians and experimentalists [1–4], fed up by the great importance of the mode coupling for passive and active integrated optical devices. The earlier works of Kogelnik [5] and Marcuse [6] deal with the approximate ideal or local mode analysis which give accurate results for the case normal to the grooves incidence.

Recently much attention has been devoted to the scattering on the surface corrugation in oblique to the grooves propagation [7–13]. Stegeman et al. [12] have proposed a precise analysis of Bragg diffraction for an oblique incidence, but the extension of the method to graded-index waveguides is not evident. In our previous papers [14, 15] we have derived explicit expressions of the coupling coefficients of obliquely incident modes of graded-index waveguides on a grating with an arbitrary groove profile. However, those formulae are valid for shallow gratings only. As the corrugation depth is increased, neither the plane wave expansion in the groove region, nor the approximate first order theories remain valid. This is quite important not only for the analysis of Bragg diffraction of waveguide modes, but to a great extent for grating couplers, where the groove depth can be comparable to the grating period and waveguide thickness, therefore the application of rigorous electromagnetic analysis becomes necessary. A theory for conical diffraction mounting has been proposed by Vincent et al. [16] which can be extended to the corrugated waveguides.

The term conical diffraction mounting is used when the incident wave vector lies out of the plane normal to the groves. The Bragg diffraction and the coupling of incident wave into waveguide modes can be represented as particular cases of conical diffraction mounting.

Recently we have applied another approach for conical diffraction mounting [17] based on the rigorous differential formalism of Chandezon et al. [18].

The aim of this paper is to present the extension of the method in [17] for mode coupling treatment. It is important to note that in contrast to the previous considerations [19–21] applied to coated gratings, now the geometry of the system is changed – in planar corrugated waveguides usually only one of the boundaries is corrugated, so that the boundary conditions are changed. In addition, for the analysis of the interaction between the modes guided by the system a homogeneous problem solution is necessary.

A detailed study of different TE-TE, TE-TM and TM-TM mode coupling and a comparison with the results of mode matching method is given in Section 4.2.

Despite of numerous works [7, 8, 10, 11, 13] there is no common opinion about the position of the angle of Brewster's law analogy in Bragg diffraction regime. In Section 4.3 we show that the angle is closed to 45° and depends on the corrugation depth.

2 Extension of the theory of conical diffraction mounting

Let us consider a multilayer waveguide represented in Fig. 1 which upper boundary is a grating with a corrugation function $y = f(x)$ of period d and depth h. From the medium 1 a plane wave with wavelength λ and wavevector $\vec{k} = (\alpha_0, -h_0, \beta_0)$ illuminates the structure.

If we denote with E_x, E_y, E_z and H_x, H_y, H_z the components of the electric $\vec{E}$ and magnetic $\vec{H}$ field vectors, respectively, and with $\vec{F}_j$ and $\vec{F}_j$ the column vectors:

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$$\vec{F}_j = \begin{bmatrix} E_{jz} \\ \omega\mu_0[H_{jx} + f'(x)H_{jy}] \\ -\omega\mu_0 H_{jz} \\ E_{jx} + f'(x)E_{jy} \end{bmatrix} \quad j = 1,2, \tag{1}$$

$$\vec{F}_j = \begin{bmatrix} E_{jz} \\ \omega\mu_0 H_{jx} \\ -\omega\mu_0 H_{jz} \\ E_{jx} \end{bmatrix} \quad j = 2, \ldots, M, \tag{2}$$

the solution of the Maxwell equations in media 1 and 2 can be searched in a matrix form:

$$\tilde{F}_j = \Omega R_j \tilde{\phi}_j[y - f(x)]\, G_j \exp(i\beta_0 z), \tag{3}$$

where

$$\begin{aligned} &\Omega_{mp} = \delta_{mp} \exp(i\alpha_m x), \\ &\tilde{\phi}_{j,mp}(u) = \delta_{mp} \exp(ir_{j;p} u), \\ &\alpha_m = \alpha_0 + mK, \quad m = 0, \pm 1, \pm 2, \ldots \\ &K = 2\pi/d. \end{aligned} \tag{4}$$

$$R_2 G_2 = R_1 G_1. \tag{5}$$

From the uniqueness of the solution in medium 2 a connection between $\tilde{F}_2(l_2)$ and $F_2(l_2)$ can be found:

$$F_2|_{y=l_2} = \Omega Q_2 \tilde{\phi}_2(l_2) G_2 \exp(i\beta_0 z). \tag{6}$$

In the calculation of (6) the following correlations between the field components, obtained from (3) and the Maxwell equations are used:

$$\begin{aligned} E_{jx} &= \left(\frac{\partial^2 E_{jz}}{\partial x\, \partial z} + i\omega\mu_0 \frac{\partial H_{jz}}{\partial y}\right) \Big/ (k^2 n_j^2 - \beta_0^2), \\ \omega\mu_0 H_{jx} &= \left(\omega\mu_0 \frac{\partial^2 H_{jz}}{\partial x\, \partial z} - ik^2 n_j^2 \frac{\partial E_{jz}}{\partial y}\right) \Big/ (k^2 n_j^2 - \beta_0^2), \end{aligned} \tag{7}$$

where $k = 2\pi/\lambda$ and n_j is the refractive index of the medium j. The explicit form of the matrix Q_2 is expressed through the components of the matrix R_2:

$$\begin{aligned} Q^{I}_{2,mp} &= \sum_q R^{I}_{2,qp}\, \mathscr{L}_{q-m}(r_{2,p}), \\ Q^{II}_{2,mp} &= \sum_q (k^2 n_2^2 r_{2,p} R^{I}_{2,qp} + \beta_0 \alpha_m R^{III}_{2,qp})\, \mathscr{L}_{q-m}(r_{2,p})/(k_2 n_2^2 - \beta_0^2), \\ Q^{III}_{2,mp} &= \sum_q R^{III}_{2,qp}\, \mathscr{L}_{q-m}(r_{2,p}), \\ Q^{IV}_{2,mp} &= \sum_q (r_{2,p} R^{III}_{2,qp} - \beta_0 \alpha_m R^{I}_{2,qp})\, \mathscr{L}_{q-m}(r_{2,p})/(k^2 n_2^2 - \beta_0^2). \end{aligned} \tag{8}$$

Here G_j is a column vector of the unknown diffraction orders amplitudes $g_{j,p}$ (both evanescent and propagating), ω is an angular frequency, μ_0 is a vacuum permittivity, δ_{mp} is the Kronecker symbol and $r_{j,p}$ and $R_{j,p}$ are the eigenvalues and the eigenvectors, respectively, of the matrix of coefficients obtained by substitution of (3) in the Maxwell equations. The boundary conditions apllied at the corrugated surface $\tilde{F}_1|_{y=f(x)} = \tilde{F}_2|_{y=f(x)}$ result in:

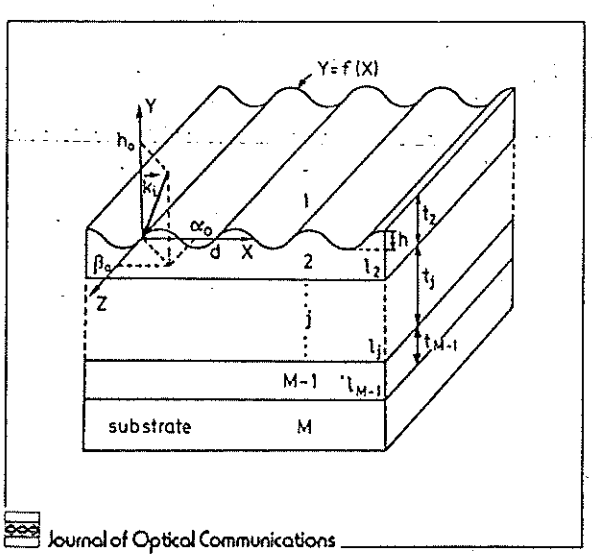

Fig. 1: Schematic representation of multilayer waveguide with corrugated surface

Here $Q_2^{I\text{-}IV}$ and $R_2^{I\text{-}IV}$ are submatrices of Q_2 and R_2 respectively:

$$R_2 = \begin{bmatrix} R_2^{I} \\ R_2^{II} \\ R_2^{III} \\ R_2^{IV} \end{bmatrix}, \quad Q_2 = \begin{bmatrix} Q_2^{I} \\ Q_2^{II} \\ Q_2^{III} \\ Q_2^{IV} \end{bmatrix} \tag{9}$$

and

$$\mathscr{L}_p(u) = \frac{1}{d} \int_0^d \exp[iuf(x) + ipKx]\, dx. \tag{10}$$

Below the groove region the field components can be developed in Rayleigh series:

$$\begin{aligned} E_{jz} &= \sum_m a'_{jm} \exp[i(\alpha_m x + h_{jm} y + \beta_0 z)], \\ -\omega\mu_0 H_{jz} &= \sum_m k n_j a''_{jm} \exp[i(\alpha_m x + h_{jm} y + \beta_0 z)], \end{aligned} \tag{11}$$

where $h_{jm}^2 = k^2 n_j^2 - \alpha_m^2 - \beta_0^2$. Using (7), F_j can be expressed as:

$$F_j = \Omega P_j \phi_j(y) A_j \exp(i\beta_0 z), \tag{12}$$

where

$$A_j = \begin{bmatrix} \vdots \\ a'_{jp} \\ \vdots \\ a''_{jp} \\ \vdots \end{bmatrix},$$

$$\phi_{j,mp}(y)=\delta_{mp}\exp(ih_{j,p}y), \tag{13}$$

$$P_j=\begin{bmatrix} D_j^I & 0 \\ D_j^{II} & D_j^{III} \\ 0 & kn_jD_j^I \\ -D_j^{III}/kn_j & D_j^{II}/kn_j \end{bmatrix}$$

and

$$D_{j,mp}^{I}=\delta_{mp},$$
$$D_{j,mp}^{II}=\delta_{mp}k^2n_j^2h_{j,p}/(k^2n_j^2-\beta_0^2), \tag{14}$$
$$D_{j,mp}^{III}=\delta_{mp}kn_j\beta_0\alpha_p/(k^2n_j^2-\beta_0^2).$$

From (5), (6) and (12) a connection between G_1 and A_2 can be found:

$$R_1G_1=R_2\bar{\phi}_2^{-1}(l_2)Q_2^{-1}P_2\phi_2(l_2)A_2. \tag{15}$$

The boundary conditions applied at the plane interfaces $l_2, \ldots, l_{M-1}$ require the continuity of F_j, i.e. from (12) it follows that:

$$P_j\phi_j(l_j)A_j=P_{j+1}\phi_{j+1}(l_j)A_{j+1}, j=2, \ldots, M-1. \tag{16}$$

Therefore, from (16) and (15) we obtain:

$$R_1G_1=R_2\bar{\phi}_2^{-1}(l_2)Q_2^{-1}P_2\phi_2(l_2)T_3\ldots T_{M-1}P_M\phi_M(l_{M-1})A_M, \tag{17}$$

where

$$T_j=P_j\phi_j(t_j)P_j^{-1}, \quad j=3, \ldots, M-1. \tag{18}$$

The relation (17) gives a connection between the field amplitudes in the upper and the lower medium. Using the out-going wave conditions, the amplitudes in the outer medium 1 and M can be devided into incident G_1^i and A_M^i and diffracted G_1^d and A_M^d field amplitudes. Regrouping the terms in (17) and introducing the vectors

$$C^d=\begin{bmatrix} G_1^d \\ A_M^d \end{bmatrix} \text{ and } C^i=\begin{bmatrix} G_1^i \\ A_M^i \end{bmatrix}$$

a linear algebraic system of equations (17) can be represented in a more convenient form:

$$SC^d=C^i, \tag{19}$$

where S is the matrix of coefficients, obtained from (17).

3 Phenomenological approach and coupled mode equations

It is well known that at certain discrete values of the wave-vector modulus $\alpha_0^2+\beta_0^2=\gamma_g^2$ the waveguide can support modes without incident wave. From a mathematical point of view the existence of non-zero solutions of the homogeneous part of (19) requires the determinant of S to be equal to zero, or with other words, S^{-1} must have a pole. In this case the rank of the matrix S is with a unity less than its order, so that the amplitude of the m-th diffracted order $a_{M,m}^d \equiv a_\nu$ depends linearly on the resonance amplitude $a_{M,0}^d \equiv a_\mu$.

For planar waveguides the pole of S^{-1} is real and equal to the waveguide mode propagation constant γ_g. At the presence of corrugation the pole becomes complex, but away from mode interaction its imaginery part remains quite small for shallow gratings. If there exists an integer m such that $\alpha_m^2+\beta_0^2=\gamma_g'^2$ (at fixed γ_g) an interaction of the waveguide modes with propagation constants γ_g and γ_g' appears at a given value of the grating vector $K_{m,0}^p$.

Two approaches are possible to determine the coupling parameters:

Near the interaction point the poles γ_g and γ_g' corresponding to the two modes are shifted in the complex plane and varying the grating period the maximum of the imaginary part of the pole is equal to the coupling coefficient [22].

We have chosen the phenomenogical approach used in the theory of grating anomalies [23]. If the grating period is varied, in the exitation point of the second pole the amplitude a_ν has a pole $K_{m,0}^p \equiv K_\nu^p$ and can be represented in the form:

$$a_\nu=\frac{C_{\nu\mu}}{K-K_\nu^p}a_\mu. \tag{20}$$

Further on a connection between $C_{\nu\mu}$ and the coupling coefficients would be obtained.

Compared to the first one, this approach has two main advantages:

i – it needs less numerical steps for the determination of the coupling coefficients,

ii – it enables to derive the coupled mode equations directly from the phenomenological formulae.

Let us suppose that the grating region is extended from $x=0$ to $x=L$, where $L \gg d$. The mode amplitudes can be represented in the following way:

$$a_\mu(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}A_\mu(K)\exp(iKx)dK,$$
$$a_\nu(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}A_\nu(K)\exp(iKx)dK, \tag{21}$$

where $A_\mu(K)$ are the Fourier components corresponding to the fixed grating vector K. Using (20) the response of the grating to the set of amplitudes $A_\nu(K)$ is:

$$a_\mu(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}A_\nu(K)\frac{C_{\mu\nu}}{K-K_\mu^p}\exp(iKx)dK. \tag{22}$$

After differentiation of (22) in x and taking into account that $C_{\mu\nu}\approx$ const. we found:

$$\frac{da_\mu(x)}{dx}=i\frac{C_{\mu\nu}}{2\pi}-\int_{-\infty}^{\infty}A_\nu(K)\frac{K}{K-K_\mu^p}\exp(iKx)\,dK$$
$$=iK_\mu^pa_\mu(x)+iC_{\mu\nu}a_\nu(x).$$

Introducing another set of mode amplitudes

$$b_\mu(x)=a_\mu(x)\exp(-iK_\mu^px)$$

and repeating the same procedure for the ν-th mode, the well known system of coupled mode equation is obtained:

$$\frac{db_\mu(x)}{dx} = iC_{\mu\nu} b_\nu(x) \exp(i\delta_{\mu\nu} x),$$
$$\frac{db_\nu(x)}{dx} = iC_{\nu\mu} b_\mu(x) \exp(-i\delta_{\mu\nu} x), \tag{23}$$

where $\delta_{\mu\nu} = K_\nu^p - K_\mu^p$.

4 Numerical results

4.1 Computer code

A computer code based on the theoretical considerations in the previous section was constructed. The eigenvalue and the eigenvector problem was solved using a standard EISPACK routines and a Gauss-Jordan scheme was applied for matrix inversion. The numerical advantages and restrictions of the method were discussed in [17, 20]. The phenomenological parameters $C_{\nu\mu}$ and K_ν^p in (20) were obtained by a least-square fit. A comparison between the amplitudes of the diffracted wave calculated numerically and from (20) is shown in Fig. 2. A very good agreement is achieved for a large domain of K except for a slight vicinity of the pole interaction point. The mode coupling provokes a shift of the propagation constant γ_g in the complex plane hence the equation $\det|S(\gamma)| = 0$ is not satisfied for real values of γ. This is illustrated in Fig. 3 where with crosses the relative deviation of the numerical results from (20) is depicted. It is obvious that the region where (20) is not valid is negligible.

For relative error less than 1% usually 4–5 calculations at different grating periods are necessary to determine the parameters $C_{\nu\mu}$ and K_ν^p by least-square fit. However, the calculations must be performed out of the domain of K where γ_g has a great imaginery part. This requirement is fulfilled when the determinant of S is approximately equal to zero.

4.2 Comparison with mode matching method

In our previous papers [14, 15] we have derived analytical formulae for coupling coefficients in a first-order in groove depth approximation. The expressions for TE-TE, TE-TM and TM-TM coupling coefficients can be unified in the following way:

$$\Gamma_{\mu\nu} = \frac{hF_m}{8} \frac{\omega\varepsilon_0(n_2^2 - n_1^2)}{\cos\theta_\mu} \vec{\mathscr{E}}_{\mu,1}^*(0) \cdot \vec{\mathscr{E}}_{\nu,2}(0), \tag{24}$$

where ε_0 is a vacuum permeability, F_m is the m-th Fourier component of the grating function f(x), corresponding to the coupling, θ_μ is the angle of propagation of the μ-th mode with respect to the positive x-axis. $\vec{\mathscr{E}}_{\mu,j}(y)$ is the electric field vector in the j-th medium, which for the two fundamental cases of polarization is equal to:

$$\vec{\mathscr{E}}_{\mu,j}^{TE} = (-\mathscr{E}_{\mu z}\sin\theta_\mu, 0, \mathscr{E}_{\mu z}\cos\theta_\mu),$$
$$\vec{\mathscr{E}}_{\mu,j}^{TM} = (\mathscr{E}_{\mu x}\cos\theta_\mu, \mathscr{E}_{\mu y}, \mathscr{E}_{\mu x}\sin\theta_\mu), \tag{25}$$

where:

$$\mathscr{E}_{\mu x} = -\frac{1}{i\omega\varepsilon_0 n_j^2} \frac{d\mathscr{H}_{\mu z}}{dy},$$
$$\mathscr{E}_{\mu y} = -\frac{\gamma_{g,\mu}}{\omega\varepsilon_0 n_j^2} \mathscr{H}_{\mu z}$$

and $\mathscr{E}_{\mu z}$ and $\mathscr{H}_{\mu z}$ are transverse TE and TM mode fields. Using (25) it can easily be shown that (24) is equivalent with (8)–(10) of [15], except for the factor 2 in the denominator of (10) in [15] which has been omitted due to authors technical mistake.

After a proper normalization of the mode fields with respect to the energy flux through the cross-section, the connection between $\Gamma_{\mu\nu}$ and $C_{\mu\nu}$ is given by:

$$\Gamma_{\mu\nu} = v_{\mu\nu} C_{\mu\nu}, \; v_{\mu\nu} = \begin{cases} \mathscr{E}_{\nu z}(l_{M-1})/\mathscr{E}_{\mu z}(l_{M-1}) & \text{for TE-TE} \\ -\sqrt{\frac{\mu_0}{\varepsilon_0}}\, \mathscr{H}_{\nu z}(l_{M-1})/n_M \mathscr{E}_{\mu z}(l_{M-1}) & \text{for TE-TM} \\ \mathscr{H}_{\nu z}(l_{M-1})/\mathscr{H}_{\mu z}(l_{M-1}) & \text{for TM-TM} \end{cases} \tag{26}$$

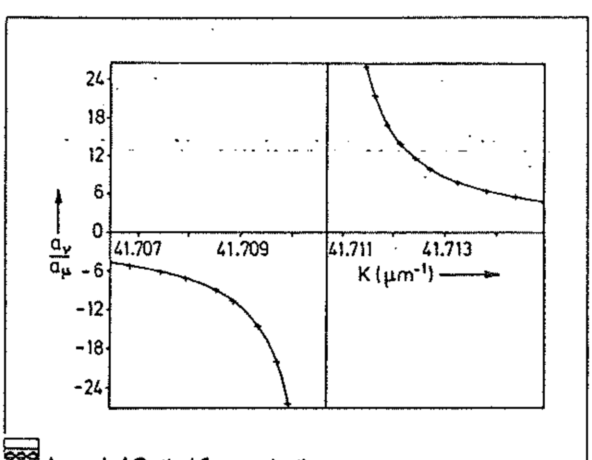

Fig. 2: Ratio a_1^{TE}/a_0^{TE} calculated numerically (with solid line) and from (20) (with crosses $C_{01} = 0.0201\,\mu m^{-1}$, $K_1^p = 41.7107\,\mu m^{-1}$) as a function of K for a waveguide with $n_1 = 1$, $n_2 = 2.3$, $n_3 = 1.6$, $t_2 = 0.3\,\mu m$, $h = 0.004\,\mu m$, $\lambda = 0.6\,\mu m$; angle of incidence $\theta = 30°$

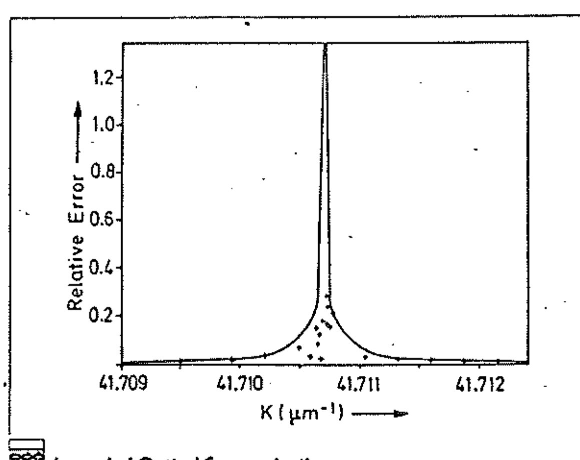

Fig. 3: Relative error dependence of the determinant of S (with solid line) and the "phenomenological" diffracted wave amplitude (with crosses) on K for the case of Fig. 2

The calculated from (24) and (26) values of the coupling coefficients for a step refractive index waveguide with a symmetrical triangular corrugation are presented in Table 1. It is worth noting that the angular dependeces of the coupling coefficients councide with the same precision.

Table 1: Comparison between the results of rigorous theory (Γ_r) and the mode matching method (Γ_n); the parameters of the waveguide and the grating are: $n_1 = 1$, $n_2 = 2.3$, $n_3 = 1.6$, $t_2 = 0.3\,\mu m$, $h = 0.004\,\mu m$, $\lambda = 0.6\,\mu m$; angle of incidence $\theta_i = 30°$

	mm^{-1}	TM_1^d	TM_0^d	TE_1^d	TE_0^d
TE_0^i	Γ_r	10.74	6.084	4.931	2.952
	Γ_n	10.67	6.121	5.003	2.970
TE_1^i	Γ_r	17.98	10.73	11.56	
	Γ_n	17.99	10.81	11.46	
TM_0^i	Γ_r	0.993	2.080		
	Γ_n	1.010	2.034		
TM_1^i	Γ_r	3.363			
	Γ_n	3.398			

4.3 Multilayer waveguides

The study of corrugated multilayer waveguides is prompted by two facts. First, multilayer waveguides are subject of continuous interest in integrated optics because of their application in modulators, heterostructures, directional couplers, etc. Some interesting effects, such as splitting of TE-TE and TM-TM modes and TE-TM degeneration are observed [24]. Second, a waveguide with an arbitrary refractive index profile can be approximated with a stack of layers with slightly different refractive indices.

The numerical and the analytical results are compared in Table 2 for a five layer waveguide. Again a very good coincidence between $\Gamma_{\mu\nu}$ and $v_{\mu\nu}C_{\mu\nu}$ is observed.

Table 2: Calculated coupling coefficients for sinusoidally corrugated multilayer waveguide with $n_1 = 1$, $n_2 = 1.54$, $n_3 = 1.53$, $n_4 = 1.52$, $n_5 = 1.51$, $t_2 = 0.6\,\mu m$, $t_3 = 0.3\,\mu m$, $t_4 = 0.1\,\mu m$, $h = 0.002\,\mu m$, $\lambda = 0.6\,\mu m$; angle of incidence $\theta_i = 30°$

	mm^{-1}	TM_0^d	TE_0^d
TE_0^i	Γ_r	$1.036 \cdot 10^{-1}$	$7.677 \cdot 10^{-2}$
	Γ_n	$1.043 \cdot 10^{-1}$	$7.806 \cdot 10^{-2}$
TM_0^i	Γ_r	$2.267 \cdot 10^{-2}$	
	Γ_n	$2.121 \cdot 10^{-2}$	

4.4 Brewster's law analogy

One of the greatest advantages of the rigorous theory is its ability to deal with deep gratings. In [14, 15] we have shown that at an angle of diffraction $\theta_\mu - \theta_\nu$ equal to 90° the $TE_\mu - TE_\mu$ coupling vanishes. However, this is valid only in the first-order approximation in the groove depth. It is interesting to study what happens when the corrugation depth is comparable to the waveguide thickness and groove spacing. A monomode waveguide is considered in order to avoid the influence of the other types of coupling. The numerical results are shown in Fig. 4. As the

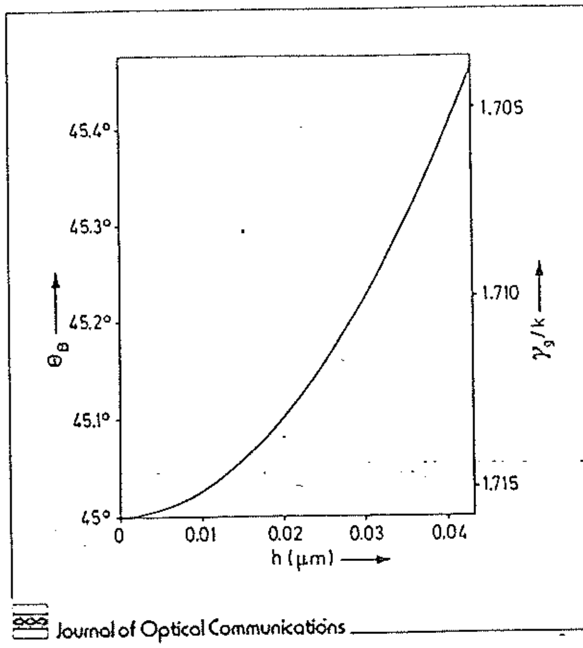

Fig. 4: Dependence of the Brewster's angle and the mode effective refractive index on the corrugation depth for a waveguide with $n_1 = 1$, $n_2 = 2.3$, $n_3 = 1.6$, $t_2 = 0.07\,\mu m$, $\lambda = 0.6\,\mu m$ and a sinusoidal grating

corrugation depth is increased, both the effective refractive index and the angle of diffraction are changing, approximately proportional to h^2, because, since θ_B and γ_g^{TE} does not depend on the sign of the groove depth, the even members in their expansion in h must be equal to zero.

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