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Enhanced light transmission by hole arrays

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Abstract

The extraordinary optical transmission of a metallic film pierced by a two-dimensional subwavelength hole array, observed by Ebbesen *et al*, is explained using rigorous electromagnetic analysis and a phenomenological approach. The analysis is based on Li's Fourier-modal method extended to crossed gratings, which reduces the diffraction problem to the search for eigenvalues and eigenvectors of a particular matrix. The computation of the eigenvalues allows us to find a new channel for light transmission through the subwavelength holes, which differs from the transmission channel in the one-dimensional case (lamellar or rectangular-rod grating). It is demonstrated that the enhanced transmission is due to the excitation of a surface plasmon on the lower metallic surface.

Keywords: Diffraction, electromagnetic theory, crossed gratings

1. Introduction

The device under study consists of a silver film deposited on a glass substrate and pierced by a two-dimensional (2D) hole array. The periods of the hole array in the two perpendicular (x and z) directions in the film plane are equal to d, which is shorter than the incident wavelength λ, from where the name 'subwavelength' comes. The film thickness is h and the holes have square cross sections with side a. The device is lighted at incidence close to the normal to the film surface. In the original experiment by Ebbesen *et al* [1], the holes had circular form with a diameter of 0.15 or 0.35 μm, $d = 0.9$ μm, $h = 0.2$ μm, and the incidence was normal to the surface. For several particular values of the wavelength, an unexpected high transmission was observed, much higher than the ratio between the hole array surface and the total silver film surface, and orders of magnitude higher than predicted by the standard aperture theory.

A great amount of theoretical effort was devoted to explain this surprising phenomenon. Several authors [2–5] attributed it to the excitation of surface plasmons, others related the effect to cavity resonances [6, 7]. However, due to the theoretical and numerical difficulties in analysing crossed (2D) gratings, all these authors made their studies using a one-dimensional (1D) classical lamellar grating. The problem was that this simplification was highly inadequate, since the channel of light transmission in the grooves of the 1D lamellar grating does not exist for 2D hole arrays [8]. It is then necessary to make a 2D analysis to correctly model the extraordinary transmission through such 2D subwavelength hole arrays (the same is also valid for superwavelength arrays). A second paper has recently reported on a fully three-dimensional theoretical study of the actual biperiodic grating [9]. In this paper, the authors analyse the coupling between the upper and lower interface surface plasmons, when the structure is symmetric (the index above is equal to the index below).

2. Fourier-modal theory

As already mentioned, we use a numerical implementation of the Fourier-modal theory extended to crossed gratings by Li [10]. The geometry is simplified by assuming square instead of circular holes. Due to the invariance of the holes in the y-direction perpendicular to the silver film surface, the permittivity is also independent of y and its 2D Fourier components in x and z have only to be computed once. The periodicity of the structure in x and y results in a quasiperiodicity of all the field components, so that the diffraction problem inside the silver layer reduces to an eigenvalue problem of a certain matrix. Any component F of the electric and magnetic fields can then be represented in the form of modes propagating or evanescent in the y-direction, with propagation constants γ_p equal to these eigenvalues [11]

$$F(x, y, z) = \sum_{m,n,p=-N}^{N} [u_p^+ \exp(i\gamma_p y) + u_p^- \exp(-i\gamma_p y)] \times \exp(i\alpha_m x + i\beta_n z) \quad (1)$$

where $u_p^\pm$ are the upward (+) and downward (−) mode amplitudes. α_m and β_n depend on the period d and on the incident angles in the following manner

$$\alpha_m = \alpha_0 + m\frac{2\pi}{d}$$
$$\beta_n = \beta_0 + n\frac{2\pi}{d} \quad (2)$$

with α_0 and β_0 being the x and y components of the incident wavevector.

When the hole dimensions are smaller than the wavelength, all the modes are evanescent, so that the field intensity below the film is governed by the modes having the lowest attenuation constants, i.e., having minimal Im (γ_p).

3. The transmission channel

To better understand the role of different modes in light transmission through the holes, we first consider a perfectly conducting material, because the modes can be calculated analytically, at least for a rectangular-rod (lamellar) grating and for a hole array with square holes. Inside the grooves of a rectangular-rod grating, one can find the propagating and evanescent modes of a plane metallic hollow waveguide. Such a waveguide can support a TEM mode which has no cut-off and can propagate whatever the distance between the waveguide plates (i.e. the groove width) may be. It has both transverse electric and magnetic field components and its constant of propagation is equal to the free-space wavenumber. When finite conductivity is taken into account, the TEM mode attenuates slightly, but it is still responsible for the light transmission through rectangular-rod (slit) gratings, as demonstrated numerically elsewhere [7].

However, the main problem of using the 1D grating model to explain the 2D hole array lies in the fact the TEM mode does not exist in cylindrical metallic hollow waveguides, contrary to the plane (hollow slab) geometry. More generally, classical waveguide theory [12] teaches that TEM modes cannot propagate in a simply-connected domain, as is the case of a cylindrical hole in a perfectly or finitely conducting material. Thus, some other channel(s) must be found. In our previous work [8] we have studied the modes inside a periodic 2D hole array in both a perfectly conducting film and in silver (refractive index equal to 0.1 + i8.94 at $\lambda = 1.388$ μm). Figure 1 presents the results. Infinite conductivity (circles) forbids completely the interaction between the modes in the adjacent holes, so that the mode constants correspond to the modes inside the hollow square metallic waveguide. For the optogeometrical parameters chosen in the introduction, all modes are below cut-off, so that they are purely evanescent, which can be observed in the figure; their propagation constants lie on the imaginary axis.

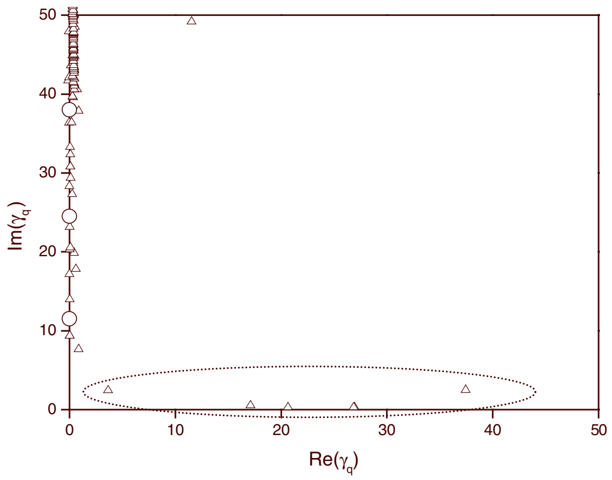

Figure 1. Eigenvalues (vertical propagation constants of the modes) of a square hole array in a silver plate: $d = 0.9$ μm; $a = 0.25$ μm; $\lambda = 1.388$ μm; circles, infinite conductivity; triangles, finite conductivity. The eigenvalues within the dashed boundary can only be found for the finite conductivity case.

The finite conductivity slightly changes the constant of propagation of these modes, but, in addition, introduces many new modes, some of them having a much smaller imaginary part of their propagation constant, so that their attenuation within the layer thickness ($h = 0.2$ μm) could be 100 times smaller, possibly leading to a 10^4 greater transmittivity. The role of each mode in the enhancement of the transmittivity will be determined by the coupling strength between the incident (and transmitted) wave and the mode. In addition, the periodicity of the holes plays some role by possibly phase-matching the horizontal wavevector components of the mode and the incident (and the transmitted) wave using equation (2) (due to the periodicity of the structure, the calculated modes in the holes are necessarily associated with a given value of the tangential component of the wavevector). Thus it is *a priori* impossible to distinguish between the contributions of the different modes in the transmission.

In order to determine which mode(s) plays the important role in the transmission, we made several numerical experiments taking $\lambda = 1.388$ μm, which is close to the last maximum in figure 1 of [1]. We first computed the mode propagation constants and the transmission, and then started to artificially double the imaginary part of the propagation constant of each mode, one after the other, and again calculated the transmission. The experiment revealed that the transmission remains almost the same, except when the propagation constant of one definite mode is changed, for which the transmission falls 40 times, from 7.5 to 0.192%. This was the mode with $\gamma_p = 0.025\,39 + i9.3813$, the mode which is the closest to the imaginary γ-axis with the lowest imaginary part.

A second numerical experiment was carried out to support this conclusion. The transmittivity T was computed as a function of the thickness of the silver film and the results are presented in figure 2 in a semi-logarithmic scale. As can be

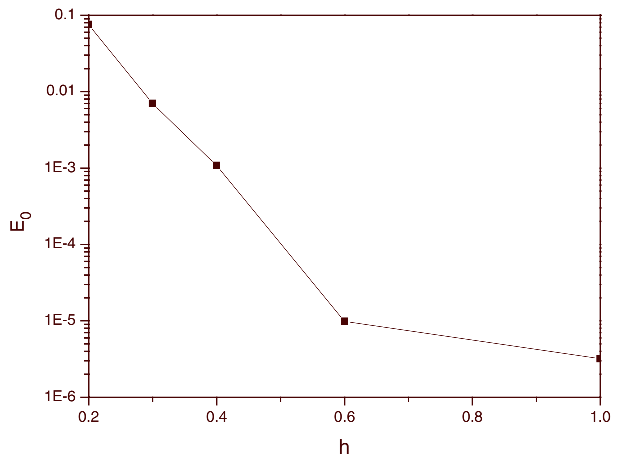

Figure 2. Zero-order transmittivity of the 2D hole array in silver as a function of the thickness h for $\lambda = 1.388$ μm and normal incidence.

seen, ln T is a linear function of h in the interval 0.2–0.6 μ m, which is a proof that the exceptional transmittivity occurs through a single channel (single mode in equation (1)). The slope of the line in figure 2 (approximately 20) pretty well agrees with the imaginary part of $\gamma_p = 0.025\,39 + i9.3813$, determined above ($\gamma_p$ is related to the attenuation of the field and, then, $2\gamma_p$ is related to the attenuation of the energy).

4. Physical nature of the enhancement of transmittivity

In the previous section we have shown that only one mode of the entire set of eigenvalues is responsible for the transmission channel which guides the incident light to the substrate through the silver film. However, this channel exists for all the wavelengths in the spectral interval considered in [1], whereas the enhanced transmission is only observed around particular wavelength values. A closer look at figure 1 of [1] reveals that the positions of the two rightmost maxima (around the wavelength of 0.9 and 1.4 μm) shows that they are situated at wavelengths close to the Rayleigh anomalies linked with the passing-off of the higher diffraction orders in the cladding ($n_{clad.} = 1$) and substrate ($n_{glass} = 1.5$). In normal incidence this passing-off condition is written

$$\frac{\lambda}{d} = n_{clad.}, n_{glass}. \tag{3}$$

Since the work of Fano [13] it is well known that in metallic gratings these anomalies, observed for the first time by Wood [14], are due to the excitation of surface plasmons propagating along the metallic–dielectric interface. For highly conducting metals, the plasmon propagation constant is slightly greater than the free wavenumber in the corresponding dielectric.

There are several approaches to obtain a better understanding of the physical nature of the resonant process. One approach is to look for poles of the scattering operator and for zeros of the transmission order amplitude in the complex plane of the different variables. This study is common for 1D gratings and has previously explained (and predicted) several curious resonance phenomena in linear [15] and non-linear optics [16], such as total absorption of light by metallic gratings and resonantly enhanced or reduced second-harmonic generation. For classical gratings there are two ways of describing the poles and the zeroes: (i) to choose the real wavelength λ and complex propagation constant α_0 along the x-axis, which is perpendicular to the groove direction [15]; (ii) to fix α_0 real and to work in the complex λ-plane.

For 2D gratings, the surface wave can be excited in many different directions in the x–z plane due to the double periodicity of the structure (see equation (2)). In addition, α_0 and β_0 are mutually independent, so that the choice of real λ would need two independent complex variables. This determines our choice to work in the complex λ-plane and to fix α_0 and β_0 real. Under normal incidence α_0 and β_0 are null and we search for complex poles λ^p of the scattering matrix S in the complex λ-plane. These poles correspond to a surface wave that can propagate in the x–z plane. A single pole in the complex λ-plane is found ($\lambda^p = 1.3878 + i0.002\,0227$). Although, in general, there are four surface waves to be excited in normal incidence, those propagating in $+x$, $-x$, $+z$ and $-z$ directions, they are all coupled by the grating to form four different standing waves, symmetrical or antisymmetrical with respect to the origin. There is only one solution symmetrical with respect to the change of signs of both x- and z-axes. The other three solutions are antisymmetrical either with respect to x or z, or both x and z and they cannot be excited in normal incidence by the incident plane wave, which is symmetrical with respect to both x and z. This is not the case in off-normal incidence, as discussed later.

If the dimensions of the holes are gradually reduced, one can plot the pole λ^p in the complex λ-plane (figure 3), and it is observed that the pole tends toward a value $\lambda^p = 1.369\,406 + i0.000\,4436$, which is the complex pole corresponding to a plasmon surface wave propagating along a plane silver–glass interface. For a plane interface between dielectric and metallic media with, respectively, refractive indices n_1 and n_2, the constant of propagation is given by the simple formula

$$k_z^p = \frac{2\pi}{\lambda} \frac{n_1 n_2}{\sqrt{n_1^2 + n_2^2}}. \tag{4}$$

When using the numerical values of silver and glass indices, for $\lambda = \lambda^p$ one obtains that $k_z^p = 2\pi/d$, i.e. the phase-matching condition $\alpha_n = \alpha_0 + n2\pi/d = \mathrm{Re}\,(k_z^p)$ is satisfied for $n = +1$ under normal incidence ($\alpha_0 = 0$). This shows that the surface plasmon is excited through the +first diffraction order of the grating illuminated at normal incidence.

The conclusion is that the pole, causing the enhanced transmission at the wavelength close to 1.39 μm is linked with this plasmon surface wave, excited via the grating periodicity. In order to quantitatively investigate the role of the pole, it is necessary to well understand that, as for 1D classical gratings [15], when the propagating diffraction orders are considered in the region of resonant guided wave excitation, the pole of the scattering matrix (i.e., the pole of the amplitude of the diffraction order) is always accompanied by a complex zero, so that when the grating tends towards a plane, the pole is compensated by the zero and no resonance anomaly is found. And indeed, numerical investigations in the case

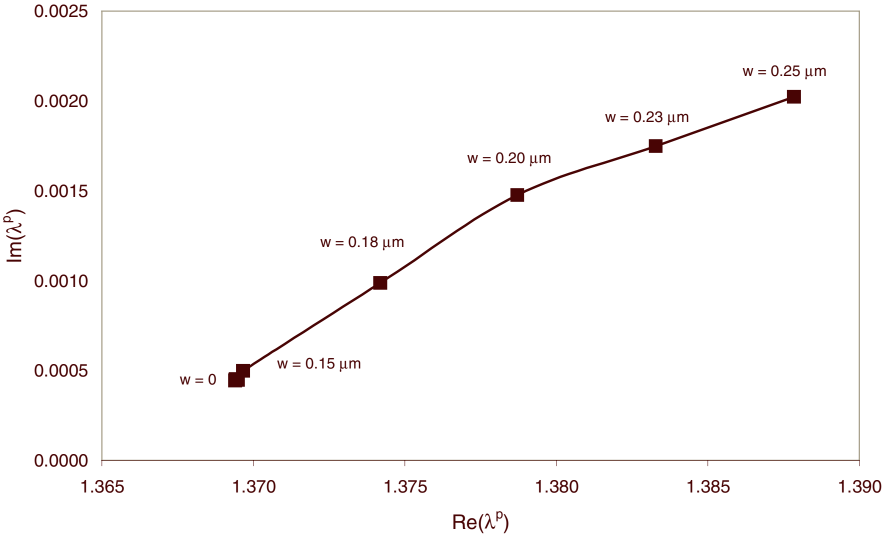

Figure 3. The trajectory of the pole λ^p in the complex λ-plane when the square hole edge w is varied from 0.25 μm down to zero.

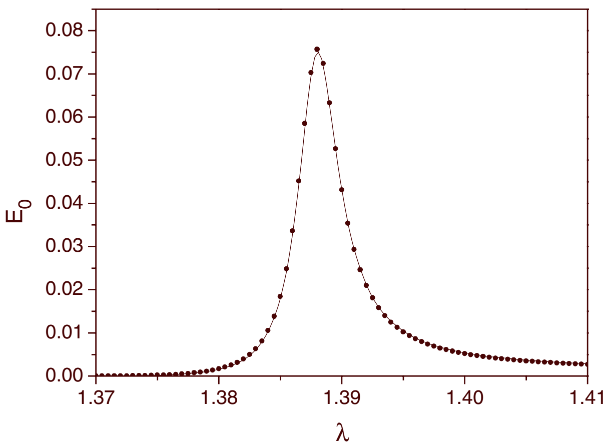

Figure 4. Comparison of the computed resonance line of the zero-order transmittivity (dots) with the values predicted from the phenomenological formula, equation (5) (solid curve). The constant c is determined by fitting the maxima values.

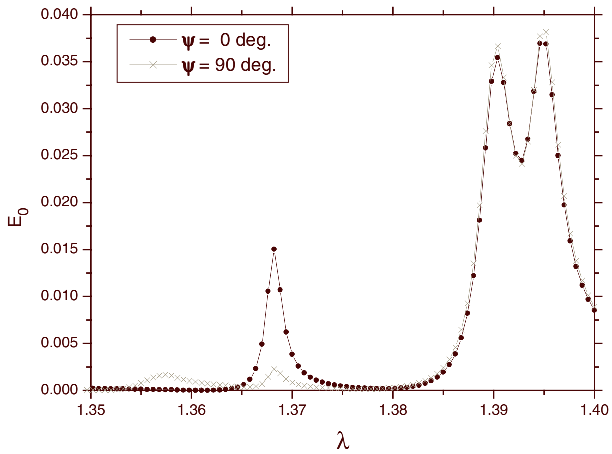

Figure 5. Spectral dependence of the zero-order transmittivity for an off-normal incidence ($\theta = 1°$, $\varphi = -150°$) and for two incident polarization angles ψ.

of the 2D grating of hole array reveal the existence of a complex zero of the transmission amplitude $\lambda^z = 1.368\,88 + \mathrm{i}0.000\,476\,45$. Using the values of the zero and the pole, figure 4 presents a comparison of the transmittivity E_0, calculated by using the rigorous electromagnetic theory and the so-called phenomenological formula

$$E_0(\lambda) = c\left|\frac{\lambda - \lambda^z}{\lambda - \lambda^p}\right|^2 \tag{5}$$

where c is a constant which does not depend on λ and the power 2 is due to the fact that the efficiency is proportional to the squared modulus of the amplitude. As is observed, the two curves match perfectly in the spectral region of enhanced transmission, which shows that the resonance enhanced transmission, experimentally observed in [1] can be explained by a plasmon excitation on the lower interface of the silver film. The channel for this excitation has already been discussed in a previous section of this paper.

When going out of normal incidence, the incident plane wave is no longer symmetric with respect to the origin, thus one can expect that the antisymmetric solutions can also be excited. Strictly speaking, these solutions no longer have such simple symmetry, because their field distribution is also affected by the off-normal incidence direction. To study this phenomenon, we chose an incident wavevector out of both the x–y and y–z planes. Its projection on the x–y plane makes an angle θ with the y-axis, while its projection on the x–z plane makes an angle φ with the x-axis. One more degree of freedom determines the polarization direction, when a linearly polarized incident plane wave is chosen. It will be characterized by the angle ψ, defined so that $\psi = 90°$ when the electric field vector lies in the plane of incidence, and $\psi = 0$ when the electric field vector is perpendicular to the plane of incidence.

In the next example we chose $\theta = 1°$ and $\varphi = -150°$. Instead of a single pole λ^p, 3 different poles are observed: $\lambda_1^p = 1.3455 + \mathrm{i}0.064\,926$, $\lambda_2^p = 1.3737 + \mathrm{i}0.006\,1341$ and $\lambda_3^p = 1.3878 + \mathrm{i}0.002\,022$, while the general arguments have led us to expect the existence of four poles. In order to clarify

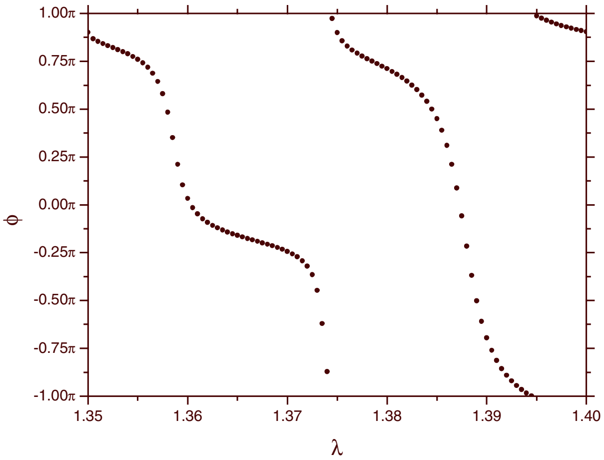

Figure 6. Spectral dependence of the phase Φ of the determinant of the scattering matrix ($\theta = 1°$, $\varphi = -150°$, $\psi = 90°$).

this point, figure 5 presents the spectral dependence of the transmittivity and, indeed, one can observe four resonance peaks, more or less pronounced, the two on the right part of the figure almost coinciding. Further numerical studies confirmed that the pole λ_3^p is a double one; at least, we were not able to separate it numerically into two different poles. Evidence can be found in figure 6, which represents the phase Φ of the determinant of the scattering matrix in the case presented in figure 5. One can observe three spectral regions where the phase varies strongly:

(i) $\lambda \cong 1.355\ \mu$m: phase shift close to π;
(ii) $\lambda \cong 1.375\ \mu$m: phase shift close to π;
(iii) $\lambda \cong 1.39\ \mu$m: phase shift close to 2π.

This shows that the first two poles are single, while the rightmost one is double.

As happens in normal incidence or for 1D gratings, the poles are accompanied by zeros of the diffracted order amplitudes. The influence of these zeros is well observed in figure 5 where they lead to a splitting of the rightmost maximum.

It is worth noticing that the poles do not depend on the incident polarization, since they are poles of the total scattering matrix, which accounts for the system response to all kinds of polarization. And indeed, the existence of the poles is well pronounced for the two orthogonal polarizations of the incident wave, as observed in figure 5. In contrast, the zeros depend on the excitation (incident) conditions, thus the different responses to the different values of ψ.

5. Conclusion

Using the eigenvalue technique, we were able to find a new channel which allows light propagation along subwavelength holes arranged in a periodic array in a highly conducting metallic film. We have proven that the previously observed extraordinary transmission is due to the resonant excitation through this channel of surface plasmons at the array lower interface between the metal and the glass substrate. The phenomenological study using complex poles of the scattering operator and complex zeros of the transmitted amplitude was able to account for this unexpected effect in both qualitative and quantitative ways.

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