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The determination of mode coupling coefficients

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Abstract. Analytical expressions for coupling coefficients for oblique incidence on a grating with an arbitrary groove profile on a planar waveguide with an arbitrary refractive index profile are obtained in the form of mode eigenfunctions values on the waveguide surface.

In our previous work [1] we investigated theoretically the coupling of modes in planar waveguides induced, in particular, by a grating. Analytical expressions for the coupling coefficients have been obtained to first order approximation in groove depth d for oblique incidence and arbitrary grating and waveguide refractive index profiles. However, for direct practical application of these formulae in integrated optical devices it is necessary to express the dependence of coupling coefficients on the waveguide parameters in the form of the mode eigenfunctions values on the waveguide–air boundary. The purpose of this work is to show how this can be done for a waveguide with an arbitrary index profile. The $\mathrm{TE}_\eta - \mathrm{TE}_\mu$ coupling coefficients are [1]

$$\Gamma_{\mu\eta}^{\mathrm{TETE}} = \frac{d}{2} F_{\mathrm{m}} C_{\mu\eta} \frac{\beta_\eta^2 - \beta_\mu^2}{\beta_\mu} \frac{\cos(\theta_\eta - \theta_\mu)}{\cos\theta_\mu}, \tag{1}$$

where

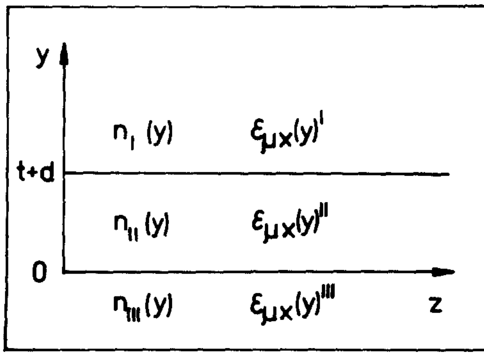
$$C_{\mu\eta} = \frac{\beta_\mu}{2\omega\mu_0} \left[\frac{\partial}{\partial d} \int_{-\infty}^{\infty} (\mathscr{E}_{\mu x}^{d=0}(y))^* \mathscr{E}_{\eta x}^{d}(y)\, \mathrm{d}y \right]_{d=0}, \tag{2}$$

and β_μ, θ_μ and $\mathscr{E}_{\mu x}^d(y)$ are the propagation constant, the angle of propagation and the eigenfunction, respectively, of the μth mode of the waveguide shown schematically in the figure. ω is the angular frequency, μ_0 the vacuum permittivity and F_{m} is the function corresponding to the coupling grating Fourier amplitude. Integrating twice in parts the expression

$$\int_{-\infty}^{\infty} \frac{\partial^2 [\mathscr{E}_{\mu x}^{d=0}(y)]^*}{\partial y^2} \frac{\partial \mathscr{E}_{\eta x}^d(y)}{\partial d} \bigg|_{d=0} \mathrm{d}y, \tag{3}$$

and taking into account the wave equation, we obtain

$$(\beta_\eta^2 - \beta_\mu^2) C_{\mu\eta} = \frac{\beta_\mu}{2\omega\mu_0} [\mathscr{E}_{\mu x}^{d=0}(y)]^* \left[\frac{\partial^2 \mathscr{E}_{\eta x}^d(y)^{\mathrm{II}}}{\partial y \partial d} - \frac{\partial^2 \mathscr{E}_{\eta x}^d(y)^{\mathrm{I}}}{\partial y \partial d} \right]_{\substack{d=0 \\ y=t}}. \tag{4}$$



Schematic representation of planar optical waveguide.

The mode eigenfunctions and their normal derivatives are continuous over the boundary I II;

$$\mathcal{E}^{d}_{\eta x}(t+d)^{\mathrm{I}}=\mathcal{E}^{d}_{\eta x}(t+d)^{\mathrm{II}}, \tag{5 a}$$

$$\left.\frac{\partial\mathcal{E}^{d}_{\eta x}(y)^{\mathrm{I}}}{\partial y}\right|_{y=t+d}=\left.\frac{\partial\mathcal{E}^{d}_{\eta x}(y)^{\mathrm{II}}}{\partial y}\right|_{y=t+d}. \tag{5 b}$$

The continuity of $\partial\mathcal{E}^{d}_{\mu x}(y)/\partial d$ and $\partial^2\mathcal{E}^{d}_{\mu x}(y)/\partial y\partial d$ requires more precise treatment, because $\mathcal{E}^{d}_{\eta x}(y=t+d)$ depends on d both directly and indirectly through $y=t+d$. Taking a derivative in d, (5) becomes

$$\left.\frac{\partial\mathcal{E}^{d}_{\eta x}(y)^{\mathrm{I}}}{\partial y}\right|_{y=t+d}+\left.\frac{\partial\mathcal{E}^{d}_{\eta x}(y)^{\mathrm{I}}}{\partial d}\right|_{y=t+d}=\left.\frac{\partial\mathcal{E}^{d}_{\eta x}(y)^{\mathrm{II}}}{\partial y}\right|_{y=t+d}+\left.\frac{\partial\mathcal{E}^{d}_{\eta x}(y)^{\mathrm{II}}}{\partial d}\right|_{y=t+d}, \tag{6 a}$$

$$\left.\frac{\partial^2\mathcal{E}^{d}_{\eta x}(y)^{\mathrm{I}}}{\partial y^2}\right|_{y=t+d}+\left.\frac{\partial^2\mathcal{E}^{d}_{\eta x}(y)^{\mathrm{I}}}{\partial y\partial d}\right|_{y=t+d}=\left.\frac{\partial^2\mathcal{E}^{d}_{\eta x}(y)^{\mathrm{II}}}{\partial y^2}\right|_{y=t+d}+\left.\frac{\partial^2\mathcal{E}^{d}_{\eta x}(y)^{\mathrm{II}}}{\partial y\partial d}\right|_{y=t+d}. \tag{6 b}$$

It is evident from (5 *b*) and (6 *a*) that $\partial\mathcal{E}^{d}_{\eta x}(y)/\partial d$ is continuous over a boundary I–II. Using (6 *b*), equation (4) becomes

$$(\beta_\eta^2-\beta_\mu^2)C_{\mu\eta}=(\beta\mu/2\omega\mu_0)k^2[n_{\mathrm{II}}^2(t)-n_{\mathrm{I}}^2(t)]\mathcal{E}^{*}_{\mu x}(t)\,\mathcal{E}_{\eta x}(t), \tag{7}$$

and the expression for the coupling coefficient takes the form

$$\Gamma^{\mathrm{TETE}}_{\mu\eta}=\frac{d}{2}F_{\mathrm{m}}\frac{k^2[n_{\mathrm{II}}^2(t)-n_{\mathrm{I}}^2(t)]}{2\omega\mu_0}\mathcal{E}^{*}_{\mu x}(t)\mathcal{E}_{\eta x}(t)\frac{\cos(\theta_\mu-\theta_\eta)}{\cos\theta_\mu}. \tag{8}$$

The coupling coefficient for TM–TM coupling can be obtained in a similar way:

$$\Gamma^{\mathrm{TMTM}}_{\mu\eta}=\frac{d}{2}F_{\mathrm{m}}\left[\frac{1}{n_{\mathrm{I}}^2(t)}-\frac{1}{n_{\mathrm{II}}^2(t)}\right]\frac{\mathcal{H}^{*}_{\mu x}(t)\mathcal{H}_{\eta x}(t)}{2\omega\varepsilon_0}\frac{\beta_\mu\beta_\eta+q_\mu q_\eta(n_{\mathrm{II}}^2(t)/n_{\mathrm{I}}^2(t))\cos(\theta_\mu-\theta_\eta)}{\cos\theta_\mu} \tag{9}$$

where $q_\mu^2=\beta_\mu^2-k^2n_{\mathrm{I}}^2$ and $\mathcal{H}_{\mu x}(t)$ is an eigenfunction of the μth TM mode taken on the waveguide surface.

The case with conversion of polarization is more complicated because of the summation over all possible modes in the coupling coefficients (see table 1 of [1]). We have calculated numerically the coupling coefficients for 9 different waveguides. The following cases have been considered: $n_{\rm I}=1$, $n_{\rm II}=1{\cdot}62$, $n_{\rm III}=1{\cdot}515$, with 5 thicknesses $t=3$, 5, 10, 15, 20 μm and $n_{\rm I}=1$, $n_{\rm II}=2{\cdot}234$, $n_{\rm III}=2{\cdot}214$, with $t=6$, 10, 20, 40 μm. These cases include both mono and multimode (up to 4 modes) waveguides. The following conclusions can be drawn from the numerical treatment:

(*a*) the summation over all possible modes cannot be omitted;
(*b*) in both mono and multimode waveguides the summation must include an integral over a continuous spectrum, otherwise the results are changed drastically;
(*c*) within a mistake of 5% the coupling coefficients can be approximated with the following formula:

$$\Gamma_{\mu\eta}^{\rm TETM}=\frac{d}{2}F_{\rm m}q_\eta n_{\rm II}^2\left(\frac{1}{n_{\rm I}^2}-\frac{1}{n_{\rm II}^2}\right)\mathscr{E}_{\mu x}^*(t)\mathscr{H}_{\eta x}(t)\frac{\sin(\theta_\mu-\theta_\eta)}{\cos\theta_\mu}. \tag{10}$$

In conclusion, it is worth noting that the expressions (8) and (9) are valid for the general case of arbitrary grating and waveguide refractive index profiles. A comparison between our results for the case of step refractive index profile and the results of Stegeman *et al.* [2] has been already made in [1].

References

[1] Popov, E., and Mashev, L., 1985, *Optica Acta*, **32**, 265.
[2] Stegeman, G. I., Sarid, D., Burke, J. J., and Hall, D. G., 1981, *J. opt. Soc. Am.*, **71**, 1497.