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Analysis of mode coupling in planar optical waveguides

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Abstract. Coupling between the modes in a planar optical waveguide induced by a boundary step discontinuity, by a single groove and by a grating with arbitrary cross section, is investigated theoretically. Analytical expressions for the coupling mode coefficients are obtained by the mode-matching method, containing the angular dependence in explicit form for both TE and TM incidence. An analogy of Brewster's law for a planar waveguide with a single step boundary discontinuity is obtained. The results for gratings are compared with the coupled-mode theory and the total field analysis.

1. Introduction

The problem of mode coupling in a planar optical waveguide on a material boundary is of great importance in integrated optics. The phenomenon is observed in strip waveguides, slab waveguide edges, thin film lenses and prisms, modulators with a buffer layer under the electrodes, gratings, etc. However, surface relief gratings are widely used as input-output couplers [1], reflectors for semiconductor lasers [2], demultiplexers and filters [3, 4] and phase-matching elements [5]. Numerical calculations of the co-linear coupling between the modes have been carried out for near-field [6] and far-field [7] radiation patterns of a tapered slab waveguide and for edge scattering in a planar waveguide [8]. Normal incidence of the waves on the grating has been considered by Marcuse [9] and Kogelnik [10]. Two approaches have been developed—an ideal-mode and a local-mode analysis [9], which give identical results for TE incidence. However, the results differ for TM incidence, because the boundary conditions are not satisfied.

The more general (and more interesting) case is that for oblique incidence of the modes on a material boundary in the waveguide. Some new effects, such as conversion of the mode polarization and the appearance of leakage and resonance effects, have been discussed elsewhere [11]. Recently much attention has been devoted to the oblique incidence of the guided waves on relief gratings [12–18]. The most commonly used approach is the perturbation analysis [12, 14–16] where the change in refractive index is introduced into the Maxwell equations as a perturbation, to give a set of coupled-mode equations for the mode amplitudes. In the case of a corrugated waveguide, however, the geometry changes the problem and must be taken into account. The vector field representation in [12, 14, 16] does not satisfy new boundary conditions at the corrugation. Explicit expressions for the coupling coefficients have been obtained up to a first order in terms of the groove depth [13], but the plane-wave expansion of the field in the grating region is valid only in some restricted cases [19].

A precise analysis of obliquely incident guided waves scattered on a surface grating is proposed by Stegeman *et al.* [17]. An additional, 'growing-wave' field is included in a plane-wave expansion in order to satisfy both the Maxwell's equations and the boundary conditions. How this method can be extended to the graded-index waveguides is not evident. The collinear or the contralinear coupling can be represented as a particular case of the oblique incidence as the angle between the direction of propagation and the direction normal to the boundary becomes zero. However, it is important to know the angular dependence of the coupling coefficients between different modes, as well as the dependence of these coefficients on the configuration parameters at various angles of incidence. The difficulties in the oblique incidence treatment follow from the fact that this case is a three-dimensional one and a set of Maxwell equations must be considered, while in a normal incidence a wave equation can be used. Furthermore, in a collinear case the boundary conditions at the transverse boundary are easily satisfied because the electromagnetic field has only three vector components for each TE and TM polarizations. However, for oblique incidence this number increases to five [11]. In this paper we present analytical results for the mode coupling of obliquely incident waves on a surface step discontinuity and on a grating with an arbitrary cross-section in a waveguide with an arbitrary refractive index profile. We use a so-called mode-matching approach, whose basic conditions are described in § 2. Evidence of this method has been demonstrated magnificently by Peng and Oliner [11].

In § 2 we give an expansion of the field in different regions and apply the mode-matching procedure at the step boundary to obtain a set of equations for the mode amplitudes. We consider step height which is small when compared to the waveguide effective thickness, because of its importance in the practical cases mentioned above. In § 3, the different types of coupling—TE–TE, TE–TM, TM–TE and TM–TM are considered. The angular dependence of the coupling coefficients is expressed in explicit form up to the first order for small step height. Mode coupling by a groove on a planar optical waveguide is discussed in § 4. The results are applied to gratings in § 5 where explicit analytical expressions for the coupling coefficients and their angular dependence are obtained. A waveguide with a step refractive index profile is examined as a special case in §6. Some peculiar consequences, including TE–TE, TE–TM and TM–TM coupling are treated in § 7. An interesting result is the existence of Brewster's law analogy in planar optical waveguides with a step discontinuity, which to our knowledge has not been previously discussed. Comparison with the total-field [17] and the local-mode [9] analysis for surface relief gratings in § 7 shows that the results are identical for normal incidence. For oblique incidence our results differ from those of Stegeman *et al.* [17].

2. Mode-matching procedure

Let us consider the configuration shown schematically in figure 1. Generally, in a planar optical waveguide (II) with an arbitrary refractive index profile there is a semi-infinite layer with a refractive index n_i and height d. We assume that the waveguide, the substrate and the superstrate media are lossless. Guided waves are incident obliquely on the step structure at an angle θ_v (figure 1). The media 1 and 2 differ in their refractive indices in the step structure region only. When the guided waves with TE or TM polarization propagate along the z direction, the step junction excites scattered waves with the same polarization [11, 20, 21]. In the case of oblique incidence the electromagnetic field has five components and the modes are neither pure TE nor pure TM. The hybrid nature of the modes involves other types of coupling [11, 20, 21], and in particular TE–TM and TM–TE conversion.

For a description of this problem we need an electromagnetic field representation, satisfying the following conditions:

(*a*) The Maxwell equations in each region.
(*b*) The boundary conditions at the boundary between regions I, II and III.
(*c*) The boundary conditions at the boundary between media 1 and 2.

To solve the boundary–value problem by the mode-matching approach, an eigensolution for the Maxwell equations and separate boundary conditions for media 1 and 2 are necessary. The general solution then can be represented as a superposition of eigensolutions with constant coefficients. These coefficients (in our case the mode amplitudes) are determined from the boundary conditions at the boundary between the two media. This formalism is quite general and is not restricted by the waveguide and step structure parameters. A set of equations, satisfying conditions (*a*) and (*b*) have solutions which are the mode fields of the infinite waveguides corresponding to medium 1 or 2 (see for example [9, 10]). The general solution in each medium 1 or 2 can be represented as a summation over ν of every TE_ν and TM_ν mode with constant amplitudes a_ν and b_ν respectively. Furthermore all possible directions of propagation of the modes must be included, i.e. an integral over the angle of propagation θ must be taken. The electric field components may be written as

$$\left.\begin{aligned}
E_x^j &= \int_0^{2\pi} \sum_\nu [a_\nu^j(\theta)\mathscr{E}_{\nu_x}^j(y)\cos\theta \exp(-\mathrm{i}\beta_\nu^j(x\sin\theta + z\cos\theta)) \\
&\qquad + b_\nu^j(\theta)\mathscr{E}_{\nu_z}^j(y)\sin\theta\exp(-\mathrm{i}\bar{\beta}_\nu^j(x\sin\theta + z\cos\theta))]\,\mathrm{d}\theta, \\
E_y^j &= \int_0^{2\pi} \sum_\nu b_\nu^j(\theta)\mathscr{E}_{\nu_y}^j(y)\exp(-\mathrm{i}\bar{\beta}_\nu^j(x\sin\theta + z\cos\theta))\,\mathrm{d}\theta \\
E_z^j &= \int_0^{2\pi} \sum_\nu [-a_\nu^j(\theta)\mathscr{E}_{\nu_x}^j(y)\sin\theta\exp(-\mathrm{i}\beta_\nu^j(x\sin\theta + z\cos\theta)) \\
&\qquad + b_\nu^j(\theta)\mathscr{E}_{\nu_z}^j(y)\cos\theta\exp(-\mathrm{i}\bar{\beta}_\nu^j(x\sin\theta + z\cos\theta))]\,\mathrm{d}\theta,
\end{aligned}\right\} \quad (1)$$

where $j=1,2$ is a medium index. Similar expressions can be obtained for the magnetic field components. Mode eigenfunctions $\mathscr{E}_{\nu_x}(y)$ and $\mathscr{H}_{\nu_x}(y)$ are solutions of the scalar wave equations for TE and TM polarization respectively. The other field components satisfy the relations

$$\left.\begin{aligned}
\mathscr{E}_{\nu_z} &= -\frac{1}{\mathrm{i}\omega\varepsilon_0 n^2(y)}\frac{\mathrm{d}\mathscr{H}_{\nu_x}(y)}{\mathrm{d}y}, & \mathscr{H}_{\nu_z} &= \frac{1}{\mathrm{i}\omega\mu_0}\frac{\mathrm{d}\mathscr{E}_{\nu_x}(y)}{\mathrm{d}y}, \\
\mathscr{E}_{\nu_y} &= -\frac{\bar{\beta}_\nu}{\omega\varepsilon_0 n^2(y)}\mathscr{H}_{\nu_x}(y), & \mathscr{H}_{\nu_y} &= \frac{\beta_\nu}{\omega\mu_0}\mathscr{E}_{\nu_x}(y),
\end{aligned}\right\} \quad (2)$$

where ε_0 and μ_0 are vacuum permeability and permittivity, ω is the angular frequency, n is the corresponding refractive index for each region, θ_ν is the angle of propagation of the νth mode, β_ν is the TE_ν propagation constant and the overbar refers to TM modes. It must be pointed out that the mode spatial distributions $\mathscr{E}_{\nu_x}$, $\mathscr{E}_{\nu_y}$, $\mathscr{E}_{\nu_z}$, $\mathscr{H}_{\nu_x}$, $\mathscr{H}_{\nu_y}$, $\mathscr{H}_{\nu_z}$ are related to the eigen coordinate system connected with the direction of propagation (figure 1). The sum over ν contains a sum over a discrete

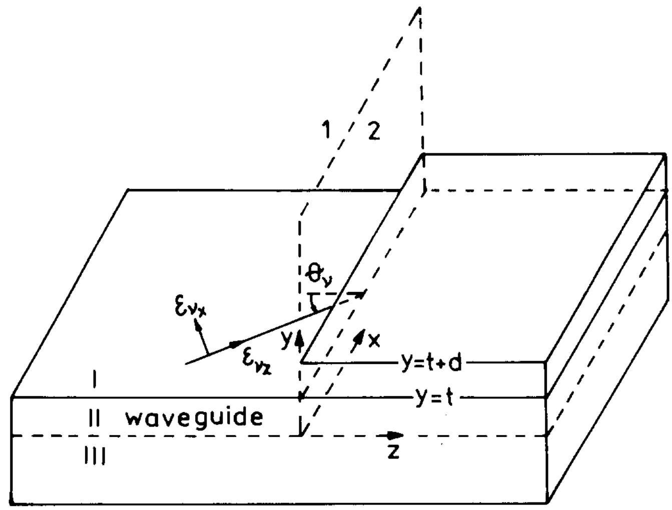

Figure 1. Planar optical waveguide with a step structure on a superstrate.

spectrum and an integral over a continuous spectrum of eigenmodes. The next step is to apply the boundary condition (c) to the field components.

Usually the satisfaction of the boundary conditions means the continuity of the field components tangential to the boundary over which the refractive index is discontinuous. This is fulfilled for condition (b). The situation is more complicated for condition (c), because the boundary between media 1 and 2 is not completely physical, i.e. the refractive index is discontinuous only in the step region of the boundary. This means that a more precise examination of the normal field components is needed. This has not been attempted before. The normal field components E_z and H_z must be continuous at the boundary $z=0$ in the regions $y>t+d$ and $y<t$ (see figure 1), because the two media are identical in these regions. In the Appendix it is shown that the continuity of E_z and H_z follows directly from the continuity of E_x, E_y, H_x, H_y and thus does not give a new set of equations. The requirement of the continuity of the tangential field components over the whole 1–2 boundary leads to this set of four equations,

$$\int_0^{2\pi} \sum_{\nu} [a_\nu^1(\theta)\mathcal{E}_{\nu_x}^1(y)\cos\theta\exp(-i\beta_\nu^1 x\sin\theta)+b_\nu^1(\theta)\mathcal{E}_{\nu_z}^1(y)\sin\theta\exp(-i\overline{\beta}_\nu^1 x\sin\theta)]\,d\theta$$
$$=\int_0^{2\pi}\sum_{\nu}[a_\nu^2(\theta)\mathcal{E}_{\nu_x}^2(y)\cos\theta\exp(-i\beta_\nu^2 x\sin\theta)$$
$$+b_\nu^2(\theta)\mathcal{E}_{\nu_z}^2(y)\sin\theta\exp(-i\overline{\beta}_\nu^2 x\sin\theta)]\,d\theta, \qquad (3\,a)$$

$$\int_0^{2\pi}\sum_{\nu} b_\nu^1(\theta)\mathcal{E}_{\nu_y}^1(y)\exp(-i\overline{\beta}_\nu^1 x\sin\theta)\,d\theta=\int_0^{2\pi}\sum_{\nu} b_\nu^2(\theta)\mathcal{E}_{\nu_y}^2(y)\exp(-i\overline{\beta}_\nu^2 x\sin\theta)\,d\theta, \quad (3\,b)$$

$$\int_0^{2\pi} \sum_\nu [b_\nu^1(\theta)\mathscr{H}_{\nu_x}^1(y)\cos\theta\exp(-i\bar{\beta}_\nu^1 x\sin\theta) + a_\nu^1(\theta)\mathscr{H}_{\nu_z}^1(y)\sin\theta\exp(-i\beta_\nu^1 x\sin\theta)]\,d\theta$$
$$= \int_0^{2\pi} \sum_\nu [b_\nu^2(\theta)\mathscr{H}_{\nu_x}^2(y)\cos\theta\exp(-i\bar{\beta}_\nu^2 x\sin\theta)$$
$$+ a_\nu^2(\theta)\mathscr{H}_{\nu_z}^2(y)\sin\theta\exp(-i\beta_\nu^2 x\sin\theta)]\,d\theta, \qquad (3\,c)$$

$$\int_0^{2\pi} \sum_\nu a_\nu^1(\theta)\mathscr{H}_{\nu_y}^1(y)\exp(-i\beta_\nu^1 x\sin\theta)\,d\theta$$
$$= \int_0^{2\pi} \sum_\nu a_\nu^2(\theta)\mathscr{H}_{\nu_y}^2(y)\exp(-i\beta_\nu^2 x\sin\theta)\,d\theta, \qquad (3\,d)$$

where the upper indexes refer to the media 1 and 2. Multiplying (3 a) by $(\beta_\mu^1/4\pi\omega\mu_0)\mathscr{E}_{\mu x}^{1*}\exp(i\beta_\mu^1 x\sin\theta_\mu^1)$ and integrating over the infinite x–y cross-section we obtain

$$(a_\mu^{1+} - a_\mu^{1-})\cos\theta_\mu^1 + \sum_\nu (b_\nu^{1+} + b_\nu^{1-})\sin\bar{\theta}_\nu^1 \chi_{\mu\nu}$$
$$= \sum_\nu (a_\nu^{2+} - a_\nu^{2-})\cos\theta_\nu^2 K_{\mu\nu}^1 + \sum_\nu (b_\nu^{2+} + b_\nu^{2-})\sin\bar{\theta}_\nu^2 K_{\mu\nu}^2, \qquad (4)$$

$$\left.\begin{aligned}
\chi_{\mu\nu} &= \frac{\beta_\mu^1}{2\omega\mu_0}\int_{-\infty}^{\infty} \mathscr{E}_{\mu x}^{1*}(y)\mathscr{E}_{\nu_z}^1(y)\,dy,\\
K_{\mu\nu}^1 &= \frac{\beta_\mu^1}{2\omega\mu_0}\int_{-\infty}^{\infty} \mathscr{E}_{\mu x}^{1*}(y)\mathscr{E}_{\nu_x}^2(y)\,dy,\\
K_{\mu\nu}^2 &= \frac{\beta_\mu^1}{2\omega\mu_0}\int_{-\infty}^{\infty} \mathscr{E}_{\mu x}^{1*}(y)\mathscr{E}_{\nu_z}^2(y)\,dy
\end{aligned}\right\} \qquad (5)$$

and the asterisk denotes complex conjugation.

In the derivation of (4) the following orthogonality relations between the mode fields and the propagation factors have been used:

$$\int_{-\infty}^{\infty} \mathscr{E}_{\mu_x}^*(y)\mathscr{E}_{\nu_x}(y)\,dy = \frac{2\omega\mu_0}{\beta_\mu}\delta_{\mu\nu}, \qquad (6)$$

$$\int_{-\infty}^{\infty} \frac{\mathscr{H}_{\mu_x}^*(y)}{n^2(y)}\mathscr{H}_{\nu_x}(y)\,dy = \frac{2\omega\varepsilon_0}{\beta_\mu}\delta_{\mu\nu}, \qquad (7)$$

and

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} \exp[-i(\beta_\nu^j\sin\theta - \beta_\mu^1\sin\theta_\mu^1)x]\,dx = \delta(\beta_\nu^j\sin\theta - \beta_\mu^1\sin\theta_\mu^1). \qquad (8)$$

Here $\delta_{\mu\nu}$ is a Kronecker symbol and δ is a Dirac δ function. In the right-hand side of (6) and (7) the energy flow is normalized to 1 W per unit cross-section perpendicular to the mode propagation direction.

From (8) we obtain Snell's law in the planar case,

$$\bar{\beta}^1_\nu \sin \bar{\theta}^1_\nu = \bar{\beta}^2_\nu \sin \bar{\theta}^2_\nu = \beta^2_\nu \sin \theta^2_\nu = \beta^1_\nu \sin \theta^1_\nu = \beta^1_\mu \sin \theta^1_\mu, \tag{9}$$

which determines the angles of reflection and transmission for each TE and TM mode.

The modes in (4) are divided into two kinds with '+' and '−' indices which correspond to the 'positive' or 'negative' direction of propagation with respect to the z axis. This is because the angle θ between the z axis and the direction of propagation is measured from the positive or negative direction of z axis (figure 1). Multiplying (3 c) by $(\bar{\beta}^1_\mu/4\pi\omega\varepsilon_0)\mathscr{H}^{1*}_{\mu x}(y)/[n^1(y)]^2 \exp(i\bar{\beta}^1_\mu x \sin \bar{\theta}^1_\mu)$ and repeating the same procedure as with (3 a), the result is

$$(b^{1+}_\mu - b^{1-}_\mu)\cos \bar{\theta}^1_\mu + \sum_\nu (a^{1+}_\nu + a^{1-}_\nu)\sin \theta^1_\nu \bar{\chi}_{\mu\nu}$$
$$= \sum_\nu (b^{2+}_\nu - b^{2-}_\nu)\cos \bar{\theta}^2_\nu \bar{K}^1_{\mu\nu} + \sum_\nu (a^{2+}_\nu + a^{2-}_\nu)\sin \theta^2_\nu \bar{K}^2_{\mu\nu}, \tag{10}$$

where

$$\left.\begin{aligned} \bar{\chi}_{\mu\nu} &= \frac{\bar{\beta}^1_\mu}{2\omega\varepsilon_0}\int_{-\infty}^{\infty} \frac{\mathscr{H}^{1*}_{\mu x}(y)}{[n^1(y)]^2}\mathscr{H}^1_{\nu z}(y)\,dy, \\ \bar{K}^1_{\mu\nu} &= \frac{\bar{\beta}^1_\mu}{2\omega\varepsilon_0}\int_{-\infty}^{\infty} \frac{\mathscr{H}^{1*}_{\mu x}(y)}{[n^1(y)]^2}\mathscr{H}^2_{\nu x}(y)\,dy, \\ \bar{K}^2_{\mu\nu} &= \frac{\bar{\beta}^1_\mu}{2\omega\varepsilon_0}\int_{-\infty}^{\infty} \frac{\mathscr{H}^{1*}_{\mu x}(y)}{[n^1(y)]^2}\mathscr{H}^2_{\nu z}(y)\,dy. \end{aligned}\right\} \tag{11}$$

Similar considerations for (3 b) and (3 d), using (2), lead to

$$\bar{\beta}^1_\mu(b^{1+}_\mu + b^{1-}_\mu) = \sum_\nu (b^{2+}_\nu + b^{2-}_\nu)\bar{\beta}^2_\nu \bar{K}^3_{\mu\nu} \tag{12}$$

and

$$\beta^1_\mu(a^{1+}_\mu + a^{1-}_\mu) = \sum_\nu (a^{2+}_\nu + a^{2-}_\nu)\beta^2_\nu K^1_{\mu\nu}, \tag{13}$$

where

$$\bar{K}^3_{\mu\nu} = \frac{\bar{\beta}^1_\mu}{2\omega\varepsilon_0}\int_{-\infty}^{\infty} \frac{\mathscr{H}^{1*}_{\mu x}(y)}{[n^2(y)]^2}\mathscr{H}^2_{\nu x}(y)\,dy \tag{14}$$

The mode-matching equations (4), (10), (12) and (13) are a system of $4N$ equations for $4N$ unknown amplitudes $\{a^{1-}_\mu, b^{1-}_\mu, a^{2+}_\mu, b^{2+}_\mu\}$, where N is the number of modes taken into account. The modes with amplitudes $\{a^{1+}_\mu, b^{1+}_\mu, a^{2-}_\mu, b^{2-}_\mu\}$ are incident on the step structure. Further we shall consider a practically interesting case of a small step height, compared to the waveguide effective thickness. In a first order approximation the expansion of all quantities characterizing medium 2 in series of d

are as follows:

$$\left.\begin{aligned}
K^1_{\mu\nu} &= \delta_{\mu\nu} + d\left[\frac{\partial}{\partial d}\frac{\beta^1_\mu}{2\omega\mu_0}\int_{-\infty}^{\infty} \mathscr{E}^{1*}_{\mu x}(y)\mathscr{E}^2_{\nu x}(y)\,\mathrm{d}y\right]_{d=0},\\
\bar{K}^1_{\mu\nu} &= \delta_{\mu\nu} + d\left[\frac{\partial}{\partial d}\frac{\bar{\beta}^1_\mu}{2\omega\varepsilon_0}\int_{-\infty}^{\infty} \frac{\mathscr{H}^{1*}_{\mu x}(y)}{[n^1(y)]^2}\mathscr{H}^2_{\nu x}(y)\,\mathrm{d}y\right]_{d=0},\\
K^2_{\mu\nu} &= \chi_{\mu\nu} + d\left[\frac{\partial}{\partial d}\frac{\beta^1_\mu}{2\omega\mu_0}\int_{-\infty}^{\infty} \mathscr{E}^{1*}_{\mu x}(y)\mathscr{E}^2_{\nu z}(y)\,\mathrm{d}y\right]_{d=0},\\
\bar{K}^2_{\mu\nu} &= \bar{\chi}_{\mu\nu} + d\left[\frac{\partial}{\partial d}\frac{\bar{\beta}^1_\mu}{2\omega\varepsilon_0}\int_{-\infty}^{\infty} \frac{\mathscr{H}^{1*}_{\mu x}(y)}{[n^2(y)]^2}\mathscr{H}^2_{\nu z}(y)\,\mathrm{d}y\right]_{d=0},\\
\bar{K}^3_{\mu\nu} &= \delta_{\mu\nu} + d\left[\frac{\partial}{\partial d}\frac{\bar{\beta}^1_\mu}{2\omega\varepsilon_0}\int_{-\infty}^{\infty} \frac{\mathscr{H}^{1*}_{\mu x}(y)}{[n^1(y)]^2}\mathscr{H}^2_{\nu x}(y)\,\mathrm{d}y\right]_{d=0}\\
&\quad + \frac{\bar{\beta}^1_\mu}{2\omega\varepsilon_0}\left(\frac{1}{[n_2(t)]^2}-\frac{1}{[n_1(t)]^2}\right)\mathscr{H}^{1*}_{\mu x}(t)\mathscr{H}^1_{\nu x}(t),
\end{aligned}\right\} \tag{15}$$

$$\left.\begin{aligned}
\beta^2_\nu &= \beta^1_\nu\left(1+\frac{d}{\beta^1_\nu}\frac{\partial\beta^2_\nu}{\partial d}\bigg|_{d=0}\right),\\
\sin\theta^2_\nu &= \sin\theta^1_\nu\left(1-\frac{d}{\beta^1_\nu}\frac{\partial\beta^2_\nu}{\partial d}\bigg|_{d=0}\right),\\
\cos\theta^2_\nu &= \cos\theta^1_\nu\left(1+\frac{d}{\beta^1_\nu}\tan^2\theta^1_\nu\frac{\partial\beta^2_\nu}{\partial d}\bigg|_{d=0}\right),
\end{aligned}\right\} \tag{16}$$

where n_i, $i=1$–3 are the refractive indices of regions I–III. For TM polarization the quantities in (16) should be overbarred. Substitution of (15) and (16) in (4), (10), (12) and (13) after regrouping the terms containing a and b leads to the required first order mode coupling system,

$$a^{1+}_\mu + a^{1-}_\mu = \sum_\nu (a^{2+}_\nu + a^{2-}_\nu)\left[\delta_{\mu\nu}(1+\xi_\nu d) + c_{\mu\nu}\frac{\beta_\nu}{\beta_\mu}d\right], \tag{17 a}$$

$$\begin{aligned}
a^{1+}_\mu - a^{1-}_\mu &= \sum_\nu (a^{2+}_\nu - a^{2-}_\nu)\left[\delta_{\mu\nu}(1+\xi_\nu\tan^2\theta_\nu d) + c_{\mu\nu}\frac{\cos\theta_\nu}{\cos\theta_\mu}d\right]\\
&\quad + d\sum_\nu \frac{\sin\bar{\theta}_\nu}{\cos\theta_\mu}\sum_\tau (b^{2+}_\tau + b^{2-}_\tau)\left[\delta_{\tau\nu}(K_{\mu\nu}-2\chi_{\mu\nu}\bar{\xi}_\nu) - \chi_{\mu\nu}(\bar{c}_{\nu\tau}+\bar{\sigma}_{\nu\tau})\frac{\bar{\beta}_\tau}{\bar{\beta}_\nu}\right]
\end{aligned} \tag{17 b}$$

and

$$b^{1+}_\mu + b^{1-}_\mu = \sum_\nu (b^{2+}_\nu + b^{2-}_\nu)\left[\delta_{\mu\nu}(1+\bar{\xi}_\nu d) + (\bar{c}_{\mu\nu}+\bar{\sigma}_{\mu\nu})\frac{\bar{\beta}_\nu}{\bar{\beta}_\mu}d\right], \tag{18 a}$$

$$\begin{aligned}
b^{1+}_\mu - b^{1-}_\mu &= \sum_\nu (b^{2+}_\nu - b^{2-}_\nu)\left[\delta_{\mu\nu}(1+\bar{\xi}_\nu\tan^2\bar{\theta}_\nu d) + \bar{c}_{\mu\nu}\frac{\cos\bar{\theta}_\nu}{\cos\bar{\theta}_\mu}d\right]\\
&\quad + d\sum_\nu \frac{\sin\theta_\nu}{\cos\bar{\theta}_\mu}\sum_\tau (a^{2+}_\tau + a^{2-}_\tau)\left[\delta_{\tau\nu}(\bar{K}_{\mu\nu}-2\bar{\chi}_{\mu\nu}\xi_\nu) - \bar{\chi}_{\mu\nu}c_{\nu\tau}\frac{\beta_\tau}{\beta_\nu}\right],
\end{aligned} \tag{18 b}$$

where

$$
\left.
\begin{aligned}
\xi_\nu &= \frac{1}{\beta_\nu^1}\frac{\partial \beta_\nu^2}{\partial d}\bigg|_{d=0},\\
\bar{\xi}_\nu &= \frac{1}{\bar{\beta}_\nu^1}\frac{\partial \bar{\beta}_\nu^2}{\partial d}\bigg|_{d=0}\\
c_{\mu\nu} &= \frac{\beta_\mu^1}{2\omega\mu_0}\left[\frac{\partial}{\partial d}\int_{-\infty}^{\infty}\mathscr{E}_{\mu x}^{1*}(y)\mathscr{E}_{\nu x}^{2}(y)\,\mathrm{d}y\right]_{d=0},\\
\bar{c}_{\mu\nu} &= \frac{\bar{\beta}_\mu^1}{2\omega\varepsilon_0}\left[\frac{\partial}{\partial d}\int_{-\infty}^{\infty}\frac{\mathscr{H}_{\mu x}^{1*}(y)}{[n^1(y)]^2}\mathscr{H}_{\nu x}^{2}(y)\,\mathrm{d}y\right]_{d=0},\\
\bar{\sigma}_{\mu\nu} &= \frac{\bar{\beta}_\mu^1}{2\omega\varepsilon_0}\left[\frac{1}{[n_2(t)]^2}-\frac{1}{[n_1(t)]^2}\right]\mathscr{H}_{\mu x}^{1*}(t)\mathscr{H}_{\nu x}^{1}(t),\\
K_{\mu\nu} &= \frac{\beta_\mu^1}{2\omega\mu_0}\left[\frac{\partial}{\partial d}\int_{-\infty}^{\infty}\mathscr{E}_{\mu x}^{1*}(y)\mathscr{E}_{\nu z}^{2}(y)\,\mathrm{d}y\right]_{d=0},\\
\bar{K}_{\mu\nu} &= \frac{\bar{\beta}_\mu^1}{2\omega\varepsilon_0}\left[\frac{\partial}{\partial d}\int_{-\infty}^{\infty}\frac{\mathscr{H}_{\mu x}^{1*}(y)}{[n^1(y)]^2}\mathscr{H}_{\nu z}^{2}(y)\,\mathrm{d}y\right]_{d=0}.
\end{aligned}
\right\}
\tag{19}
$$

Equations (17) and (18) have been obtained for a boundary $z=0$. The field amplitudes $a_\nu^{j\pm}$ and $b_\nu^{j\pm}$ for an arbitrary boundary $z=\Lambda$ must be replaced by $a_\nu^{j\pm}\exp(\mp \mathrm{i}\beta_\nu^j\Lambda\cos\theta_\nu^j)$ and $b_\nu^{j\pm}\exp(\mp \mathrm{i}\bar{\beta}_\nu^j\Lambda\cos\bar{\theta}_\nu^j)$.

3. Evaluation of the mode amplitudes

Let us consider the case when only one TE_η mode with amplitude a_η^{1+} is incident on the step discontinuity. All other incident modes are taken equal to zero so that

$$a_\mu^{2-}=b_\mu^{2-}=b_\mu^{1+}=0,\quad a_\mu^{1+}=\delta_{\mu\eta}a_\eta^{1+}. \tag{20}$$

The amplitudes of the incident a_η^{1+} and the transmitted a_η^{2+} modes in (17) are of the zeroth order in terms of d, while the reflected modes and the transmitted modes with mode number different from η are of higher order. Substituting (20) in (17) and omitting the terms of higher than the first order, the amplitudes can be expressed in explicit form:

$$
\left.
\begin{aligned}
a_\eta^{2+} &= a_\eta^{1+}[1-d(c_{\eta\eta}+\tfrac{1}{2}\xi_\eta(1+\tan^2\theta_\eta))]\\
a_\eta^{1-} &= a_\eta^{1+}(d/2)\xi_\eta(1-\tan^2\theta_\eta)
\end{aligned}
\right\}\quad \text{for } \mathrm{TE}_\eta\text{–}\mathrm{TE}_\eta \text{ conversion,} \tag{21}
$$

$$
\left.
\begin{aligned}
a_\mu^{2+} &= -a_\eta^{1+}\frac{d}{2}c_{\mu\eta}\left(\frac{\beta_\eta}{\beta_\mu}+\frac{\cos\theta_\eta}{\cos\theta_\mu}\right)\\
a_\mu^{1-} &= a_\eta^{1+}\frac{d}{2}c_{\mu\eta}\left(\frac{\beta_\eta}{\beta_\mu}-\frac{\cos\theta_\eta}{\cos\theta_\mu}\right)
\end{aligned}
\right\}\quad \text{for } \mathrm{TE}_\eta\text{–}\mathrm{TE}_\mu \text{ conversion } (\mu\neq\eta), \tag{22}
$$

and

$$b_\mu^{1-}=b_\mu^{2+}=-\frac{d}{2}a_\eta^{1+}\frac{\sin\theta_\eta}{\cos\bar{\theta}_\mu}\left(\bar{K}_{\mu\eta}-2\bar{\chi}_{\mu\eta}\xi_\eta-\beta_\eta^2\sum_\nu\bar{\chi}_{\mu\nu}\frac{c_{\nu\eta}}{\beta_\nu^2}\right)$$

$$\text{for } \mathrm{TE}_\eta\text{–}\mathrm{TM}_\mu \text{ conversion.} \tag{23}$$

The same procedure applied to TM incidence leads to:

$$\left.\begin{aligned} b_\eta^{2+} &= b_\eta^{1+}\left[1-d\left(\bar{c}_{\eta\eta}+\frac{\bar{\sigma}_{\eta\eta}}{2}+\bar{\xi}_\eta\frac{1+\tan^2\bar{\theta}_\eta}{2}\right)\right] \\ b_\eta^{1-} &= b_\eta^{1+}\frac{d}{2}[\bar{\xi}_\eta(1-\tan^2\bar{\theta}_\eta)+\bar{\sigma}_{\eta\eta}] \end{aligned}\right\} \text{ for } \mathrm{TM}_\eta\text{–}\mathrm{TM}_\eta \text{ conversion,} \quad (24)$$

$$\left.\begin{aligned} b_\mu^{2+} &= -b_\eta^{1+}\frac{d}{2}\left[\bar{c}_{\mu\eta}\left(\frac{\bar{\beta}_\eta}{\bar{\beta}_\mu}+\frac{\cos\bar{\theta}_\eta}{\cos\bar{\theta}_\mu}\right)+\bar{\sigma}_{\mu\eta}\frac{\bar{\beta}_\eta}{\bar{\beta}_\mu}\right] \\ b_\mu^{1-} &= b_\eta^{1+}\frac{d}{2}\left[\bar{c}_{\mu\eta}\left(\frac{\bar{\beta}_\eta}{\bar{\beta}_\mu}-\frac{\cos\bar{\theta}_\eta}{\cos\bar{\theta}_\mu}\right)+\bar{\sigma}_{\mu\eta}\frac{\bar{\beta}_\eta}{\bar{\beta}_\mu}\right] \end{aligned}\right\} \text{ for } \mathrm{TM}_\eta\text{–}\mathrm{TM}_\mu \text{ conversion } (\mu\neq\eta) \quad (25)$$

and

$$a_\mu^{1-} = a_\mu^{2+} = -b_\eta^{1+}\frac{d}{2}\frac{\sin\bar{\theta}_\eta}{\cos\theta_\mu}\left(K_{\mu\eta}-2\chi_{\mu\eta}\bar{\xi}_\eta-\bar{\beta}_\eta^2\sum_\nu\chi_{\mu\nu}\frac{\bar{c}_{\nu\eta}+\bar{\sigma}_{\nu\eta}}{\bar{\beta}_\nu^2}\right)$$
$$\text{for } \mathrm{TM}_\eta\text{–}\mathrm{TE}_\mu \text{ conversion.} \quad (26)$$

The mode-matching system (17) and (18) is a linear inhomogenous algebraic system for unknown amplitudes. The linearity of the inhomogenous part leads directly to the fulfilment of the superposition principle. Therefore, when a superposition of modes is incident on the step boundary, scattered mode amplitudes can be represented as a linear combination of the scattered modes amplitudes for a single mode incidence.

4. Mode coupling by a groove

Consider now a guided wave obliquely incident on a rectangular groove placed on the top of a planar optical waveguide (figure 2). The incident wave excites all possible modes at the boundary 1–2. In a first-order approximation the amplitudes of the scattered modes are of order d-times of the incident wave amplitude. On the second boundary 2–3 the modes scattered at the first boundary excite modes with amplitudes of order d^2. A diagrammatic representation of the mode coupling is shown in figure 3. In the first order approximation the second order coupling in figure 3 is neglected. Applying equations (21–26) for the boundaries 1–2 and 2–3 and omitting the terms of higher than the first order in d, for the amplitudes of the transmitted and reflected modes we obtain:

$$\left.\begin{aligned} a_\eta^{1-} &= a_\eta^{1+}\frac{d}{2}\xi_\eta(1-\tan^2\theta_\eta)(\exp(-2i\beta_\eta\cos\theta_\eta\Lambda_1)-\exp(-2i\beta_\eta\cos\theta_\eta\Lambda_2)) \\ a_\eta^{3+} &= a_\eta^{1+}\exp(-id\xi_\eta\beta_\eta\cos\theta_\eta(1+\tan^2\theta_\eta)(\Lambda_2-\Lambda_1)) \end{aligned}\right\} \quad (27)$$

for TE_η–TE_η coupling,

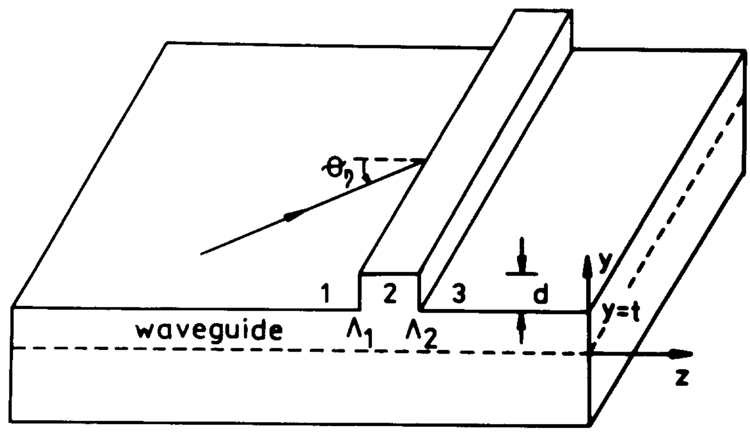

Figure 2. Schematic diagram of the planar optical waveguide with a surface groove.

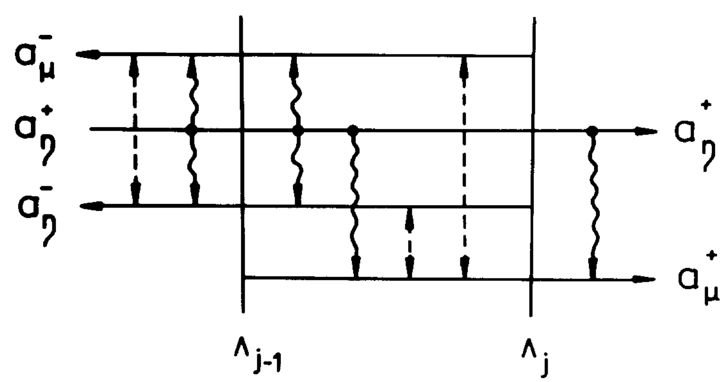

Figure 3. Mode coupling diagram on rectangular groove, with (——) zeroth order coupling, ($\sim$) first order coupling and (---) second order coupling.

$$\left.\begin{aligned} a_\mu^{1-} &= a_\eta^{1+}\frac{d}{2}c_{\mu\eta}\left(\frac{\beta_\eta}{\beta_\mu}-\frac{\cos\theta_\eta}{\cos\theta_\mu}\right)[\exp(-\mathrm{i}(\beta_\eta\cos\theta_\eta+\beta_\mu\cos\theta_\mu)\Lambda_1) \\ &\qquad -\exp(-\mathrm{i}(\beta_\eta\cos\theta_\eta+\beta_\mu\cos\theta_\mu)\Lambda_2)] \\ a_\mu^{3+} &= -a_\eta^{1+}\frac{d}{2}c_{\mu\eta}\left(\frac{\beta_\eta}{\beta_\mu}+\frac{\cos\theta_\eta}{\cos\theta_\mu}\right)[\exp(-\mathrm{i}(\beta_\eta\cos\theta_\eta-\beta_\mu\cos\theta_\mu)\Lambda_1) \\ &\qquad -\exp(-\mathrm{i}(\beta_\eta\cos\theta_\eta-\beta_\mu\cos\theta_\mu)\Lambda_2)] \end{aligned}\right\} \quad (28)$$

for TE_η–TE_μ coupling ($\mu\neq\eta$)

$$\left.\begin{aligned} b_\mu^{1-} &= -a_\eta^{1+}\frac{d}{2}\frac{\sin\theta_\eta}{\cos\bar{\theta}_\mu}\bar{R}_{\mu\eta}[\exp(-\mathrm{i}(\beta_\eta\cos\theta_\eta+\bar{\beta}_\mu\cos\bar{\theta}_\mu)\Lambda_1) \\ &\qquad -\exp(-\mathrm{i}(\beta_\eta\cos\theta_\eta+\bar{\beta}_\mu\cos\bar{\theta}_\mu)\Lambda_2)] \\ b_\mu^{3+} &= -a_\eta^{1+}\frac{\sin\theta_\eta}{\cos\bar{\theta}_\mu}\frac{d}{2}\bar{R}_{\mu\eta}[\exp(-\mathrm{i}(\beta_\eta\cos\theta_\eta-\bar{\beta}_\mu\cos\bar{\theta}_\mu)\Lambda_1) \\ &\qquad -\exp(-\mathrm{i}(\beta_\eta\cos\theta_\eta-\bar{\beta}_\mu\cos\bar{\theta}_\mu)\Lambda_2)] \end{aligned}\right\} \quad (29)$$

for TE_η–TM_μ coupling, where

$$\bar{R}_{\mu\eta}=\bar{K}_{\mu\eta}-2\bar{\chi}_{\mu\eta}\xi_\eta-\beta_\eta^2\sum_\nu\bar{\chi}_{\mu\nu}\frac{c_{\nu\eta}}{\beta_\nu^2}. \quad (30)$$

Similar expressions are obtained for TM_η–TM_μ, TM_η–TM_η and TM_η–TE_μ coupling by multiplication of the amplitudes (24–26) with the corresponding to the type of polarization exponentials.

The groove with an arbitrary profile (figure 4) is divided into rectangles, each with an infinitesimal base and height d_j, which is small in comparison to the waveguide effective thickness. For each rectangle the change in the mode amplitude is proportional to

$$d_j[\exp(-\mathrm{i}\Delta_{\mu\eta}\Lambda_{j-1})-\exp(-\mathrm{i}\Delta_{\mu\eta}\Lambda_j)], \quad (31)$$

where

$$\Delta_{\mu\eta}=\beta_\eta\cos\theta_\eta\mp\beta_\mu\cos\theta_\mu. \quad (32)$$

The symbols + and − refer to positive and negative directions of propagation. After summation over all rectangles in (31), the amplitude change of the guided wave scattered by the whole groove becomes proportional to

$$\mathrm{i}\Delta_{\mu\eta}\int_{\Lambda_0}^{\Lambda_0+\Lambda}[f(z)-t]\exp(-\mathrm{i}\Delta_{\mu\eta}z)\,\mathrm{d}z, \quad (33)$$

because the width of the rectangles is infinitesimal. Where $y=f(z)$ (the groove profile function) and

$$d_j=f(\Lambda_{j-1})-t. \quad (34)$$

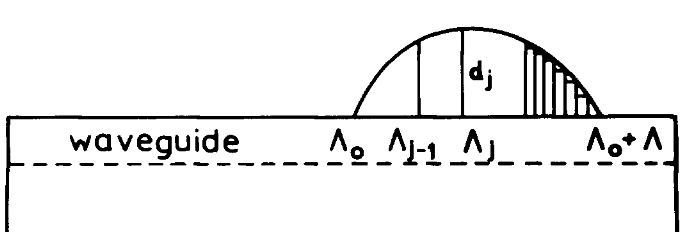

Figure 4. Approximation of the groove profile with rectangular slides.

Similar consideration for the transmission amplitude leads to

$$a_\eta^{3+} = a_\eta^{1+} \exp(-\mathrm{i}e_\eta) \tag{35}$$

where

$$e_\eta = \beta_\eta \xi_\eta \cos\theta_\eta (1+\tan^2\theta_\eta) \int_{\Lambda_0}^{\Lambda_0+\Lambda} [f(z)-t]\,\mathrm{d}z.$$

For a TM case the corresponding quantities in (31), (32), (33) and (35) should be overbarred.

5. Mode coupling by a grating

The geometry of the problem under consideration is shown in figure 5. A TE_η or TM_η guided wave is incident at an angle θ_η on the relief surface grating with a profile function

$$y = f(z) = t + \mathrm{d}\varepsilon(z) \tag{36}$$

From § 4 it is evident that the first-order approximation in groove depth is equivalent to a small change of the incident wave amplitude. This is true for grating single groove coupling; however, the amplitude of the wave diffracted by the whole grating can be comparable with the incident wave in the case of phase synchronism. Using the periodicity of the grating, equation (33) can be represented as

$$\mathrm{i}d\Delta_{\mu\eta} \sum_m F_m[\exp(\mathrm{i}\delta_{\mu\eta,m}\Lambda)-1]/\mathrm{i}\delta_{\mu\eta,m}\Lambda, \tag{37}$$

where

$$\left.\begin{aligned} F_m &= \Lambda^{-1}\int_0^\Lambda \varepsilon(z)\exp(-\mathrm{i}mKz)\,\mathrm{d}z, \quad && K = 2\pi/\Lambda, \\ \delta_{\mu\eta,m} &= mK - \Delta_{\mu\eta}, && m = 0, \pm 1, \pm 2, \ldots \end{aligned}\right\} \tag{38}$$

We shall consider further the interesting practical case of Bragg diffraction, when the only significant term in the sum (37) which remains is one which deviates only slightly from the Bragg condition

$$\boldsymbol{\beta}_\mu = \boldsymbol{\beta}_\eta + m\mathbf{K}. \tag{39}$$

In this case (37) becomes $\mathrm{i}d\Delta_{\mu\eta}F_m \exp \mathrm{i}\delta_{\mu\eta,m}z$ and substituting $\Delta(a,b)_\mu/\Lambda$ with $d(a,b)_\mu/\mathrm{d}z$ in (27 29) we obtain the coupled-mode equations

$$\left.\begin{aligned} \frac{\mathrm{d}\,c_\mu^j(z)}{\mathrm{d}z} &= \mathrm{i}\Gamma_{\mu\eta}^{jl} c_\eta^l(z)\exp \mathrm{i}\delta_{\mu\eta,m}z, \\ \frac{\mathrm{d}\,c_\eta^l(z)}{\mathrm{d}z} &= \mathrm{i}\Gamma_{\eta\mu}^{lj} c_\mu^j(z)\exp(-\mathrm{i}\delta_{\mu\eta,m}z), \end{aligned}\right\} \tag{40}$$

where $c_\mu^1 = a_\mu$, $c_\mu^2 = b_\mu$ and the coupling coefficients Γ are given in the table, with

$$R_{\mu\eta} = K_{\mu\eta} - 2\chi_{\mu\eta}\bar{\xi}_\eta - \bar{\beta}_\eta^2 \sum_\nu \chi_{\mu\nu}\frac{\bar{c}_{\nu\eta}+\bar{\sigma}_{\nu\eta}}{\bar{\beta}_\nu^2}.$$

Equations (40) are valid for both reflection and transmission cases. The angles θ in the table are measured from the positive direction of z axes (i.e. for reflected waves

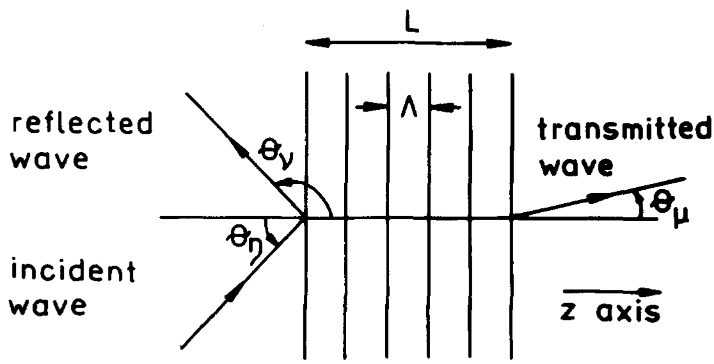

Figure 5. Schematic representation of the surface relief grating.

Coupling coefficients for different kinds of coupling.

Coupling	Coupling coefficients
TE_η–TE_η	$\Gamma^{1,1}_{\eta,\eta}=dF_m\xi_\eta\beta_\eta\dfrac{\cos 2\theta_\eta}{\cos\theta_\eta}$
TE_η–TE_μ	$\Gamma^{1,1}_{\mu\eta}=\dfrac{d}{2}F_m c_{\mu\eta}\dfrac{\beta_\eta^2-\beta_\mu^2}{\beta_\mu}\dfrac{\cos(\theta_\eta-\theta_\mu)}{\cos\theta_\mu}$
TE_η–TM_μ	$\Gamma^{2,1}_{\mu\eta}=\dfrac{d}{2}F_m\bar{R}_{\mu\eta}\bar{\beta}_\mu\dfrac{\sin(\bar{\theta}_\mu-\theta_\eta)}{\cos\bar{\theta}_\mu}$
TM_η–TE_μ	$\Gamma^{1,2}_{\mu\eta}=\dfrac{d}{2}F_m R_{\mu\eta}\beta_\mu\dfrac{\sin(\theta_\mu-\bar{\theta}_\eta)}{\cos\theta_\mu}$
TM_η–TM_η	$\Gamma^{2,2}_{\eta,\eta}=dF_m\bar{\beta}_\eta\left[\bar{\xi}_\eta\dfrac{\cos 2\bar{\theta}_\eta}{\cos\bar{\theta}_\eta}+\bar{\sigma}_{\eta,\eta}\cos\bar{\theta}_\eta\right]$
TM_η–TM_μ	$\Gamma^{2,2}_{\mu\eta}=\dfrac{d}{2}F_m\left[\bar{c}_{\mu\eta}\dfrac{\bar{\beta}_\eta^2-\bar{\beta}_\mu^2}{\bar{\beta}_\mu}\dfrac{\cos(\bar{\theta}_\eta-\bar{\theta}_\mu)}{\cos\bar{\theta}_\mu}+\bar{\sigma}_{\mu\eta}\bar{\beta}_\eta\dfrac{\sin(\bar{\theta}_\mu-\bar{\theta}_\eta)}{\sin\bar{\theta}_\eta}\right]$

the angles are obtuse). For the reflection case coupling coefficients $\Gamma_{\mu\eta}$ and $\Gamma_{\eta\mu}$ have opposite signs and the solution of (40) is represented by hyperbolic functions. For the transmission case the coefficients have the same signs and the solution is represented by trigonometric functions [10].

6. Waveguide with a step refractive index profile

The expressions for Γ in the table are valid, to a first-order approximation in d, for a waveguide with an arbitrary refractive index profile and for a grating with an arbitrary cross-section. For a waveguide with a step refractive index profile the coupling coefficients for TE–TE and TM–TM coupling can be obtained in closed form from the wave equations by integration in parts. The calculations are tedious so we give only the final results,

$$\left.\begin{aligned}\Gamma_{\mathrm{TE}_\mu \mathrm{TE}_\eta} &= \frac{d}{2}\frac{\beta_\mu}{2\omega\mu_0} F_m k^2[n_2^2 - n_1^2]\mathscr{E}^*_{\mu_x}(t)\mathscr{E}_{\eta_x}(t)\frac{\cos(\theta_\eta - \theta_\mu)}{\beta_\mu \cos\theta_\mu},\\ \Gamma_{\mathrm{TM}_\mu \mathrm{TM}_\eta} &= \frac{d}{2}\frac{\bar{\beta}_\mu}{2\omega\varepsilon_0} F_m\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)\mathscr{H}^*_{\mu_x}(t)\,\mathscr{H}_{\eta_x}(t)\frac{\bar{\beta}_\mu\bar{\beta}_\eta + q_\mu q_\eta \dfrac{n_2^2}{n_1^2}\cos(\bar{\theta}_\eta - \bar{\theta}_\mu)}{\bar{\beta}_\mu \cos\bar{\theta}_\mu}\end{aligned}\right\} \quad (41)$$

Here $q_\mu^2 = \bar{\beta}_\mu^2 - k^2 n_1^2$, k is the wave-vector, n_1 is the superstrate refractive index, n_2 is the waveguide refractive index and $\mathscr{E}_{\mu_x}(t)$, $\mathscr{H}_{\mu_x}(t)$ are the values of the mode eigenfunction at the waveguide–superstrate boundary.

7. Discussion

Despite the amount of work on this subject there is no common opinion regarding the angular dependence of the TE_η–TE_η coupling coefficients for the case of an obliquely incident wave on a grating [12, 13, 16, 18]. From the table it is clear that TE_η–TE_η coupling vanishes if the angle of incidence is $\pi/4$. This is the well-known Brewster's law analogy for Bragg reflection by planar gratings. It is important to notice that this phenomenon is a result of a corresponding effect on a single step boundary. From (21) we see that the coupling also vanishes at a step discontinuity if the angle between the reflected and the transmitted modes is 90°. This phenomenon corresponds to Brewster's law in bulk optics. Such an analogy, however, exists only for a small step height, of first order in d. In the general case system equations (4, 10, 12 and 13) must be considered, so that the excitation of each mode is performed through all possible modes. Higher order coupling (figure 3) gives rise to the indirect energy transfer from the incident to the reflected wave through other modes. Another interesting point is that the angular dependence of the coupling coefficients is not influenced by the waveguide or grating parameters.

Stegeman *et al.* [17] have presented a review of the results for the coupling coefficients obtained by different methods. In the case of normal incidence our results for TE–TE and TM–TM coupling in a step index waveguide coincide with those of Stegeman *et al.* and those of the local mode approach [9]. However, for the case of oblique incidence and the above mentioned kinds of coupling our results differ from those of Stegeman *et al.* by the cosine in the denominator of the angular dependence term (see the table). This term is responsible for interaction length $l = \Lambda \cos^{-1}\theta_\eta$ of the ηth mode in the single grating groove. Unlike normal incidence where the mode coupling is accomplished with a polarization conservation [9, 10], in oblique incidence a polarization conversion is possible because of the hybrid nature of the modes. If, for example, a TE guided wave hits a boundary at an angle θ, the $\mathscr{E}_{\nu_x}$ component transverse to the propagation direction excites the longitudinal electric vector component in the scattered modes which corresponds to a TM polarization.

The backscattering case with polarization conversion (TE–TM and TM–TE coupling) has been discussed in [17] and results similar to ours have been obtained without the cosine denominator shown in the table. It is important to note that the mode fields $\mathscr{E}_{\nu_x}(y)$ and $\mathscr{E}_{\nu_z}(y)$ in [17], as well as $\mathscr{H}_{\nu_x}(y)/n^2$ and $\mathscr{H}_{\nu_z}(y)$ are taken to be mutually orthogonal, thus the overlapping integrals, in our notations $\chi_{\mu\nu}$ and $\bar{\chi}_{\mu\nu}$, are cancelled. In fact this is inaccurate. The orthogonality relations (6) and (7) refer to the TE and TM eigenfunctions separately, and there is no such orthogonality relation between TE and TM eigenfunctions. Orthogonality between TE and TM modes propagating in different directions in an infinite slab waveguide is satisfied due to the different propagation factors. The existence of a boundary cancels this orthogonality. In collinear and contralinear coupling TE and TM fields have mutually transverse vectors, but in the case of oblique incidence the vectors are non-transverse and the overlapping of mode fields causes energy exchange between all possible modes. Direct evaluation for the case of waveguide with a step refractive index profile confirms that $\chi_{\mu\nu}$ and $\bar{\chi}_{\mu\nu}$ are non-zero for both $\mu=\nu$ and $\mu\neq\nu$

$$\left.\begin{aligned}\chi_{\mu\nu}&=-\mathrm{i}\frac{\bar{\beta}_\nu^2}{\beta_\mu^2-\bar{\beta}_\nu^2}\frac{\beta_\mu}{2k^2}\left[\mathscr{E}^*_{\mu x}(t)\mathscr{H}_{\nu x}(t)\left(\frac{1}{n_2^2}-\frac{1}{n_1^2}\right)+\mathscr{E}^*_{\mu x}(0)\mathscr{H}_{\nu x}(0)\left(\frac{1}{n_3^2}-\frac{1}{n_2^2}\right)\right],\\ \bar{\chi}_{\mu\nu}&=\mathrm{i}\frac{\beta_\nu^2}{\bar{\beta}_\mu^2-\beta_\nu^2}\frac{\bar{\beta}_\mu}{2k^2}\left[\mathscr{E}_{\nu x}(t)\mathscr{H}^*_{\mu x}(t)\left(\frac{1}{n_2^2}-\frac{1}{n_1^2}\right)+\mathscr{E}_{\nu x}(0)\mathscr{H}^*_{\mu x}(0)\left(\frac{1}{n_3^2}-\frac{1}{n_2^2}\right)\right].\end{aligned}\right\}\quad(42)$$

The summation over all possible modes in the right-hand side of (30) can be abbreviated to a summation over modes adjacent to the incident and the scattered mode.

Appendix

From the Maxwell equations we have

$$\left.\begin{aligned}n^2E_z&=\frac{1}{\mathrm{i}\omega\varepsilon_0}\left(\frac{\partial H_y}{\partial x}-\frac{\partial H_x}{\partial x}\right),\\ H_z&=\frac{1}{\mathrm{i}\omega\mu_0}\left(\frac{\partial E_x}{\partial x}-\frac{\partial E_x}{\partial x}\right).\end{aligned}\right\}\quad(\mathrm{A}\,1)$$

The continuity of E_y and H_y and Snell's law (9) ensure the continuity of $\partial E_y/\partial x$ and $\partial H_y/\partial x$. Consider now the continuity requirement for $\partial E_x/\partial y$ and $\partial H_x/\partial y$ at $z=0$

$$\left.\frac{\partial E_x^1}{\partial y}\right|_{z=0}=\left.\frac{\partial E_x^2}{\partial y}\right|_{z=0},\quad(\mathrm{A}\,2)$$

$$\left.\frac{\partial H_x^1}{\partial y}\right|_{z=0}=\left.\frac{\partial H_x^2}{\partial y}\right|_{z=0}.\quad(\mathrm{A}\,3)$$

Multiplying (A 2) by

$$(\beta_\mu^1/4\pi\omega\mu_0)\exp(\mathrm{i}\beta_\mu^1x\sin\theta_\mu^1)\int\mathscr{E}^{1*}_{\mu x}(y)\,\mathrm{d}y$$

and (A 3) by

$$(\bar{\beta}_\mu^1/4\pi\omega\varepsilon_0)\exp(\mathrm{i}\bar{\beta}_\mu^1x\sin\bar{\theta}_\mu^1)\int(\mathscr{H}^{1*}_{\mu x}(y)/[n^1(y)]^2)\,\mathrm{d}y,$$

after integration in parts over the infinite x–y cross section we get (4) and (10). Therefore the normal field components E_z and H_z are continuous at the boundary $z=0$ in the regions $y<t$ and $y>t+d$, where the refractive index of the medium does not change.

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