Fuzzy Triangle Contour Characterization by Subspace Based Methods of Array Processing

Haiping Jiang, Julien Marot, Salah Bourennane, and Caroline Fossati
Institut Fresnel/GSM, Ecole Centrale Marseille
D. U. de Saint-Jérôme, av. Normandie Niemen, 13397, Marseille, France
phone: + (33) 4 91 28 82 02, email: julien.marot@fresnel.fr
web: www.fresnel.fr/perso/marot

***Abstract*—Fuzzy paradigm was considered from several aspects in image segmentation. For the first time, we derive a signal processing model out of an image which contains a fuzzy contour. We propose to adapt subspace-based methods of array processing which are originally dedicated to multiple incoherently distributed sources, to retrieve the orientation and spread parameters of fuzzy contours. A set of experiments performed on hand-made and real-world images shows that the proposed methods estimate accurately the expected orientation and spread parameters of fuzzy contours, and exhibit a small computational load.**

I. Introduction

Fuzzy contours occur very often in images, owing to object movements, light transmission environment, etc. Several methods have been proposed for solving this problem. One can distinguish two categories of methods: those which perform contour-based segmentation, and those which perform region-based segmentation. Firstly, contour-based segmentation methods consider fuzzy contours as textured regions in the image, that is, a set of pixels which has a transverse width of more than one pixel. These methods aim at determining a mean position of the pixels of the textured region. In particular a new model for active contours based on techniques of curve evolution, which was adapted from the level set paradigm, was proposed to segment contours ”without edges” [1].

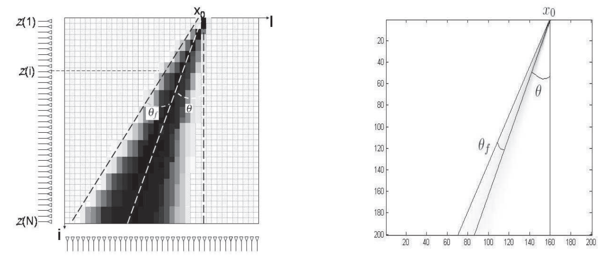
Secondly, region-based segmentation methods rely on the theory of fuzzy sets, to define membership functions and classify the pixels. In particular, a fuzzy paradigm was adopted in the frame of mathematical morphology to characterize the repartition of objects in an image by ”fuzzy” relationships [2]. In this fuzzy paradigm, one could localize an object at left or at the right-hand side of another object. This led in particular to applications for medical 3D image segmentation. Also, unsupervised segmentation was adapted to classification by a fuzzy version of hidden Markov chains [3]. This work considers that a fuzzy membership function is added to a crisp membership to characterize the values taken by the Markov process and thereby classify pixels in an image.

Array processing methods were adapted to contour characterization [4], [5]: it was shown that a specific signal generation scheme yields an array processing signal model out of an image which contains contours with a width of one pixel. High resolution methods could then be applied to distinguish close contours by considering them as punctual sources. In this paper, we propose a novel approach to characterize fuzzy contours, that is, contours which are no longer one pixel wide but characterized by a spread parameter. For this, we derive a novel signal model. One of the attraction is that it permits to characterize entirely a ”triangular-like” fuzzy contour with three parameters of orientation, spread, and offset. We adapt a subspace-based method of array processing to provide an estimate of the orientation and spred parameters.

Section II states the fuzzy contour retrieval problem, section III derives an array processing model out of signals generated from the image. Section IV adapts subspace-based methods of array processing to estimate orientation and spread parameters of the fuzzy contours. Section V presents experimental results which validate the proposed models and methods.

II. Problem statement

In this section, we provide the models that we adopt for the processed image, for the gray level distribution of the contours which are present in the image, and for the technique which permits to generate a signal out of the image content. Let $I(i,l)$ be an $N \times L$ recorded image (see Fig. 1(a)). For example, a camera records a scene where a phenomenon generates a light beam. We consider that $I(i,l)$ is compound of a fuzzy contour and an additive uniformly distributed noise. The fuzzy contour is supposed to have main orientation θ and center offset x_0. The pixel values are supposed to be small enough to be neglected at a distance θ_f on each side of the main orientation of the contour. The gray level evolution of the fuzzy contour around its main orientation in every row can



(a) (b)

Fig. 1. (a) linear antenna model in an image containing a fuzzy contour; (b) hand-made fuzzy triangle contour characterized by the main orientations θ and offsets x_0 in the image

be, in a general manner, described by a Gaussian evolution depending on a spread parameter σ. If we define the gaussian attenuation threshold of the pixel as δ_T, the relationship among θ_f, σ, and δ_T is:

$$\delta_T = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{\theta_f^2}{2\sigma^2}}.$$

Below this threshold value, the pixel values are supposed to be negligible.

If there are multiple fuzzy contours in the image, every contour is characterized by the main orientation θ_k and the angle spread θ_f (see Fig. 1(b)). Every fuzzy contour obeys Gaussian distribution with variance ${\sigma_k}^2$. We expect that such a contour model facilitates the transfer of array processing methods to the considered parameter estimation issue. In order to set the link between image data representation and sensor array processing methods [4], array sensors are supposed to be placed in front of each row of the image. Each sensor receives the signal only from its corresponding row in the matrix. All the pixels in the image are assumed to propagate narrow-band electromagnetic waves with zero initial phases. Furthermore, we assume that the waves emanating from pixels in a given row of the image matrix are confined to travel only along that row towards the corresponding sensor.

We adopt the signal generation scheme proposed in [4]:

$$z(i) = \sum_{l=1}^{L} I(i,l)\; e^{-j\mu l},\; i = 1, \ldots, N \tag{1}$$

where μ is an *a priori* set propagation parameter. In the next section, we show that the adopted contour model and signal generation process yield an array processing signal model, handled by subspace-based methods.

III. Array processing signal model

Firstly, we assume that the image contains only one fuzzy contour of main orientation θ, angle spread $2\theta_f$, offset x_0, and standard variance σ for the Gaussian distribution of the pixel values. On the i^{th} sensor we get:

$$z(i) = \sum_{\breve{\theta}=-\theta_f}^{\theta_f} e^{-j\mu(x_0-(i-1)tan(\theta+\breve{\theta}))} \cdot \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{\breve{\theta}^2}{2\sigma^2}} \tag{2}$$

For small values of $\breve{\theta}$, we get the following approximation with a Taylor series expansion:

$$\tan(\theta + \breve{\theta}) \simeq \tan\theta + \frac{1}{\cos^2\theta}\breve{\theta} \tag{3}$$

Combining Eqs. (2) and (3) yields:

$$z(i) = \frac{1}{\sqrt{2\pi}\sigma} e^{-j\mu x_0} e^{j\mu(i-1)\tan\theta} \sum_{\breve{\theta}=-\theta_f}^{\theta_f} e^{\frac{j\mu(i-1)\breve{\theta}}{\cos^2\theta}} \cdot e^{-\frac{\breve{\theta}^2}{2\sigma^2}} \tag{4}$$

When σ is small enough (note that it is coherent with the fact that θ_f is small enough), we can turn the considered discrete calculation into a continuous case calculation:

$$z(i) \simeq \frac{1}{\sqrt{2\pi}\sigma} e^{-j\mu x_0} e^{j\mu(i-1)\tan\theta} \int_{\breve{\theta}=-\infty}^{\breve{\theta}=+\infty} e^{-\frac{\breve{\theta}^2}{2\sigma^2} + \frac{j\mu(i-1)\breve{\theta}}{\cos^2\theta}} d\breve{\theta} \tag{5}$$

A general formula provides the equality:

$$\int_{y=-\infty}^{y=+\infty} e^{-ay^2+jby} dy = \sqrt{\frac{\pi}{a}} e^{-\frac{b^2}{4a}} \tag{6}$$

It is easy to express Eq. (5) by:

$$z(i) = e^{-j\mu x_0} \cdot e^{j\mu(i-1)tan(\theta)} \cdot e^{-\frac{\mu^2\sigma^2(i-1)^2}{2\cos^4\theta}} \tag{7}$$

Equation (7) is the signal generated on the i^{th} sensor in the case where there exists only one fuzzy contour in the image.

Secondly, we consider the case where the image contains:

- d fuzzy contours, with orientations θ_k, offsets x_{0k}, and standard variance σ_k $(k = 1, \ldots, d)$;
- identically distributed noise pixels.

The expression of the received signal by i^{th} sensor becomes:

$$z(i) = \sum_{k=1}^{d} e^{-j\mu x_{0k}} \cdot e^{j\mu(i-1)tan\theta_k} \cdot e^{-\frac{\mu^2\sigma_k^2(i-1)^2}{2\cos^4\theta_k}} + n(i) \tag{8}$$

where $n(i)$ is a noise term originated by the noise pixels during the signal generation process. The expression of the signal components in Eq. (8) permits to adopt the notations coming from array processing. We define:

1) When the continuous approximation holds, the source amplitude components are expressed as:

$$s(k) = e^{-j\mu x_{0k}}, k = 1, \cdots, d \tag{9}$$

2) the steering vector associated with the k-th contour as: $\mathbf{c}(\eta_k) = [c_1(\eta_k), c_2(\eta_k), \cdots, c_i(\eta_k), \cdots, c_N(\eta_k)]^T$, where $\eta_k = [\theta_k, \sigma_k]^T$, with $c_i(\eta_k) = e^{j\mu(i-1)\tan\theta_k} \cdot e^{-\frac{\mu^2\sigma_k^2(i-1)^2}{2\cos^4\theta_k}}$.
3) the noise vector $\mathbf{n} = [n(1), n(2), \ldots, n(N)]^T$.

These notations permit to express the signal generated out of the image in a matrix form:

$$\mathbf{z} = \mathbf{C}(\eta)\mathbf{s} + \mathbf{n} \tag{10}$$

where $\mathbf{z} = [z(1), z(2), \ldots, z(N)]^T$,
$\mathbf{s} = [s(1), s(2), \cdots, s(d)]^T$, $\mathbf{C}(\eta) = [\mathbf{c}(\eta_1), \mathbf{c}(\eta_2), \cdots, \mathbf{c}(\eta_d)]$.
Equation (10) shows that, by adopting the signal generation scheme of Eq. (1) and the proposed model for fuzzy triangle contours, we can make an analogy between the signals generated out of the image and an array processing signal model. Therefore, we predict that array processing methods can yield the parameters of the expected contours.

IV. Subspace based methods of array processing for orientation and spread parameter estimation

A. Main orientation and spread estimation: DSPE

In this subsection, we propose to adapt a method coming from array processing and originally dedicated to distributed source characterization [6], [7]. An array processing method can be applied to the generated signal provided in Eq. (10), to characterize the contours in the image by retrieving their parameters. Subspace-based parameter estimation methods such as MUSIC [8] assume that several realizations of a signal are available. However, the processed image is a deterministic

data and provides only one signal of length N through the signal generation process of Eq. (1). To adapt a subspace-based method such as MUSIC, we have to simulate artificially multiple signal measurements out of a single signal by splitting the array (of length N) into smaller overlaying sub-arrays (of length M). Each sub-array provides a signal realization. Therefore, out of one snapshot, we get several signal realizations. This is called spatial smoothing technique [4]. There exist a constraint on M, and a relationship between N, M and the number of snapshot T: $d < M \leq N - d + 1$; and $M = N - T + 1$. For details, refer to [4]. At the same time, Eq. (10) is rewritten as $\mathbf{z}_t = \mathbf{C}(\eta)\mathbf{s}_t + \mathbf{N}_t$, where each steering vector $\mathbf{c}(\eta_k)$ of length M is defined as:
$\mathbf{c}(\eta_k) = [c_1(\eta_k), c_2(\eta_k), \cdots, c_i(\eta_k), \cdots, c_M(\eta_k)]^T$, with
$c_i(\eta_k) = e^{j\mu(i-1)\tan\theta_k} \cdot e^{-\frac{\mu^2\sigma_k^2(i-1)^2}{2\cos^4\theta_k}}$, and
$\mathbf{s}_t = [s_1(t), s_2(t), \cdots, s_d(t)]^T$ where
$s_k(t) = \sqrt{2\pi}G\sigma_k e^{-j\mu x_{0k}} e^{\frac{-\mu^2\sigma_k^2}{2}} e^{j(t-1)\mu tan\theta_k}$,
$t = 1, 2, \cdots, T$.
The covariance matrix of $\mathbf{z}(t)$ is defined by:

$$\mathbf{R}_{zz} = \frac{1}{T}\sum_{t=1}^{T}\mathbf{z}_t\mathbf{z}_t^H \tag{11}$$

where $(.)^H$ denotes Hermitian transpose. We perform eigenvalue decomposition of $\mathbf{R}_{zz}$.

$$\mathbf{R}_{zz} = [\mathbf{U}_1 \quad \mathbf{U}_2]\ \mathbf{\Lambda}\ \mathbf{U} \tag{12}$$

In the literature, we can find two types of distributed sources, namely, decorrelated, also called incoherently distributed (ID), and coherently distributed (CD) sources [9]. An interesting property of the spatial smoothing technique is that it decorrelates the sources [4]. Therefore, in this study, we can consider that all sources are decorrelated. As we are ensured to have decorrelated sources thanks to the spatial smoothing process, the columns of matrix $\mathbf{U}_1\,(\mathrm{M}\times\mathrm{d})$ span the signal subspace, the columns of matrix $\mathbf{U}_2\,(\mathrm{M}\times(\mathrm{M}-\mathrm{d}))$ span the noise subspace, and $\mathbf{\Lambda} = diag\,(\lambda_1, \lambda_2, \cdots \lambda_d)$ where λ_i is the eigenvalue associated with the i^{th} eigenvector. Hence, $\mathbf{U}_2$ is orthogonal to the steering vectors $\mathbf{c}\,(\theta_k)\,,\ k = 1, \ldots, d$. We estimate the η_k parameters ($k = 1, \ldots, d$) through the bidimensional search procedure of the maxima of the pseudo spectrum given by the DSPE (Distributed Signal Parameter Estimator) method [6], [9]:

$$\mathrm{DSPE}\,(\eta_k) = \frac{\|\ \mathbf{c}\,(\eta)\ \|^2}{\|\ \mathbf{c}^H\,(\eta)\cdot\mathbf{U}_2\ \|^2} \tag{13}$$

where $\mathbf{c}\,(\eta)$ is a model for the signal subspace vectors. Thus, DSPE method provides the orientation and spread values, through the relationship: $\eta_k = [\theta_k, \sigma_k]^T,\ \forall\ k = 1, \ldots, d$. In the next section, we illustrate the performance of the proposed method for the estimation of the orientation and spread parameter values in several application cases.

B. Offset estimation: variable speed generation scheme

To estimate all offset values, we adopt a variable speed generation scheme [4], and a specific "dechirping" procedure: we set the propagation parameter as a value which depends on the row index. Parameter μ becomes $\alpha(i-1)$. Then, for each η_k estimated at subsection IV-A, we apply a specific signal transformation with the knowledge of θ_k and σ_k. For all $k = 1, \ldots, d$ successively, we get:

$$w(i) = z(i)/(e^{j\alpha(i-1)^2 tan\theta_k} \cdot e^{-\frac{\alpha^2\sigma_k^2(i-1)^4}{2\cos^4\theta_k}}) \tag{14}$$

For instance, when $k = 1$, we aim at estimating the first offset value x_{01}:

$$w(i) = e^{-j\alpha(i-1)x_{01}} + n'(i) \tag{15}$$

where $n'(i)$ is a noise term resulting from the transformation of the original noise term by the dechirping procedure.
Equation (15) shows that any subspace-based method such as TLS-ESPRIT [4], [5] can be applied to retrieve the offset value x_{01}. The same process is applied for all $k = 1, \ldots, d$. At this point, we estimated all offset values x_{0k}, $k = 1, \ldots, d$.

V. RESULTS

In this section, we illustrate the propose method on hand-made and real-world images. All experiments are performed on a computer equipped by 2.83GHz 2 Quad CPU and 4G memory. Images have size 200×200 pixels. Adequate parameter values found experimentally, which are used for signal generation, are $\mu = 2.3\ 10^{-2}$ and $\alpha = 2.5\ 10^{-3}$, the size of the sub-arrays used for the spatial smoothing procedure is $M = 4$.

A. Hand-made images

We propose a statistical study which characterizes the performance of the proposed method. The proposed method is run for several triplets of values (θ; σ; x_0). The results obtained are provided in Table I, which provides the estimated values ($\hat{\theta}$; $\hat{\sigma}$; $\hat{x}_0$), and the bias (E_θ; E_σ; $E_{x_{0k}}$) defined by: $E_\theta = |\hat{\theta} - \theta|$, $E_\sigma = |\hat{\sigma} - \sigma|$, and $E_{x_0} = |\hat{x}_0 - x_0|$. These results show that the proposed method always provides a very accurate estimation of the orientation θ, and that the best estimation results for σ are obtained for values between 1.5 and 2.4. The bias on the estimated offset values x_0 is always equal to or less than 1 pixel. Note that the case $\sigma = 0$ was considered, and leads to an estimate $\hat{\sigma} = 0.36$. The result contour appears then as a 1-pixel wide contour. As an illustration of the proposed algorithm,

TABLE I
ESTIMATED VALUES $\hat{\theta}$; $\hat{\sigma}$; $\hat{x}_0$ AND MEAN ERROR VALUES E_θ; E_σ; E_{x_0} IN ($^\circ$;PIXELS;PIXELS) OBTAINED WITH THE PROPOSED METHOD, VERSUS CONTOUR CHARACTERISTICS (θ; σ; x_0).

(θ; σ; x_0)	($\hat{\theta}$; $\hat{\sigma}$; $\hat{x}_0$)	(E_θ; E_σ; E_{x_0})
(10; 0; 50)	(11.7; 0.36; 49)	(1.7; 0.36; 1)
(9; 0.5; 48)	(8.6; 0.74; 49)	(0.4; 0.24; 1)
(11; 1; 51)	(10.9; 1.47; 50.5)	(0.1; 0.47; 0.5)
(10; 1.2; 80)	(10.9; 1.3; 80)	(0.0; 0.1; 0)
(10; 1.5; 50)	(9.9; 1.76; 49.5)	(0.1; 0.26; 0.5)
(12; 1.7; 45)	(11.9; 1.87; 45)	(0.1; 0.17; 0)
(5; 1.9; 60)	(4.9; 1.92; 60)	(0.1; 0.02; 0)
(13; 2; 39)	(13.0; 1.94; 39)	(0.0; 0.06; 0)
(-12; 2.2; 53)	(-12.0; 2.10; 53)	(0.0; 0.10; 0)
(-10; 2.4; 36)	(-9.9; 2.41; 36)	(0.1; 0.01; 0)
(10; 3; 50)	(9.9; 2.19; 50)	(0.1; 0.81; 0)

Figs. 2(a,d) provide the processed image, containing one or two fuzzy contour(s) whose characteristics are to be estimated.

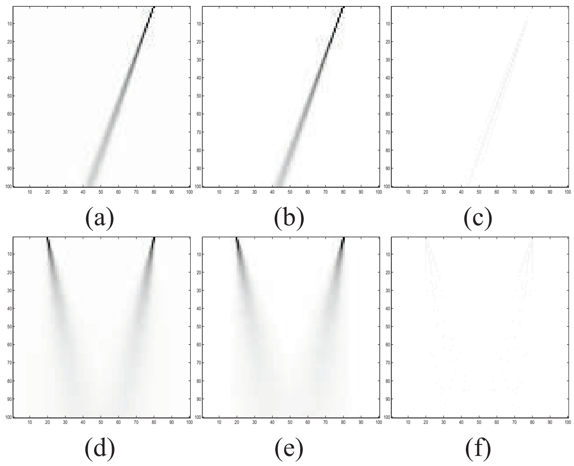


Fig. 2. Estimation of (θ; σ; x_0): (a) processed (20°; 1.2; 80); (b) result (10°; 1.01; 80); (c) difference processed - result; (d) processed (10°; 5.1; 80) and (−10°; 5.1; 20); (e) result (10°; 5.01; 80) and (−10°; 5.01; 20); (f) difference processed - result

Figs. 2(b,e) provide the result image, containing the contour drawn from the estimated contour parameters. Figs. 2(c,f) provide the difference image. The difference images are not entirely white, which is due to the slight bias on the estimation of σ. This bias may be due to approximations of Eqs. (3) and (5): the adequation between signal model and generated signal cannot be strictly fulfilled. The whole computational time need to estimate the orientation and spread values is 2.30 sec. In comparison, the computational load which is needed to generate the signal is negligible, and the time needed for the offset estimation step is 0.9 sec.

B. Real-world images

Figs. 3 and 4 illustrate the proposed algorithm on a real-world image. Our goal is to study the beam which is produced by a space instrument.

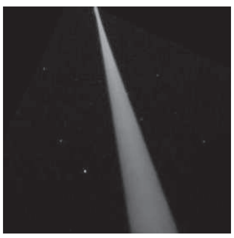


Fig. 3. Light beam characterization: original real-world image

We seek for the main orientation and spreading of the light beam. Fig. 4 focuses on the part of interest of the image (see Fig. 4(a)), and provides the image simulated with the estimated parameters (see Fig. 4(b)). The estimated parameters are $\hat{\theta} = -19°$; $\hat{\sigma} = 6.01$ pixels; $\hat{x}_0 = 112$ pixels.

We notice that the visual aspect of Fig. 4(b) is very close to the aspect of Fig. 4(a), which means that the contour characteristics were accurately estimated.

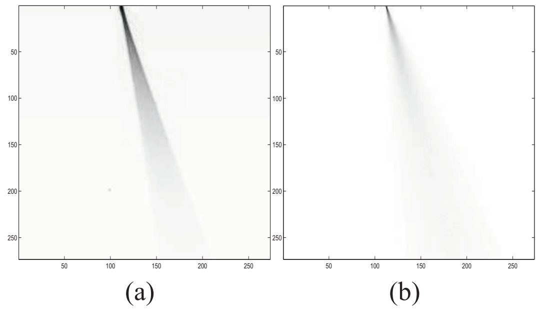


Fig. 4. Light beam characterization: (a) processed; (b) result (−19°; 6.01; 112)

These experiments show that there exists an interval $\sigma = [1.5, \ldots, 3]$ where the approximations of Eqs. (3) and (5) are both valid: the expected parameters are retrieved with a small bias. Moreover, the proposed method handles the case of 1-pixel wide contours.

VI. Conclusion

We show in this paper that an array processing signal model can be adequately adapted to the content of an image containing a fuzzy contour when a signal generation scheme is applied to this image. We show that if a spatial smoothing technique is applied to the signal generated out of the image, the sources in the signal model are decorrelated. Therefore, this allows us to adapt the DSPE (Distributed Signal Parameter Estimator) method to retrieve the characteristics of the fuzzy contours present in an image: the main orientation and the spread parameter. Experimental results obtained on hand-made images and real-world photographs proved the efficacy and the interest of the proposed method.

References

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