FAST SUBSPACE-BASED SOURCE LOCALIZATION METHODS

J. Marot, C. Fossati, and S. Bourennane

GSM - Institut Fresnel / CNRS-UMR 6133 – Ecole Centrale Marseille,
D.U. de Saint Jérôme F-13397 MARSEILLE Cedex 20, FRANCE.

ABSTRACT

Source localization is based on the spectral matrix algebraic properties. Propagator, and Ermolaev-Gershman (EG) noneigenvector algorithms exhibit a low computational load. Propagator is based on spectral matrix partitioning. EG algorithm obtains an approximation of noise subspace using an adjustable power parameter of the spectral matrix and choosing a threshold value. In this paper, we aim at demonstrating the usefulness of QR and LU factorizations of the spectral matrix to improve these methods. Experiments show that the modified propagator and EG algorithms based on factorized spectral matrix lead to better localization results, compared to the existing methods.

1. INTRODUCTION

Propagator, and Ermolaev-Gershman (EG) methods are noneigenvector algorithms which exhibit low computational load. These algorithms are efficient in non-noisy or high signal to noise ratio (SNR) environments. However both algorithms shall be improved. Propagator is not robust to noise; EG algorithm requires the knowledge of a threshold value between largest and smallest eigenvalues, which are not available as eigendecomposition is not performed.

In this paper, we propose new versions of the propagator and EG localization methods [1, 3] which employ a factorized spectral matrix and which are efficient in noisy situations. To this end, we use the upper triangular matrices obtained by the LU or QR factorizations of the spectral matrix.

We exploit the benefit of the factorization algorithm regarding the new rearrangement of the elements of the spectral matrix in the resulting upper triangular matrices **R** or **U**. All the signal information is focused in the upper-left corner block matrix of size equal to the number of sources. This block matrix contains the largest diagonal elements of the factorized matrix. In other words, it concentrates the signal information which is scattered in all spectral matrix elements. This concentration improves the robustness to noise of propagator method.

2. PROBLEM STATEMENT

Consider an array of N sensors receiving the signals generated by P $(P<N)$ narrow-band sources in the presence of an additive noise. The received signal vector is sampled and the FFT algorithm is used to transform the data into the frequency domain, we present these samples by [1]:

$$\mathbf{x}(f)=\mathbf{A}(f)\mathbf{s}(f)+\mathbf{n}(f) \tag{1}$$

In the rest of the paper the frequency f is omitted. In Eq. (1) $\mathbf{x}$ is the Fourier transform of the array output vector, $\mathbf{s}=[s_1,\ldots,s_P]^T$ is the signal source vector and $\mathbf{n}=[n_1,\ldots,n_N]^T$ is the additive noise vector.

The $(N\times P)$ matrix $\mathbf{A}=[\mathbf{a}(\theta_1),\ldots,\ \mathbf{a}(\theta_P)]$ is the transfer matrix of the sources-sensors array system with respect to a chosen reference point. The steering vectors $\mathbf{a}(\theta_i)$ where θ_i, $i=1,\ldots,P$, is the DOA of the i^{th} source measured with respect to the normal of the array. For a linear uniform array with N sensors the steering vector is

$$\mathbf{a}(\theta_i)=\frac{1}{\sqrt{N}}\left[1,\ e^{-j\varphi_i},\ e^{-2j\varphi_i},\ldots,\ e^{-(N-1)j\varphi_i}\right]^T$$

Where $\varphi_i=2\pi f\frac{d}{c}\sin(\theta_i)$; d is the sensor spacing and c is the wave propagation velocity. Assume that the signals and the additive noises are stationary and ergodic zero mean complex-valued random processes. In addition, the noises are assumed to be uncorrelated between sensors, and to have identical variance σ^2 in each sensor. It follows from these assumptions that the spatial $(N\times N)$ spectral matrix of the observation vector is given by:

$\mathbf{\Gamma}=\mathbf{A}\mathbf{\Gamma}_s\mathbf{A}^H+\mathbf{\Gamma}_n$, where $\mathbf{\Gamma}=E\left[\mathbf{x}\mathbf{x}^H\right]$, $\mathbf{\Gamma}_s=E[\mathbf{s}\mathbf{s}^H]$,
$\mathbf{\Gamma}_n=E\left[\mathbf{n}\mathbf{n}^H\right]=\sigma^2\mathbf{I}$,

where $\mathrm{E}\,[.]$ denotes the expectation operator and $\mathbf{I}$ is the $(N\times N)$ identity matrix. In the following, the propagator and EG algorithms are presented and improved.

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3. OVERVIEW OF PROPAGATOR AND EG ALGORITHMS

We present in this section two noneigenvector methods, Propagator and "Ermolaev and Gershman" methods.

3.1 Propagator method

Propagator method [1] relies on the partition of the transfer matrix **A**. Providing that **A** is full rank P, and the first rows are linearly independent, there exists a $P\times(N-P)$ matrix $\mathbf{\Pi}_\Gamma$ called propagator operator, such that [5] $\tilde{\mathbf{A}} = \mathbf{\Pi}_\Gamma^H \overline{\mathbf{A}}$, where $\overline{\mathbf{A}}$ and $\tilde{\mathbf{A}}$ are the $P\times P$ and $(N-P)\times P$ block matrices respectively obtained by partitioning the transfer matrix: $\mathbf{A} = [\overline{\mathbf{A}}^T \tilde{\mathbf{A}}^T]^T$, define the $N\times(N-P)$ matrix $\mathbf{D}_\Gamma$: $\mathbf{D}_\Gamma = [\mathbf{\Pi}_\Gamma^T \quad -\mathbf{I}_{N-P}]^T$, where $\mathbf{I}_{N-P}$ is the $(N-P)\times(N-P)$ identity matrix. We get $\mathbf{D}_\Gamma^H\mathbf{A} = \mathbf{\Pi}_\Gamma^H\overline{\mathbf{A}} - \tilde{\mathbf{A}} = \mathbf{0}$. In other words the (*N-P*) columns of $\mathbf{D}_\Gamma$ are orthogonal to the columns of **A**. This means that the subspace spanned by the columns of the matrix $\mathbf{D}_\Gamma$ is the same as the subspace spanned by the noise subspace given by the eigenvectors associated with the (*N-P*) smallest eigenvalues of matrix $\mathbf{\Gamma}$. We then obtain the DOAs of the sources by the peak positions in the so-called spatial spectrum [1]: $F_{\Pr}(\theta) = \left|\mathbf{a}(\theta)^H \mathbf{D}_\Gamma \mathbf{D}_\Gamma^H \mathbf{a}(\theta)\right|^{-1}$. Propagator algorithm is based on the noise subspace spanned by the columns of matrix $\mathbf{D}_\Gamma$. The computation of matrix $\mathbf{D}_\Gamma$ requires a prior knowledge of matrix $\mathbf{A}$, which is unknown. However, matrix $\mathbf{D}_\Gamma$ can be estimated only from the received data [1].

The estimation of propagator is performed in this way:
We define the data matrix **X** containing all K signal realizations as: $\mathbf{X}=[\mathbf{x}_1,\ldots,\mathbf{x}_K]$.
Matrix **X** is partitioned (in the same way as $\mathbf{A}$) as $\mathbf{X} = [\overline{\mathbf{X}}^T \tilde{\mathbf{X}}^T]^T$. The resulting spectral matrix will be expressed as follows:

$$\mathbf{\Gamma} = \begin{pmatrix} \mathbf{\Gamma}_{11} + \sigma^2\mathbf{I}_p & \mathbf{\Gamma}_{11}\mathbf{\Pi}_\Gamma \\ \mathbf{\Pi}_\Gamma^H\mathbf{\Gamma}_{11} & \mathbf{\Pi}_\Gamma^H\mathbf{\Gamma}_{11}\mathbf{\Pi}_\Gamma + \sigma^2\mathbf{I}_{N-p} \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{G}_{11} & \mathbf{\Gamma}_{12} \\ \mathbf{\Gamma}_{21} & \mathbf{G}_{22} \end{pmatrix} \qquad (2)$$

where $\mathbf{\Gamma}_{11}$ and $\mathbf{\Gamma}_{12}$ are, respectively, $(P\times P)$ and $(P\times(N-P))$ matrices, using the partition of matrix **A** we have: In non-noisy environment $(\boldsymbol{\sigma}^2 = 0)$, the relation $\mathbf{\Gamma}_{12} = \mathbf{\Gamma}_{11}\mathbf{\Pi}_\Gamma$ is used to estimate $\mathbf{\Pi}_\Gamma$:

$$\mathbf{\Pi}_\Gamma = \mathbf{\Gamma}_{11}^{-1}\mathbf{\Gamma}_{12} \qquad (3)$$

In the presence of noise, Eq. (3) is no longer valid. An estimation of the matrix $\mathbf{\Pi}_\Gamma$ is provided by minimizing the cost function $J(\mathbf{\Pi}_\Gamma) = \|\mathbf{\Gamma}_{12} - \mathbf{G}_{11}\mathbf{\Pi}_\Gamma\|^2$, where $\|.\|$ is the Frobenius norm. The optimal solution is given by: $\mathbf{\Pi}_\Gamma = \mathbf{G}_{11}^{-1}\mathbf{\Gamma}_{12}$. In practice the data are generally impaired and the SNR value is not always high. Then, the performance of propagator method depends on the signal information contained in the block matrix $\mathbf{G}_{11}$ respect to the noise and its linear dependency with the block matrix $\mathbf{\Gamma}_{12}$.

3.2 Ermolaev Gershman algorithm

The basic principle of EG method is to obtain the projector on the noise subspace by:

$$\mathbf{V}_{en}\mathbf{V}_{en}^H = \lim_{m\to\infty} \left(\left(\frac{1}{\lambda_s} \mathbf{\Gamma} \right)^m + \mathbf{I} \right)^{-1}$$

where '*e*' stands for "Ermolaev". The threshold value λ_s is between λ_P and λ_{P+1}. In practice parameter *m* can be set to 10.

4. IMPROVED ALGORITHMS

4.1 Propagator method.

We insert an LU decomposition step in Propagator method to improve its robustness to noise. The properties of the upper triangular matrix are used to minimize the influence of model errors.

Assume that spectral matrix $\mathbf{\Gamma}$ bears LU factorization, then it is expressed as [4, 5]:

$$\mathbf{\Gamma} = \mathbf{LU} = \begin{pmatrix} \mathbf{L}_{11} & \mathbf{0} \\ \mathbf{L}_{21} & \mathbf{I}_{N-P} \end{pmatrix} \begin{pmatrix} \mathbf{U}_{11} & \mathbf{U}_{12} \\ \mathbf{0} & \mathbf{U}_{22} \end{pmatrix} \qquad (4)$$

we have:

$$\mathbf{\Gamma} = \begin{pmatrix} \mathbf{L}_{11}\mathbf{U}_{11} & \mathbf{L}_{11}\mathbf{U}_{12} \\ \mathbf{L}_{21}\mathbf{U}_{11} & \mathbf{L}_{21}\mathbf{U}_{12} + \mathbf{U}_{22} \end{pmatrix} \qquad (5)$$

Using Eqs. (2), (3) and (5), we have:

$$\mathbf{L}_{11}\mathbf{U}_{12} = \mathbf{L}_{11}\mathbf{U}_{11}\mathbf{\Pi}_U. \quad (6)$$

Finally, the novel estimate of the propagator operator using LU factorization is: $\mathbf{\Pi}_U = \mathbf{U}_{11}^{-1}\mathbf{U}_{12}$. Following similar calculations with the QR factorization, and same partitioning of matrices $\mathbf{Q}$ and $\mathbf{R}$, we obtain: $\mathbf{\Pi}_R = \mathbf{R}_{11}^{-1}\mathbf{R}_{12}$. The useful signal components are concentrated in matrices $\mathbf{U}_{11}$ and $\mathbf{U}_{12}$. This yields a better robustness to noise compared to the case where the classical propagator method is applied.

Let the matrices

$\mathbf{D}_U = \left[\mathbf{\Pi}_U^T \quad -\mathbf{I}\right]^T$ and $\mathbf{D}_R = \left[\mathbf{\Pi}_R^T \quad -\mathbf{I}\right]^T$.

It follows that the DOAs of the sources are given by the positions of the maxima of the following functions:

$$F_{U-\mathrm{Pr}}(\theta) = \left[\mathbf{a}^H(\theta)\mathbf{D}_U\mathbf{D}_U^H\ \mathbf{a}(\theta)\right]^{-1},$$
$$F_{R-\mathrm{Pr}}(\theta) = \left[\mathbf{a}^H(\theta)\mathbf{D}_R\mathbf{D}_R^H\ \mathbf{a}(\theta)\right]^{-1}$$

4.2. Ermolaev Gershman algorithm

Following the algebra results published in [4, 5], we have: $\lambda_P > \|\mathbf{U}_{22}\| > \lambda_{P+1}$, and $\lambda_P > \|\mathbf{R}_{22}\| > \lambda_{P+1}$.

Then we can choose either $\lambda_s = \lambda_s^U = \|\mathbf{U}_{22}\|$ or $\lambda_s = \lambda_s^R = \|\mathbf{R}_{22}\|$.

5. ALGORITHM COMPLEXITIES

The main advantage of the methods presented in this paper, namely propagator and Ermolaev and Gershman methods is their low computational load.

	Traditional method	Proposed method
Propagator	$N^2P+P^2N+P^3$	$P^2(N-P+1)$
Ermolaev and Gershman	N^3	$N^3/3+N^2$

The proposed methods are based on the LU or QR factorization which requires considerably less computations than eigendecomposition. This result is interesting for large arrays with few sources which is often the case in underwater acoustics.

6. SIMULATION RESULTS

In the following simulations, a linear antenna of N=15 equispaced sensors is used, sensors being distant by $d = \frac{c}{2f_0}$, where f_0 is the source frequency and c is the velocity of the propagation. Eight uncorrelated source signals of equal power have DOA values: 5°, 10°, 20°, 25°, 35°, 40°, 50° and 55°, and are temporally stationary zero-mean with the same central frequency $f_0 = 115\,\mathrm{Hz}$. The additive noise is not correlated with the signals and it is also assumed white. SNR is defined by: $SNR = 10\log_{10}\left(s/\sigma^2\right)$, where s is the power of the source and σ^2 is the noise variance.

6.1. Propagator method.

We have considered several simulations with different SNR values. Firstly, the employed propagator methods are calculated with SNR = 0 dB. The number of sources is taken equal to 8, and supposed to be known.

It has been shown that, in the presence of an additive noise, the performances of the standard propagator are considerably degraded. However, the results obtained show that these degradations are not significant when the proposed propagator algorithms are used even if the values of SNR are relatively low. Indeed Figs. 1, 2 and 3 show that only the proposed methods have localized all the sources when the SNR is equal to 0 dB.

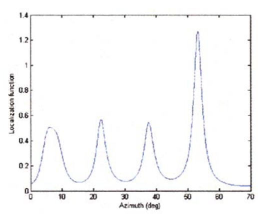

Fig. 1. $\mathbf{\Pi}_\Gamma$-Propagator SNR=0dB.

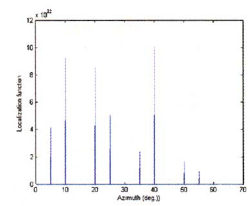

Fig. 2. $\mathbf{\Pi}_U$-Propagator, SNR=0 dB.

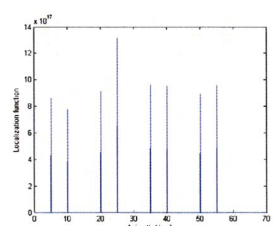

Fig. 3. $\mathbf{\Pi}_R$-Propagator with SNR= 0 dB.

The previous results have shown that even in the presence of noise the propagator algorithms localize all sources when LU or QR factorization is used.

6.2. Ermolaev Gershman algorithm

In order to compare the performance of the considered algorithms based on our thresholds λ_s^U or λ_s^R to one based on the threshold value λ_s arbitrarily chosen between λ_P and λ_{P+1}, several experiments with the same experimental conditions as in the previous subsection are carried out with m =10. Figs. 4 and 5 exemplify the obtained localization results.

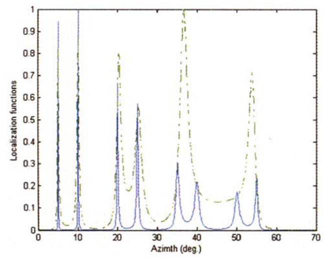

Fig. 4. EG algorithm as a function of the threshold values λ_S^U and λ_s ; with m=10 and SNR= -5 dB.

__: λ_S^U , -.-: λ_s .

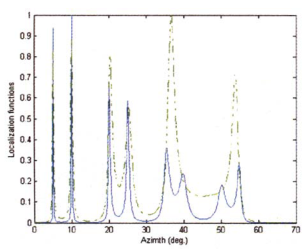

Fig. 5. EG method as a function of the threshold values λ_S^R and λ_s ; with m=10 and SNR= -5 dB.

__: λ_S^R , -.-: λ_s .

Fig. 6 presents the standard deviation values for modified EG and propagator methods, and Cramér Rao bound for EG method.

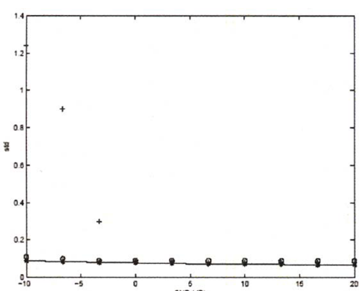

Fig. 6 standard deviation as a function of SNR:EG method(+), propagator method (o), EG Cramér Rao bound (-);

The results obtained show that the rank revealing triangular factorizations improve DOA localization, by enabling the choice of a convenient threshold value.

7. CONCLUSION

In this paper, we have improved two noneigenvector high resolution methods, namely the propagator method and the Ermolaev and Gershman algorithm. The improvement of propagator method and Ermolaev and Gershman algorithms is based on LU or QR factorization of the spectral matrix. This leads to an efficient localization of the narrow-band sources even if the SNR is low. Actually, the upper triangular matrices contain the information enabling source localization. The existing propagator method is a least square solution and still very sensitive to noise. On the opposite, the modified propagator method is calculated accurately from the upper triangular matrix even in the presence of noise. A major problem of the Ermolaev and Gershman algorithm is the estimation of the threshold. New and analytical thresholds are proposed to apply Ermolaev and Gershman algorithm. The threshold values are estimated thanks to the norm of the block matrix of the upper-right corner triangular matrix. The resulting algorithm for the localization without eigendecomposition is an approximation method, but the numerical results show its high accuracy even when the SNR is low.

8. REFERENCES

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