ARRAY PROCESSING APPROACH FOR OBJECT SEGMENTATION IN IMAGES

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ABSTRACT

Thanks to a specific formalism for signal generation, it is possible to transpose an image processing problem to an array processing problem. For straight line characterization, the existing method Subspace-based Line DEtection (SLIDE) works on virtual signals generated on a linear antenna. In this paper we propose to retrieve circular and nearly circular contours in images. We propose a novel method for radius estimation, and we extend the estimation of circles to the retrieval of circular-like distorted contours. For this purpose we develop a new model for virtual signal generation : we simulate a circular antenna, so that a high resolution method can be employed for radius estimation. An application to biomedical imaging is proposed.

***Index Terms*—** Antenna arrays, Biomedical image processing, Image segmentation, Optimization methods, Signal processing antennas

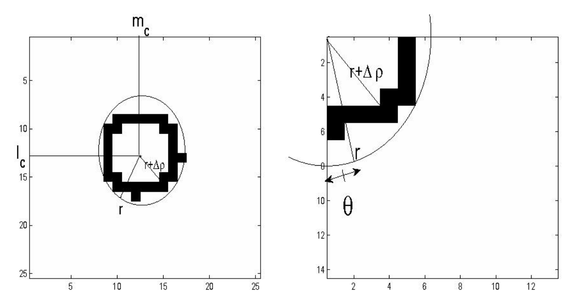
1. INTRODUCTION

Circular features are often sought in digital image processing. Several methods have been proposed for solving this problem. Generalized Hough transform [1] in particular is applied to biometrics, but its computational load is elevated. Contour-based snakes methods [2] detect objects with concavities. Fast array processing methods were adapted to the retrieval of contours in images [3, 4]. An existing method estimates the coordinates of the center of a circle [5], through a signal generation process upon a linear antenna [3]. In this paper we propose a new approach which employs a circular antenna. Section 2 sets the problem of circle retrieval and radius estimation, and explains how to generate a linear phase signal from the image, by means of a circular antenna; we show that the generated signal suits a high resolution method. In Section 3 we extend the work concerning circular contours to the case of any circular-like contour, by means of the Gradient optimization method [4]. In Section 4 we expose the results obtained by our methods, and propose a comparison with the generalized Hough transform.

2. PROBLEM SETTING AND SIGNAL GENERATION

2.1. Problem setting

The purpose of this paper is to estimate the radius of a circle, and the distortions between a closed contour and a circle that fits this contour. We propose to employ a circular antenna that enables a particular signal generation. We show that the phase characteristics of the signals which are generated fit classical high resolution and optimization methods. Fig. 1(a) presents a binary image I. An object in the image is made of edge pixels with value $'1'$, over a background of zero-valued pixels. The object is fitted by a circle with radius value r and center coordinates (l_c, m_c). Fig. 1(b) shows a sub-image extracted from the original image, such that its top left corner is the center of the circle. We associate this sub-image with a set of polar coordinates (ρ, θ), such that each pixel of the expected contour in the sub-image is characterized by the coordinates $(r + \Delta\rho, \theta)$, where $\Delta\rho$ is the shift between the pixel of the contour and the pixel of the circle that fits the contour and which has same coordinate θ. $\Delta\rho$ can get either positive or negative values.



(a) (b)

Fig. 1. (a) An image containing a contour fitted by a circle with center coordinates (l_c, m_c) ; (b) Bottom right quarter of the contour and pixel coordinates in the polar system (ρ, θ) having its origin on the center of the circle. r is the radius of the circle. $\Delta\rho$ is the value of the shift between a pixel of the contour and the pixel of the circle having same coordinate θ.

In previous work [6, 4] the optimization method that is set retrieves the phase shift between a linear phase model and the phase of a signal which is generated from the image. The phase shift corresponding to each component of the signal generated on a linear antenna is proportional to the pixel shift between an approximately linear contour composed of one pixel per row or column and an initialization straight contour. In this paper, we retrieve contours which are no longer approximately linear but approximately circular. Contours which are approximately circular are supposed to be made of more than one pixel per row for some of the rows of the image and more than one pixel per column for some columns of the image. Therefore the principles of signal generation which are relevant for the retrieval of approximately linear contours are no longer relevant for the retrieval of approximately circular contours.

2.2. Signal generation

We set an analogy between the problem of the estimation of a circular contour in an image and the problem of the estimation of a wavefront in array processing. Our basic idea is to obtain a linear phase signal from an image containing a contour which is a quarter of circle. The signals which are virtually generated upon the antenna have a phase that is constant or varies linearly as a function of the index of the sensor. A quarter of circle with radius r and a circular antenna are represented on Fig. 2.

We explain here how to generate signal components along several directions in the image, corresponding to different values of θ in the polar coordinates system of the sub-image. The antenna is associated with the sub-image containing any quarter of the expected contour. It is a quarter of circle centered on the top left corner, and passing through the bottom right corner of the sub-image. Such an antenna is adapted to the sub-images containing each quarter of the expected contour (see Fig. 2). Every sub-image but one is rotated such that its top left corner is the estimated center. A squared image is obtained by zero-padding. Therefore the antenna has radius $R_{antenna}$ such that $R_{antenna} = \sqrt{2} * N_{subimage}$ where $N_{subimage}$ is the number of rows or columns in the sub-image supposed to be squared shaped. When we consider the sub-image which includes the right bottom part of the expected contour, we have the relation : $N_{subimage} = max(N-l_c, N-m_c)$ where l_c and m_c are the vertical and horizontal coordinates of the center of the expected contour in a cartesian set centered on the top left corner of the whole processed image (see Fig. 1). Coordinates l_c and m_c are estimated by the method proposed in [5], which is based on the generation of signals on a linear antenna by a variable speed propagation scheme.

The directions adopted for signal generation go from the top left corner of the sub-image to the corresponding sensor. If the antenna is composed of S sensors, there are S different signal components.

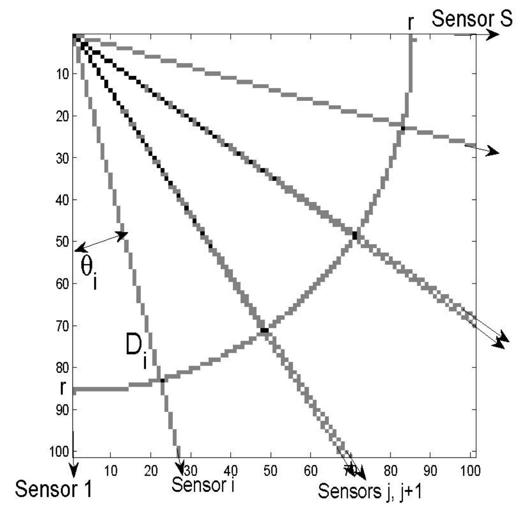


Fig. 2. A sub-image is extracted from the processed image : its top left corner is the center of the expected circle of radius r. The sub-image is associated with a circular array composed of S sensors.

Let us consider $\mathbf{D_i}$, the line that makes an angle θ_i with the vertical axis and passes through the top left corner of the sub-image. The i^{th} component $z(i)$ ($i = 1, \ldots, S$) of the signal $\mathbf{z}$ generated out of the image is obtained through the following computation :

$$z(i) = \sum_{(l,m)\in \mathbf{D_i}} \sum_{l,m=1}^{l,m=N_{subimage}} I(l,m) exp(-j\mu\sqrt{l^2+m^2}), \quad (1)$$

The integer l (resp. m) indexes the lines (resp. the columns) of the image. Parameter μ is the constant propagation parameter [7]. Each sensor indexed by i is associated with a line $\mathbf{D_i}$ having an orientation $\theta_i = \frac{(i-1)*\pi/2}{S}$. The presence of the term $I(l,m)$ indicates that only pixels that have value different from 0 are taken into account for signal generation. The constraint $(l,m) \in \mathbf{D_i}$, means that for the index i, only the pixels that belong to the line with orientation θ_i are taken into account. The minimum number of sensors that enables a perfect characterization of any contour is the number of pixels that would be virtually aligned on a quarter of circle having radius $\sqrt{2} * N_{subimage}$. Therefore the minimum number S of sensors is $\sqrt{2} * N_{subimage}$. In the most general case there exist more than one circle for one center. Therefore we use a variable speed propagation scheme [7] toward the circular antenna. This turns our problem of radius estimation into a problem of frequency estimation. Now, it is well-known that high resolution methods are able to distinguish several close-valued frequencies. In particular, TLS-ESPRIT method has exhibited a good behavior in the application of array processing to straight line detection [3]. The number d of concentric circles is estimated by an MDL criterion [6]. We set $\mu = \alpha(i-1)$, for each sensor indexed by $i = 1, \ldots, S$. From Eq. (1), the signal received on each sensor is :

$$z(i) = \sum_{k=1}^{d} exp(-j\alpha(i-1)r_k) + n(i), \ i = 1, \ldots, S \quad (2)$$

where $r_k, k = 1, \ldots, d$ are the values of the radius of each circle, and $n(i)$ is a noise term due to outlier pixels. All components $z(i)$ form the observation vector $\mathbf{z}$. The signal model of Eq. (2) suits the frequency estimation method TLS-ESPRIT [3]. The estimated radius values are used in next section to initialize an optimization method.

3. OPTIMIZATION METHOD FOR THE ESTIMATION OF NEARLY CIRCULAR CONTOURS

The optimization methods proposed in [6, 4] assume that one component of the generated signal is associated with only one unknown for the optimization method, the pixel shift between the initialization contour and the expected contour at one row (or column) of the image. We propose to employ a circular antenna and to retrieve the shift values between an initialization circle and the expected contour, along several directions in the image. These directions pass through the center of the initialization circle and have several orientation values. We work successively on each quarter of circle, and retrieve the distortions between one quarter of the initialization circle and the part of the expected contour that is located in the same quarter of the image. As an example, in Fig. 1, the right bottom quarter of the considered image is represented in Fig. 1(b). Our optimization strategy is the following :
A contour in the considered sub-image can be described in a set of polar coordinates by : $\{\rho(i), \theta(i), \ i = 1, \ldots, S\}$. We aim at estimating the S unknowns $\rho(i), \ i = 1, \ldots, S$ that characterize the contour, forming a vector :

$$\rho = [\rho(1), \rho(2), \ldots, \rho(S)]^T. \quad (3)$$

The basic idea is to consider that ρ can be written as : $\rho = [r + \Delta\rho(1), r + \Delta\rho(2), \ldots, r + \Delta\rho(S)]^T$ (see Fig. 1), where r is the radius of a circle that fits the expected contour. The optimization method that we employ aims at estimating $\{\Delta\rho(i), \ i = 1, \ldots, S\}$, that is, the shifts between the initialization circle and the expected contour.
By making an analogy with Eq. (2), the components of signal $\mathbf{z}$ generated out of the image containing the expected contour are written :

$$z(i) = exp(-j\mu\rho(i)), \ \forall i = 1, \ldots, S \quad (4)$$

Equation (4) is obtained from Eq. (2) by considering $d = 1$ and replacing the constant r_1 by a radial coordinate $\rho(i)$, that can be different for each sensor i. So we try to recreate the signal defined in Eq. (4) from which we ignore the S parameters. We start from an initialization vector ρ_0, characterizing a quarter of circle that fits the expected distorted contour in the considered sub-image. The S components of ρ_0 are equal to r, the radius value that was previously estimated : $\rho_0 = [r, r, \ldots, r]^T$. Then, with q indexing the steps of this recursive algorithm, we aim at minimizing

$$J(\rho_q) = \left|\left|\mathbf{z} - \mathbf{z}_{estimated\ for\ \rho_q}\right|\right|^2 \quad (5)$$

where $||.||$ represents the norm induced by the usual scalar product of $\mathbb{C}^S$. The components of $\mathbf{z}_{estimated\ for\ \rho_q}$ are defined in the same way as the components of $\mathbf{z}$ as a function of the components of ρ_q, and the components of ρ_q are obtained from the components of ρ_0 by adding a shift : $\rho_q = [r + \Delta\rho_q(1), r + \Delta\rho_q(2), \ldots, r + \Delta\rho_q(S)]^T$. For this purpose we use the fixed step gradient method. The vectors of the series are obtained by the relation :

$$\forall q \in \mathbb{N} : \ \ \rho_{q+1} = \rho_q - \lambda\nabla(J(\rho_q)) \quad (6)$$

where $0 < \lambda < 1$ is the step for the descent. The recurrence loop is

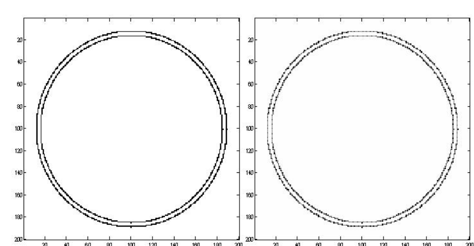
$$\rho_q \longrightarrow \mathbf{z}_{estimated\ for\ \rho_q} \longrightarrow J(\rho_q) \quad (7)$$

When q tends to infinity, the criterion J tends to zero and $\rho_q(i) = r + \Delta\rho(i) = \rho(i), \ \forall\, i = 1, \ldots, S$.
We denote by $\widehat{\rho}$ the vector containing all estimated values $\rho_q(i), \ i = 1, \ldots, S$, with q tending to infinity.

4. RESULTS OBTAINED BY THE PROPOSED METHODS

The proposed methods are applied to hand-made and real-world images having $N = 200$ columns and rows. We choose a number of sensors $S = 400$ for each quarter of image, which is larger than the minimum acceptable value. The procedures for center and radius estimation are run with propagation parameter $\alpha = 1.35 * 10^{-2}$. When TLS-ESPRIT method is run the length of each sub-array is, as recommended [7], $M = \sqrt{S} = 20$.

4.1. Estimation of the radius of two close circles



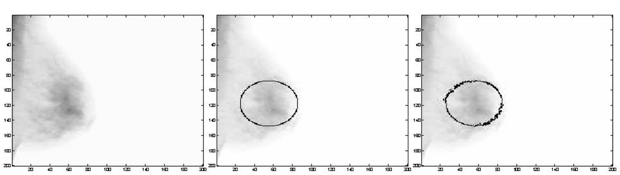
(a) (b)

Fig. 3. Center and radius estimation, two circles : (a) Processed, (b) Result (superimposed), with the proposed method for radius estimation.

Our signal generation process, associated with a high resolution method, solves two close-valued offsets (see Fig. 3). The expected radius values are 84 and 88 pixels. The estimated radius values obtained with the proposed method are 84.12 and 88.21 pixels, and the required computational time is 0.359 sec. The slight bias may come from the signal generation process. The generalized Hough transform finds the radius values with the *a priori* knowledge of the center and the number of circles. Then it can be compared with our method. Generalized Hough transform provides the estimated values : 83.94 and 87.97 pixels, and the required computational time is 14.5 sec. Visually there is no difference between the results of both methods.

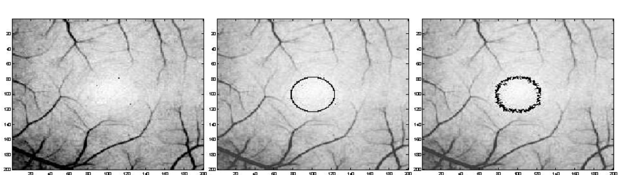
4.2. Distorted circle fitting

We now use our optimization method for biomedical applications. For the initialization step, parameters are the same as in subsection 4.1. The signal generation scheme that is performed before applying the optimization method is run with constant propagation parameter $\mu = 5 * 10^{-3}$. Gradient algorithm is run with a descent step parameter $\lambda = 0.02$, and 3000 iterations are necessary. Fig. 4 concerns medical images : fibroglandular disc localization in a mammographic image. Fig. 5 illustrates the localization of the foveal avascular zone in a digital retinal angiogram. Figs. 4(a) and 5(a) give the processed image. Figs. 4(b) and 5(b) give the initialization circle superimposed to the processed image. Figs. 4(c) and 5(c) give the result obtained with Gradient method. Part of the small bias remaining after the initialization is cancelled. Computational times are respectively 2.1 sec. for the initialization and 16.1 sec. for the optimization method.



(a) (b) (c)

Fig. 4. (a) Processed image, (b) Initialisation, (c) Result obtained with the proposed optimization method.



(a) (b) (c)

Fig. 5. (a) Processed image, (b) Initialisation, (c) Result obtained with the proposed optimization method.

5. CONCLUSION

We have shown in this paper how array processing and optimization methods can be applied to estimate distorted circular contours in images. In particular we propose a circular antenna for the generation of linear phase signals when circular contours are expected. This facilitates the application of high resolution methods and optimization algorithms for the estimation of distorted circles in images. We proposed a method for the estimation of several radius values that can be close to each other, and adapted an optimization strategy to the retrieval of distorted circles. Simulation results and an application to biomedical images proved that our methods work well with hand-made and real-world images.

6. REFERENCES

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