FUZZY CONTOUR CHARACTERIZATION BY SUBSPACE BASED METHODS OF ARRAY PROCESSING AND DIRECT METHOD

Haiping Jiang, Julien Marot, Caroline Fossati, and Salah Bourennane

Institut Fresnel/GSM, Ecole Centrale Marseille
D. U. de Saint-Jérôme, av. Normandie Niemen, 13397, Marseille, France
phone: + (33) 4 91 28 82 02, email: julien.marot@fresnel.fr
web: www.fresnel.fr/perso/marot

ABSTRACT

Blurred contours are often sought in digital image processing. Most of the existing techniques try to detect blurred contours by fuzzy techniques. In this article, we propose to model the blurred contours by generating the virtual signals from the image, and provide a new viewpoint to detect blurred contour. Especially, the blurred contour with exponential distribution can be presented by three parameters. Then, array processing methods are adopted to estimate these parameters, and optimization method is also considered when the spread parameter is estimated. Experiments finally prove that the proposed methods estimate the parameters of blurred contours accurately and with a reduced computational load.

1. INTRODUCTION

Blurred contours occur very often in images, owing to object movements, light transmission environment, etc. Several methods have been proposed for solving this problem. One can distinguish two categories of methods: those which perform contour-based segmentation, and those which perform region-based segmentation. Firstly, contour-based segmentation methods consider blurred contours as textured regions in the image, that is, a set of pixels which has a transverse width of more than one pixel. These methods aim at determining a mean position of the pixels of the textured region. In particular a recent model for active contours based on techniques of curve evolution, which was adapted from the level set paradigm, was proposed to segment contours ”without edges” [1].

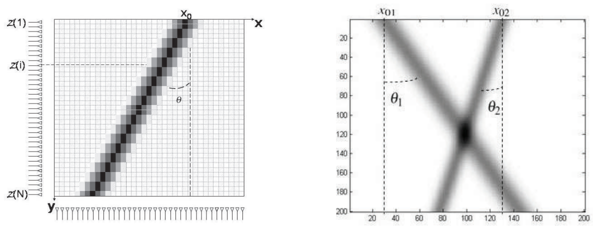
Secondly, region-based segmentation methods rely on the theory of fuzzy sets, to define membership functions and classify the pixels. In particular, a blurred paradigm was adopted in the frame of mathematical morphology to characterize the repartition of objects in an image by ”fuzzy” relationships [2]. In this fuzzy paradigm, one could localize an object at left or at the right-hand side of another object. This led in particular to applications for medical 3D image segmentation [2]. Also, unsupervised segmentation was adapted to classification by a fuzzy version of hidden Markov chains [3]. In [3], a fuzzy membership function is added to a crisp membership to characterize the values taken by the Markov process and thereby classify pixels in an image.

Array processing methods, and in particular high resolution methods [4], were adapted to contour characterization [5, 6]: a specific signal generation scheme yields an array processing signal model out of an image with 1-pixel wide contours. High resolution methods could then be applied to distinguish close contours by considering them as punctual sources. In this paper, we propose a novel approach to characterize blurred contours, that is, contours which are no longer one pixel wide but characterized by a spread parameter. For this, we derive a novel signal model. The advantage of the proposed model: it permits to characterize entirely a linear blurred contour with three parameters orientation, offset, and spread. We adapt subspace-based methods of array processing to provide an estimate of the orientation and offset parameters. Then, DIRECT optimization method is adapted to retrieve the spread parameters.

Section 2 states the blurred contour retrieval problem, section 3 derives an array processing model out of signals generated from the image. Section 4 adapts subspace-based methods of array processing to estimate orientation and offset parameters of the blurred contours. Section 5 proposes a method for the estimation of the blurred contour spread parameters. Section 6 proposes a refined offset estimation. Section 7 summarizes the proposed methods and section 8 presents experimental results.

2. PROBLEM STATEMENT

In this section, we provide the models that we adopt for the processed image, for the gray level distribution of the contours which are present in the image, and for the technique which permits to generate a signal out of the image content. Let $I(i,l)$ be an $N \times C$ recorded image (see Fig. 1(a)). We consider that $I(i,l)$ is compound of a blurred contour and an additive uniformly distributed noise. The blurred contour is supposed to have width $2X_f$, main orientation θ, and center offset x_0. The contour gray level variation around its main orientation in every row can be, in a general manner, described by a decreasing exponential function; for example, a Gaussian evolution depending on a spread parameter σ: $g(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}} = G\, e^{-\frac{x^2}{2\sigma^2}}$.



(a) (b)

Figure 1: (a) linear antenna model in an image containing a blurred line; (b) blurred contours characterized by the main orientations θ_k and offsets x_{0k} in the image

If there are multiple blurred contours in the image, every contour is characterized by the main orientation θ_k and the offset x_{0k} (see Fig. 1(b)). Every blurred contour obeys Gaussian distribution with the variance $\sigma_k{}^2$. We expect that such a contour model facilitates the transfer of array processing methods to the considered parameter estimation issue. In order to set the link between image data representation and sensor array processing methods [7], array sensors are supposed to be placed in front of each row or column of the image. Each sensor receives the signal only from its corresponding row in the matrix. All the pixels in the image are assumed to propagate narrow-band electromagnetic waves with zero initial phases. Furthermore, we assume that the waves emanating from pixels in a given row of the image matrix are confined to travel only along that row towards the corresponding sensor.

We adopt the signal generation scheme proposed in [7]:

$$z(i) = \sum_{l=1}^{C} I(i,l)\, e^{-j\mu l},\ i = 1,\ldots,N \qquad (1)$$

In the next section, we show that the adopted models and signal generation process yield an array processing signal model, which is handled by subspace-based methods of array processing.

3. ARRAY PROCESSING SIGNAL MODEL

Firstly, we assume that the image contains only one blurred contour of width $2X_f$, main orientation θ, offset x_0, and spread parameter σ. The signal generated on the i^{th} sensor is then expressed as:

$$\begin{aligned} z(i) &= G\ \textstyle\sum_{x=1}^{X_f} e^{-j\mu(x_0+x-(i-1)tan(\theta))} e^{-\frac{x^2}{2\sigma^2}} \\ &+ G\ \textstyle\sum_{x=1}^{X_f} e^{-j\mu(x_0-x-(i-1)tan(\theta))} e^{-\frac{x^2}{2\sigma^2}} \\ &+ G\, e^{-j\mu(x_0-(i-1)tan(\theta))} \end{aligned} \qquad (2)$$

That is:

$$\begin{aligned} z(i) = &\ G\ \textstyle\sum_{x=-X_f}^{X_f} e^{-j\mu(x_0+x-(i-1)tan(\theta))} e^{-\frac{x^2}{2\sigma^2}} \\ = &\ G\, e^{-j\mu x_0} e^{j\mu(i-1)tan(\theta)} \textstyle\sum_{x=-X_f}^{X_f} e^{-j\mu x} e^{-\frac{x^2}{2\sigma^2}} \end{aligned} \qquad (3)$$

where G is the maximum gray level value in the image, and X_f is the half-width of the contour. If σ is small enough compared with the number of columns in the image, we can turn the considered discrete calculation into a continuous case calculation. Eq. (3) becomes:

$$z(i) \approx G\, e^{-j\mu x_0} e^{j\mu(i-1)tan(\theta)} \int_{x=-\infty}^{+\infty} e^{-j\mu x} e^{-\frac{x^2}{2\sigma^2}}\, dx \qquad (4)$$

A general formula provides the equality:

$$\int_{x=-\infty}^{+\infty} e^{-ax^2+jbx} dx = \sqrt{\frac{\pi}{a}} e^{-\frac{b^2}{4a}} \qquad (5)$$

It is easy to express eq. (4) by

$$z(i) = \sqrt{2\pi} G\, e^{-j\mu x_0} e^{j\mu(i-1)tan(\theta)} \sigma e^{-\frac{\mu^2\sigma^2}{2}} \qquad (6)$$

Eq. (6) is the signal in the i-th row in the case where there exists only one blurred contour in the image.

Secondly, we consider the case where the image contains:

- d blurred contours, with orientations θ_k, offsets x_{0k}, and spread parameters σ_k $(k = 1,\ldots,d)$;
- identically distributed noise pixels.

The expression of the received signal by i^{th} sensor becomes:

$$z(i) = \sqrt{2\pi} G\ \textstyle\sum_{k=1}^{d} e^{-j\mu x_{0k}} e^{j\mu(i-1)tan(\theta_k)} \sigma_k e^{-\frac{\mu^2\sigma_k{}^2}{2}} + n(i) \qquad (7)$$

where $n(i)$ is a noise term originated by the noise pixels during the signal generation process. The expression of the signal components in Eq. (7) permits to adopt the notations coming from array processing. We define:

1. the source amplitude associated with the k-th contour as: $s(k) = G\ \ e^{-j\mu x_{0k}} \sum_{x=-X_f}^{X_f} e^{-j\mu y} e^{-\frac{x^2}{2\sigma_k{}^2}},\ \ k = 1,\cdots,d$. When the continuous approximation holds, the source amplitude components are expressed as:
$$s(k) = \sqrt{2\pi} G e^{-j\mu x_{0k}} \sigma_k e^{-\frac{\mu^2\sigma_k{}^2}{2}}, k = 1,\cdots,d \qquad (8)$$
2. the steering vector associated with the k-th contour as: $\mathbf{c}(\theta_k) = [c_1(\theta_k), c_2(\theta_k),\cdots,c_i(\theta_k),\cdots,c_N(\theta_k)]^T$, with $c_i(\theta_k) = e^{j\mu(i-1)\tan(\theta_k)}$.
3. the noise vector $\mathbf{n} = [n(1), n(2),\ldots,n(N)]^T$.

These notations permit to express the signal generated out of the image in a matrix form:

$$\mathbf{z} = \mathbf{C}(\theta)\mathbf{s} + \mathbf{n} \qquad (9)$$

where:
$\mathbf{z} = [z(1), z(2),\ldots,z(N)]^T$, $\mathbf{C}(\theta) = [\mathbf{c}(\theta_1), \mathbf{c}(\theta_2),\cdots,\mathbf{c}(\theta_d)]$, $\mathbf{s} = [s(1), s(2),\cdots,s(d)]^T$.

Eq. (9) shows that, by adopting the signal generation scheme of Eq. (1) and the proposed model for blurred contours, we can make an analogy between the signals generated out of the image and an array processing signal model. Therefore, we expect that array processing methods can yield the parameters of the expected contours.

4. SUBSPACE BASED METHODS OF ARRAY PROCESSING FOR ORIENTATION AND OFFSET ESTIMATION

4.1 Estimation of the blurred contour main orientation

An array processing method can be applied to the generated signal provided in eq. (9), to characterize the contours in the image by retrieving their parameters. However, for the signal generated from the image, which is time-independent, we can not get any sample series, so a subspace-based parameter estimation method such as MUSIC can not be directly adapted to the image signal [8]. We have to simulate artificially multiple signal measurements out of a single sample array data by splitting the array (of length N) into smaller overlaying sub-arrays (of length M). This is called spatial smoothing technique. There exist a constraint on M, and a relationship between N, M and the number of snapshot P: $d < M \leq N-d+1$; and $M = N-P+1$. For details, refer to [7]. From the observation vector $\mathbf{z}$ we obtain P overlapping sub-vectors. By grouping all sub-vectors obtained in matrix form, we obtain

$$\mathbf{Z}_P = [\mathbf{z}_1,\cdots,\mathbf{z}_P] \qquad (10)$$

where

$$\mathbf{z}_p = \mathbf{C}_M(\theta)\mathbf{s}_p + \mathbf{n}_p,\ p = 1, \cdots, P. \tag{11}$$

Each column of $\mathbf{C}_M(\theta)$ is a vector of length M expressed as: $\mathbf{c}(\theta_k) = [c_1(\theta_k), c_2(\theta_k), \cdots, c_i(\theta_k), \cdots, c_M(\theta_k)]^T$, with $c_i(\theta_k) = e^{j\mu(i-1)tan(\theta_k)}$, and
$\mathbf{s}_p = [s_p(1), s_p(2), \cdots, s_p(d)]^T$ where
$s_p(k) = \sqrt{2\pi}G\sigma_k e^{-j\mu x_{0k}} e^{\frac{-\mu^2\sigma_k^2}{2}} e^{j(p-1)\mu tan(\theta_k)}$, $p = 1, 2, \cdots, P$.
The covariance matrix of all sub-vectors of Eq. (10) is defined by:

$$\mathbf{R}_{zz} = \frac{1}{P}\sum_{p=1}^{P} \mathbf{z}_p {\mathbf{z}_p}^H \tag{12}$$

where $(.)^H$ denotes Hermitian transpose. We operate the singular value decomposition (SVD) of $\mathbf{R}_{zz}$.

$$\mathbf{R}_{zz} = [\mathbf{U}_1\ \ \mathbf{U}_2]\ \mathbf{\Lambda}\ \mathbf{V} \tag{13}$$

For independent sources, the columns of matrix $\mathbf{U}_1\,(\mathrm{M}\times\mathrm{d})$ span the signal subspace, the columns of matrix $\mathbf{U}_2\,(\mathrm{M}\times(\mathrm{M}-\mathrm{d}))$ span the noise subspace, and $\mathbf{\Lambda} = diag(\lambda_1, \lambda_2, \cdots \lambda_d)$ where λ_i is the eigenvalue associated with the i^{th} eigenvector. Hence, $\mathbf{U}_2$ is orthogonal to the steering vectors $\mathbf{c}(\theta_k)$, $k = 1, \ldots, d$. We estimate the θ_k parameters ($k = 1, \ldots, d$) through the maxima of the pseudo spectrum [8] given by

$$\mathrm{MUSIC}(\theta_k) = \frac{1}{\|\ \mathbf{c}^H(\theta)\cdot\mathbf{U}_2\ \|^2} \tag{14}$$

where $\mathbf{c}^H(\theta)$ is a model for the signal subspace vectors.

4.2 Estimation of the blurred contour offset

Once the orientation values are known, the offset values can be estimated by variable speed generation scheme [6] and TLS-ESPRIT algorithm [7]. Variable speed propagation scheme consists in setting $\mu = \alpha(i-1)$. Eq. (7) becomes: $z(i) =$

$$\sqrt{2\pi}G\ \sum_{k=1}^{d} e^{-j\alpha(i-1)x_{0k}} e^{j\alpha(i-1)^2 tan(\theta_k)} \sigma_k e^{-\frac{(\alpha(i-1))^2{\sigma_k}^2}{2}} + n(i) \tag{15}$$

We can consider for instance the first orientation $\theta = \theta_1$.

As θ_1 value has been estimated, we can divide $z(i)$ by the term $e^{j\alpha(i-1)^2 tan(\theta_1)}$. We obtain:

$$w(i) = z(i)/e^{j\alpha(i-1)^2 tan(\theta_1)} =$$

$$\sqrt{2\pi}G\ e^{-j\alpha(i-1)x_{01}}\sigma_1 e^{-\frac{(\alpha(i-1))^2{\sigma_1}^2}{2}} + n'(i) \tag{16}$$

where $n'(i)$ is a noise term resulting from the influence noisy pixels and all but the first contour.
At this point, the value of σ_1 is not known and we propose an approximation which permits to get momentarily a gross estimate of x_{01} without the prior knowledge of σ_1. If the propagation parameter α is chosen such that $\alpha(i-1) << 1$, $\forall\, i = 1, \ldots, N$, we can adopt the following approximation: $w(i) \approx \tilde{w}(i) =$

$$\sqrt{2\pi}G\ e^{-j\alpha(i-1)x_{01}}\sigma_1 + n(i) \tag{17}$$

The signal $\tilde{\mathbf{w}} = [\tilde{w}(1), \tilde{w}(2), \ldots, \tilde{w}(N)]$ fits the model required by the frequency estimation TLS-ESPRIT method [7] which retrieves the first offset value x_{01} from Eq. (17).

The division process of Eq. (16) and the adaptation of TLS-ESPRIT method are repeated for each value $k = 1, \ldots, d$. At this point a gross estimate of the offset values is available, which will be used to estimate the spread parameter values.

5. DIRECT METHOD FOR SPREAD PARAMETER ESTIMATION OF THE BLURRED CONTOURS

In this subsection we propose a least-square criterion which involves the signal generated out of the image and the signal model of Eq. (9). This criterion depends on the parameters of all contours. We adapt DIRECT optimization method to retrieve the spread parameter values by minimizing this criterion.

5.1 Least-squares criterion derivation

The contour orientations estimated by MUSIC algorithm are used to compute the steering matrix $\mathbf{C}(\theta)$ (see Eq. (9)). The source vector $\mathbf{s}$ depends not only on the offset parameters x_{0k} ($k = 1, \ldots, d$), but also on the spread parameters σ_k ($k = 1, \ldots, d$). Therefore we propose to retrieve the components of the source vector s, through the following criterion minimization:

$$\hat{\mathrm{s}} = \underset{\mathbf{s}}{argmin}||\mathbf{Z} - \mathbf{C}\mathbf{s}||^2 \tag{18}$$

It is easy to show that the density function of the measurement noise is Gaussian if the outliers are identically distributed over the image [7]. Therefore, the above least-squares problem provides the maximum likelihood estimate for the source vector. The relationship between the source vector components and the spread parameter values is given by (see eq. (8)):

$$s(k) = f(\sigma_k) = \sqrt{2\pi}G\sigma_k e^{-j\mu x_{0k}} e^{-\frac{\mu^2{\sigma_k}^2}{2}} \tag{19}$$

We denote by $\sigma = [\sigma_1, \ldots, \sigma_d]^T$ the vector containing all spread parameter values, and by $\mathbf{f}(\sigma) = [f(\sigma_1), \ldots, f(\sigma_d)]^T = [s(1), \ldots, s(d)]^T$ the source vector. We denote by $\hat{\sigma} = [\hat{\sigma}_1, \ldots, \hat{\sigma}_d]^T$ the vector containing the estimates of all spread parameter values. From eqs. (18) and (19), we get:

$$\hat{\sigma} = \underset{\sigma}{argmin}||\mathbf{Z} - \mathbf{C}\mathbf{f}(\sigma)||^2 \tag{20}$$

which can be expressed as:

$$\hat{\sigma} = \underset{\sigma}{argmin}(J(\sigma)) \tag{21}$$

where J denotes the criterion to be minimized. To solve eq. (21) and minimize criterion J, we adopt a recurrence loop to modify recursively vector $\hat{\sigma}$. The series vectors are obtained from the relation $\forall q \in \mathbb{N}$:

$$\hat{\sigma}^q \rightarrow \mathbf{f}(\hat{\sigma}^q) \rightarrow J(\hat{\sigma}^q) \tag{22}$$

When q tends to infinity, the criterion J tends to zero and $\hat{\sigma}_k^q = \sigma_k$, $\forall\ k = 1, \ldots, d$. To carry out this recurrence loop, we adopt the robust DIRECT (DIviding RECTangles) optimization method [9]. DIRECT method is initialized by $\hat{\sigma}^0$, and a research space which is an acceptable interval for each value. Vector $\hat{\sigma}^0$ and the research space are *a priori* fixed by the user. The main property of DIRECT is that it is able to obtain the global minimum of a function. DIRECT normalizes the research space in a hypercube and evaluates the solution which is located at the center of this hypercube. Then, some solutions are evaluated and the hypercube is divided into smaller cubes, supporting the zones where the evaluations are small. When the required number of iterations $q = It$ is reached, DIRECT provides the estimated vector of spread parameters $\hat{\sigma}^{It} = [\sigma_1, \sigma_2, \ldots, \sigma_d]$.

6. REFINED ESTIMATION OF THE OFFSET VALUES

The knowledge of all spread values σ_k, $k = 1, \ldots, d$ permits to avoid the approximation made in section 4, which led to signal components $\tilde{w}(i)$ out of signal components $w(i)$ (see Eq. (17)). Starting from the expression of $w(i)$ in Eq. (16), we derive the signal $\omega(i)$, $i = 1, \ldots, d$:
$\omega(i) =$

$$w(i)/(\sqrt{2\pi}\sigma_1 e^{-\frac{(\alpha(i-1))^2\sigma_1{}^2}{2}}) = G\, e^{-j\alpha(i-1)x_{01}} + n'(i) \quad (23)$$

where $n'(i)$ is a noise term resulting from the influence of all but the first contour. The signal components $\omega(i)$ fit TLS-ESPRIT method, which is applied d times to retrieve the exact offset values $x_{0k},\ k = 1, \ldots, d$.

7. SUMMARY OF THE PROPOSED ALGORITHM

An outline of the proposed blurred contour estimation method is given as follows:

- find out the mean position of the pixels of the contour:
  - choose μ as a constant value, and estimate the orientations θ_k $(k = 1, \ldots, d)$ by MUSIC method;
  - choose μ as a variable value $\mu = \alpha(i-1)$, and estimate the offsets x_{0k} $(k = 1, \ldots, d)$ by TLS-ESPRIT method;
- estimate the spread parameters σ_k $(k = 1, \ldots, d)$ through the minimization of a least-squares criterion by DIRECT optimization method;
- obtain a refined estimation of x_{0k} $(k = 1, \ldots, d)$, with the knowledge of the previously estimated σ_k values.

8. EXPERIMENTAL RESULTS

8.1 Hand-made images

In all numerical experiments, we consider images of size $N \times N$ where $N = 200$. At the same time, all experiments are performed on a computer equipped with 2.83GHz 2 Quad CPU and 4Go memory. As concerns parameter μ, [7] provides a study that gives the maximum value of an estimated orientation. Adequate parameters are $\mu = 10^{-1}$, to estimate the orientation parameters with MUSIC method, and $\mu = 10^{-3}$, to estimate the spread parameters with DIRECT methods. The value of α is set as 2.5 10^{-3}.

• Case 1: the spread parameters of two blurred contours have both large values.

Fig. 2 exemplifies the case where the spread parameters are 8 and 8 respectively, the center offsets of two blurred contours $x_{01} = 150$ and $x_{02} = 40$, and the main orientation of two contours are $\theta_1 = 10°$ and $\theta_2 = -10°$. The proposed method is compared with Chan and Vese's level set method [1], which meant to delimitate blurred contours.

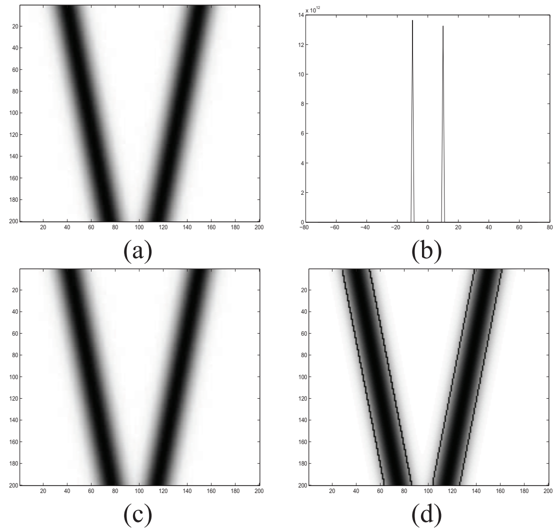


Figure 2: (a) processed image with two blurred contours; (b) pseudo spectrum when MUSIC algorithm is exploited; (c) estimation by the proposed method; (d) superposition of the initial image and Chan and vese result

Orientation values are estimated as 10° and −10°. For this, signal generation and MUSIC algorithm lasts 0.069 *sec*. Offset values are estimated as 150 and 40. For this, TLS-ESPRIT algorithm lasts 0.17 *sec*. The spread parameters are estimated as 8.01 and 7.98 by 15 iterations of DIRECT in 0.053 sec. From Fig. 2(d), we can see that Chan and Vese method provides a boundary for the two expected contours. Namely, the method converges, and the active contour stops inside the blurred boundaries of the object. However, we denote that the proposed method characterizes the whole contour including grey level variation, whereas the levelset method considers local properties to stop the evolution of the active contour.

• Case 2: concurrence between blurred contour and high-contrast contour in the image.

In the following experiment, we try to detect the image including blurred contour and high-contrast, one pixel wide contour. The main orientation of blurred line is 10°, its offset is 150, and the spread parameter σ is 8. For the high-contrast straight line, its orientation and offset are −10° and 40.

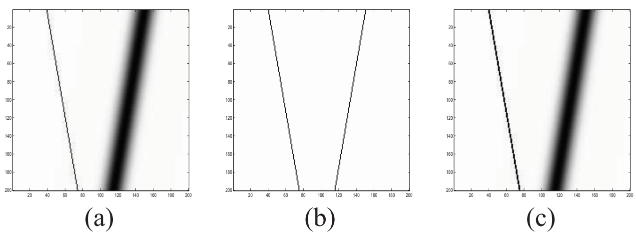


Figure 3: (a) processed image; (b) contour center pixels;(c) superposition of the initial image and the center pixel estimation

From Fig. 3, the estimated orientation of blurred contour is 10°. The offset is estimated as 149.6 pixels. The estimated spread parameter is 8. The detection of high-contrast line contour is characterized by the estimated orientation $-10°$ and offset 40.3. The estimated spread parameter is found to be 0.01. So the bias on the estimated parameters is always 1% or less.

8.2 Real-world images

We consider the estimation of the cinematic parameters of multiple objects in an image. In [10], a technique is proposed for estimating the parameters of two-dimensional (2-D) uniform motion of multiple moving objects in a scene, based on long-sequence image processing and the application of a multi-line fitting algorithm.

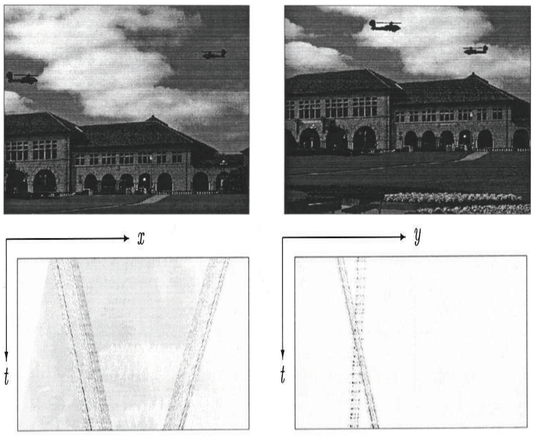


Figure 4: Motion estimation: image sequence and trajectories [10]

A specific formalism and a line detection algorithm yields the cinematic parameters of the objects [10]. However the proposed method does not take into account the variation along time of the cinematic parameters of the objects. Fig. 4 shows the first and last images among a set of 100 images representing two helicopters moving on a fixed background with a supposedly uniform speed. Estimated cinematic parameters [10] are $v_{1x} = 1.1$ pixels/frame and $v_{1y} = -0.2$ pixels/frame; $v_{2x} = -1.5$ pixels/frame and $v_{2y} = 0.9$ pixels/frame. This estimation would be valid for a punctual object and a constant speed. The trajectory is not a straight line because of the size of the object and the slight speed variations.

We propose to measure the imprecision due to the size of the object and the speed variation. Fig. 5 is the bottom right image of Fig. 4. By estimating the spread parameter σ for the two contours, we deduce the accuracy of the estimation of the cinematic parameters. In this case the estimated spread parameters are $\sigma_1 = 10$ and $\sigma_2 = 15$, which yields a maximum bias on the trajectory slopes and thereby on the speeds of $\Delta v_{1y} = \sigma_1/N = 5\%$ and $\Delta v_{2y} = \sigma_2/N = 7.5\%$.

9. CONCLUSION

We show in this paper how array processing and DIRECT optimization method can be applied to characterize blurred contours in images. In particular we propose a model for blurred contours. We show that, with an adequate signal generation technique, we obtain signals which follow an array processing model. We adapt subspace-based methods of array processing to retrieve the orientation and a gross estimate of the offset of the blurred contours. We propose an

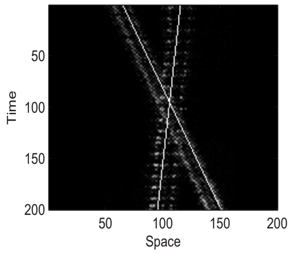


Figure 5: Trajectory characterization

adequate optimization strategy to retrieve the spread parameters of all contours. Then, we obtain the exact offset values with the knowledge of the spread parameters. Experimental results obtained on hand-made images and real-world machine vision images proved the efficacy and the interest of the proposed method.

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