Phase Distortion Estimation by DIRECT and Spline Interpolation Algorithms

Julien Marot and Salah Bourennane

Abstract—**An important cause of performance loss in source localization in underwater acoustics is that towed flexible antennas deviate from the assumed rectilinear shape. In this work, the localization of sources in presence of phase errors is studied. Cancellation of phase errors is necessary to solve the source-localization problem. A previous work led to interesting results for antennas composed of a few sensors. We propose here a novel algorithm which is adapted to the antennas composed of a large number of sensors, keeping a small computational load. Our method is based on an orthogonality property between signal and noise subspaces and a novel optimization method: the robust DIviding RECTangles algorithm accelerated by spline interpolation. The performances of the proposed method are illustrated by applying it to the characterization of three sources.**

Index Terms—**Array processing, distorted wavefront, estimation, global optimization, spline interpolation.**

I. Introduction

Giving the parameters of wavefronts impinging on a distorted antenna has been the purpose of various previous studies [1]–[3]. In this paper, we consider a distorted antenna and wavefronts that are distorted because of the inhomogeneity of the propagation medium [2]. Previous works considered antennas with a relatively low number of sensors [2]–[4]. Contrary to these works, we consider antennas composed of a large number of sensors. The proposed method must still lead to small computational times, to permit an easy real-time implementation. The proposed method is particularly useful when the number of sensors is much greater than the number of sources, which is usually the case in practice [3].

Section II presents the problem statement. The method proposed is described in Section III, where we estimate phase distortions by an optimization method. In Section IV, we evaluate the performances of the proposed method by an example with three wavefronts and a statistical study.

II. Problem Statement

Fig. 1 represents distorted wavefronts impinging on a distorted antenna (sensors $1, 2, \ldots, \mathrm{N}$). A limitation of the high-resolution methods based on the estimation of a covariance matrix is their inability to perform satisfactorily when a received wavefront is different from the assumed linear-phase wavefront model. In [2], a statistically efficient weighted signal subspace fitting algorithm is proposed, that exhibits low root-mean-square (rms) error values. We consider the case where the antenna is composed of many sensors. In order to keep small computational times, we propose an accelerated version of the DIviding RECTangles (DIRECT) algorithm [5] that gives valuable results in image processing [6].

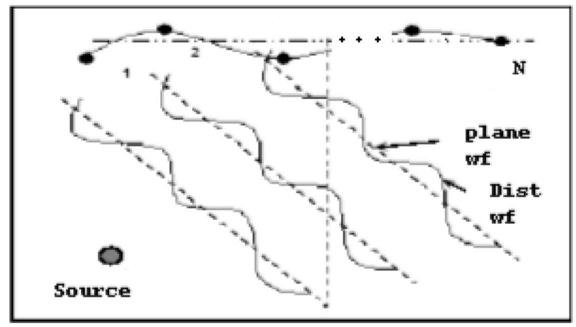

Fig. 1. Distorted wavefronts received on a distorted antenna composed of a large number of sensors.

III. Retrieval and Cancellation of Phase Distortions

A. Phase Shift Retrieval

We consider a d-spacing linear array of N identical sensors which receive the narrowband signals from P sources, centered at frequency f_0, in the presence of an additive noise. Using vector notations, the Fourier transforms of the outputs of the array can be written as [1]–[4] $\mathbf{z}(f_0) = \mathbf{A}(\boldsymbol{\phi})\mathbf{s}(f_0) + \mathbf{n}(f_0)$, where the $N \times P$ matrix $\mathbf{A}(\boldsymbol{\phi}) = [\mathbf{a}(\boldsymbol{\phi}_1), \mathbf{a}(\boldsymbol{\phi}_2), \ldots, \mathbf{a}(\boldsymbol{\phi}_P)]$ is the transfer matrix of the source sensor-array system [4]. The p^{th} column of $\mathbf{A}(\boldsymbol{\phi})$, called the directional vector of the p^{th} source, is $\mathbf{a}(\boldsymbol{\phi}_p) = \left[e^{-j\varphi_{p1}}, e^{-j\varphi_{p2}}, \ldots, e^{-j\varphi_{pN}}\right]^T$, where

$$\varphi_{pi} = \varphi_{pi}^{lin} + \Delta\varphi_{pi} \tag{1}$$

and $\varphi_{pi}^{lin} = 2\pi f_0 (d/c)(i-1)\sin(\theta_p)$. θ_p is the direction-of-arrival of the p^{th} source, c is the wave speed, and $\Delta\varphi_{pi}$ is an additive phase shift value, also called distortion, that can be due to the deformation of the array and to the perturbation of the impinging wavefronts. The vector containing all phase values for source p is denoted by $\boldsymbol{\phi}_p = [\varphi_{p1}, \varphi_{p2}, \ldots, \varphi_{pN}]^T$. $\mathbf{z}(f_0) = [z_1(f_0), z_2(f_0), \ldots, z_N(f_0)]^T$, $\mathbf{s}(f_0) = [s_1(f_0), s_2(f_0), \ldots, s_P(f_0)]^T$ and $\mathbf{n}(f_0) = [n_1(f_0), n_2(f_0), \ldots, n_N(f_0)]^T$ are the Fourier transforms of the array outputs, the source signals and noise vectors, respectively. We consider that K snapshots are

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available: $\{\mathbf{z}_1(f_0), \mathbf{z}_2(f_0), \ldots, \mathbf{z}_K(f_0)\}$. The high-resolution method proposed in [4] uses the eigendecomposition of the cross-spectral matrix of the received signals $\mathbf{\Gamma}_{zz}(f_0) = (1/K)\Sigma_{l=1}^{K}\mathbf{z}_l(f_0)\mathbf{z}_l^H(f_0)$, where ^{H} denotes transpose conjugate. By denoting $\mathbf{V}(f_0)$ the matrix containing the vectors of the noise subspace [4] associated with the $(N-P)$ smallest eigenvalues, we get [4]: $\mathbf{V}(f_0)^H\mathbf{A}(\boldsymbol{\phi}) = \mathbf{0}$. In the rest of the paper the frequency f_0 is omitted. This orthogonality property is used in the MUltiple SIgnal Classification (MUSIC) algorithm [4]. The inverse of $|\mathbf{V}(f_0)^H\mathbf{a}(\boldsymbol{\phi})|_F$, where $|.|_F$ denotes Frobenius norm, is called pseudospectrum. Firstly, using the MUSIC algorithm, initial values of directions-of-arrival are estimated. Secondly, we calculate for each initial direction-of-arrival $\widehat{\theta}_{0p}, p = 1, \ldots, P_0$ $(P_0 \le P)$ a vector of phase values $\widehat{\boldsymbol{\phi}}_p^{\,0} = \left[\widehat{\varphi}_{p1}^{lin}, \widehat{\varphi}_{p2}^{lin}, \ldots, \widehat{\varphi}_{pN}^{lin}\right]^T$, where

$$\widehat{\varphi}_{pi}^{lin} = 2\pi f_0 \frac{d}{c}(i-1)\sin(\widehat{\theta}_{0p}). \tag{2}$$

We use the DIRECT algorithm initialized by vector $\boldsymbol{\phi}_p^{\,0}$, in order to calculate the expected phase values

$$\boldsymbol{\phi}_p = \operatorname{argmin}(|\mathbf{V}^H\mathbf{a}(\boldsymbol{\phi}_p)|_F) \tag{3}$$

DIRECT performs global optimization [5]. The computational load of the DIRECT algorithm grows rapidly when the number of sensors, or equivalently the number of unknown phase values, increases. At any iteration m, we accelerate the DIRECT algorithm by reducing the number of retrieved unknowns and then we propose spline interpolation [6] to obtain the N phase values of $\widehat{\boldsymbol{\phi}}_p^{m}$: we interpolate a subset of values of $\widehat{\boldsymbol{\phi}}_p^{m}$, which are retrieved by the DIRECT algorithm. The more interpolation nodes, the more accurate the estimation. The series of vectors $\widehat{\boldsymbol{\phi}}_p^{m}$ converges when m tends to infinity [5], towards a vector $\widehat{\boldsymbol{\phi}_p}$ such that $|\mathbf{V}^H\mathbf{a}(\widehat{\boldsymbol{\phi}_p})|_F$ is minimum.

B. Cancel the Phase Shifts in the Received Signals

Then for each wavefront, the optimization method is run anew and initialized by the approximate value of the corresponding direction-of-arrival. For each source p, phase distortions of the received signals are canceled to obtain signals $\mathbf{z}_{l,\text{processed}}^p$, $l = 1\ldots,K$ that suit perfectly the method based on the orthogonality between signal and noise subspaces [4]. For one wavefront, phase distortions do not depend only on the physical shape of the antenna but also on the orientation of the wavefront. At this point an approximate value of several directions-of-arrival is available. We get the vector of estimated phase distortions

$$\widehat{\boldsymbol{\Delta\phi}}_p = \left[\widehat{\Delta\varphi}_{p1}, \widehat{\Delta\varphi}_{p2}, \ldots, \widehat{\Delta\varphi}_{pN}\right]^T \tag{4}$$

corresponding to the current wavefront using (1). We can cancel the phase distortions and compute the signals to be processed defined by

$$\mathbf{z}_{l,processed}^p = \mathbf{D}^p(\widehat{\boldsymbol{\Delta\phi}}_p)\,\mathbf{z}_l,\ l = 1, \ldots, K \tag{5}$$

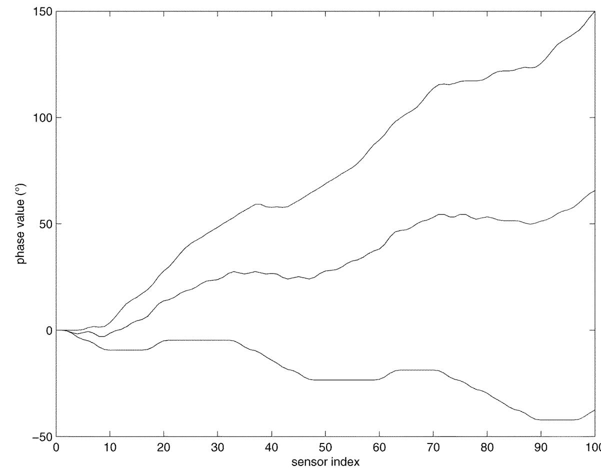

Fig. 2. Evolution of the phase of the three impinging wavefronts as a function of the sensor index.

where

$$\mathbf{D}^p(\widehat{\boldsymbol{\Delta\phi}}_p) = diag[e^{j\widehat{\Delta\varphi}_{p1}}, e^{j\widehat{\Delta\varphi}_{p2}}, \ldots, e^{j\widehat{\Delta\varphi}_{pN}}]. \tag{6}$$

Then the signals $\mathbf{z}_{l,processed}^p$, $l = 1, \ldots, K$ are employed in the high-resolution method. This permits to distinguish several close directions-of-arrival. This process is repeated iteratively for each direction-of-arrival: At each iteration it, for one direction-of-arrival index p, the following criterion must hold to pursue the algorithm $|\widehat{\theta}_p^{\,it} - \widehat{\theta}_p^{\,it-1}| > \epsilon$, *a priori* fixed threshold. That is, we consider we reached the accurate estimate of the p^{th} direction-of-arrival when the estimated value does not vary from an iteration to another.

C. Proposed Algorithm

In the following, we summarize the proposed algorithm to improve the MUSIC algorithm in the presence of phase distortions:

1) Estimate grossly initial values of several directions-of-arrival $\widehat{\theta}_{0p}, p = 1, \ldots,\ P_0, (P \le P_0)$ with MUSIC [4].

2) For each $\widehat{\theta}_{0p}$ repeat the following process, until the estimate of the direction-of-arrival does not vary from an iteration to another:

a) Estimate $\widehat{\boldsymbol{\phi}_p}$ by DIRECT algorithm associated with spline interpolation: Retrieve the phase shifts between the vector $\widehat{\boldsymbol{\phi}_p^0}$ of phase values corresponding to a plane wavefront [see (2)], and a vector of phase values described by (1).

b) Cancel the phase shifts in the received signal realizations [see (5)].

c) apply the MUSIC method [4] to the signal realizations obtained by (5), only in a reduced angle interval around the first estimation $\widehat{\theta}_{0p}$ of θ_p. Owing to phase cancellation, several close sources may be resolved inside the considered interval.

3) Regroup the P_0 pseudospectra calculated around the P_0 initial directions-of-arrival of the high-resolution method. Each maximum argument is the direction-of-arrival of one source.

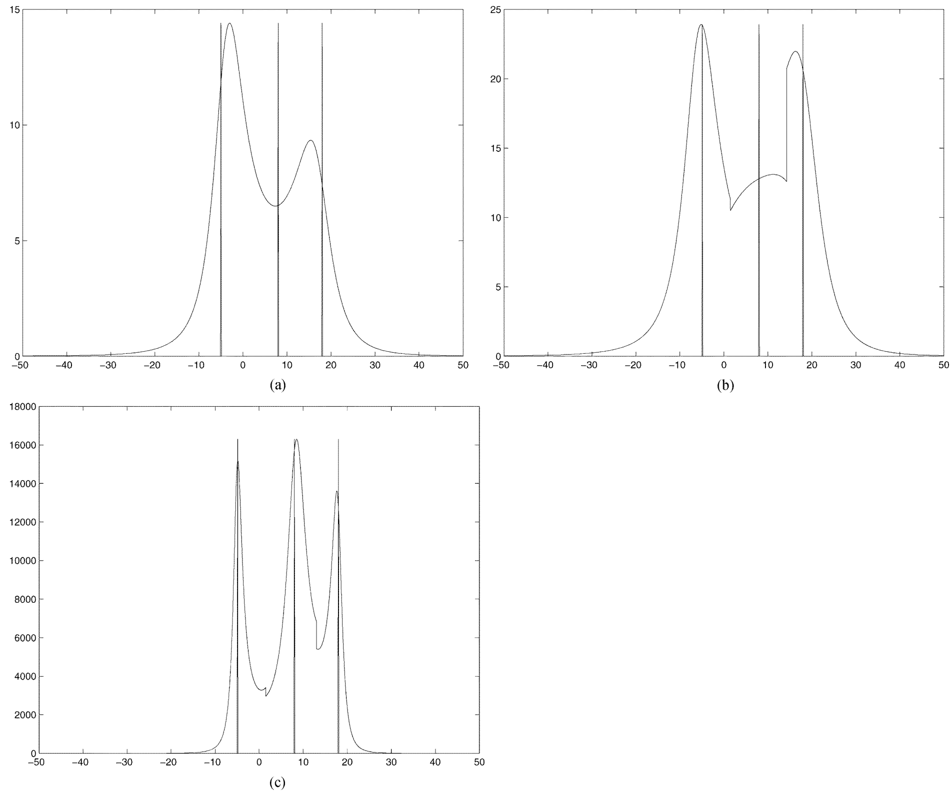

Fig. 3. Pseudospectra of MUSIC, represented in the interval $[-50°, 50°]$, superimposed to the expected values (vertical lines): (a) without cancellation of phase distortions; (b) and (c) with the estimation with 10 (resp. 25) iterations of DIRECT and cancellation of the phase distortions.

IV. Simulation Results

An antenna of $N = 100$ equi-spaced sensors with inter-element spacing $d = c/32f_0$ is used. Such an inter-element spacing avoids any phase indetermination for any angle. Time signals collected upon each sensor are 5 s and 1000 samples long. The additive noise is independent from the signals and with diagonal cross-spectral matrix $\mathbf{\Gamma}_n = \sigma^2\mathbf{I}$, $\mathbf{I}$ being Identity matrix. The signal-to-noise ratio (SNR) is defined as the ratio of the power of each source signal to the average power of the noise. We consider the configuration with $P = 3$ sources impinging on the array at $\theta_1 = -5.0°$, $\theta_2 = 8.0°$, and $\theta_3 = 18.0°$. The distribution of the time delay of the received signal at the sensor i is given, for $(i = 1, \ldots, N)$, $(p = 1, \ldots, P)$, by

$$\tau_{p_i} = \frac{d}{c}\left((i-1)\sin(\theta_p) + \alpha_p \sin\left(\frac{2\pi(i-1)}{T_p}\right) + t_i\right) \quad (7)$$

where $[t_1, \ldots, t_N]^T$ is a Gaussian vector with zero-mean used as perturbation, α_p is a distortion amplitude factor, T_p is a distortion period factor. Fig. 2 shows the three impinging wavefronts, including the phase shifts provoked by the distortions of the antenna. Fig. 3 shows the pseudospectra obtained with MUSIC, for an experiment with $\mathrm{SNR} = 16$ dB. Source localization is improved when we estimate the shape of the antenna with 25 iterations of DIRECT in our optimization method and when we perform phase cancellation: see Fig. 3(c), compared with that obtained without phase cancellation [see Fig. 3(a)], and compared to the case when only ten iterations of DIRECT are run [see Fig. 3(b)]. When phase cancellation is not done, the pseudospectrum of MUSIC exhibits only two maxima. The eigendecomposition of the cross-spectral matrix of signal realizations leads to two dominant eigenvalues. Therefore, we assume that the dimension of the signal subspace is $P_0 = 2$. The values obtained for the directions-of-arrival are $\widehat{\theta}_{01} = -4.0°$ and $\widehat{\theta}_{02} = 16.5°$. Then we initialize our recursive procedure with $\widehat{\theta}_{01}$, and $\widehat{\theta}_{02}$. When our algorithm is applied and phase cancellation is done for each direction-of-arrival, the values obtained for the directions-of-arrival are $\widehat{\theta_1} = -4.9°$, $\widehat{\theta_2} = 8.2°$, and $\widehat{\theta_3} = 17.6°$. One supplementary source is resolved in place of the single source with direction $\widehat{\theta}_{02}$, thanks to phase cancellation. For one iteration of the procedure of estimation of the phase distortions, computational time is 25 s for our optimization method with ten spline interpolation nodes and 25 iterations of DIRECT, on a 3.0-GHz Pentium-4 processor.

TABLE I

SNR (dB)	30	34	38	42	46
ME (°)	0.13	0.08	0.06	0.04	0.02
RMSE (°)	0.09	0.06	0.03	0.01	0.005

We propose a statistical study, considering the same wavefront parameters as above, performing 1000 trials. Mean error over the values of θ_1, θ_2, θ_3 is: $ME = (1/3000)\ \sum_{p=1}^{3}(\sum_{j=1}^{1000}\left|\widehat{\theta}_{p_j} - \theta_p\right|)$, where j indexes the trials and $\widehat{\theta}_{p_j}$ is the estimation obtained at the j^{th} trial for the p^{th} direction-of-arrival. Rms error is defined by $RMSE = \sqrt{(1/3000)\ \sum_{p=1}^{3}\sum_{j=1}^{1000}(\widehat{\theta}_{p_j} - \theta_p)^2}$. The statistical results obtained are always acceptable (see Table I).

V. Conclusion

A novel algorithm for the characterization of distorted wavefronts and the improvement of resolution in the presence of phase distortions is proposed. We focused on the case of antennas with many sensors. To keep low computational times, we proposed a version of "DIRECT" algorithm accelerated by spline interpolation to retrieve distorted wavefronts impinging on a distorted antenna. We have shown that, taking into account phase distortions and canceling them, the estimation of the directions-of-arrival of several wavefronts is possible and accurate.

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