Optimisation
Master TSI

Optimisation bio-inspirée

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J. Marot CM 12 h TD 8 h
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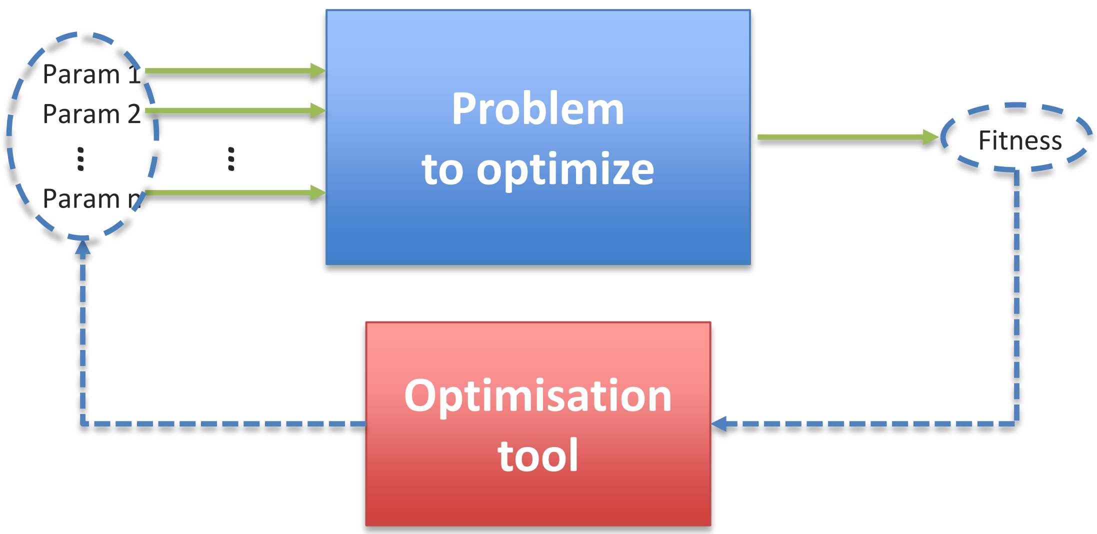
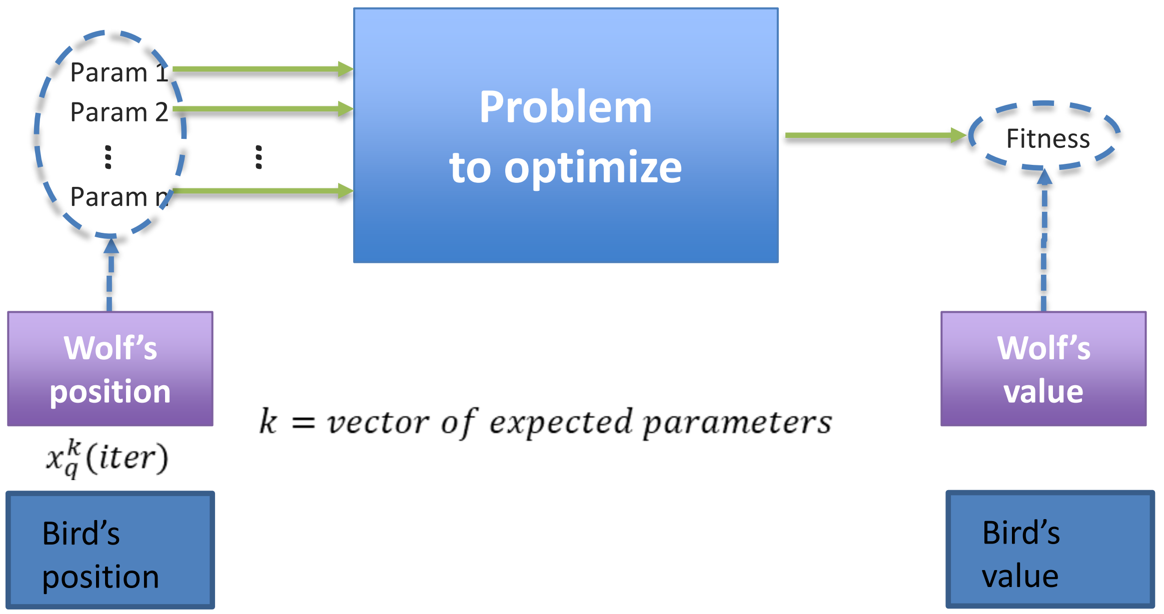
1. Definitions

Optimization problem:

Maximizing or minimizing a function or a problem by systematically choosing input values from within an allowed set and computing the value of the function.



Bio-inspired algorithm
Swarm optimization



1. Definitions

Bio-inspired optimization:

Optimization methods inspired from natural process.

Good capacity in solving computationally expensive problems, with limited function evaluation.

ex: Genetic Algorithm (GA)

Swarm optimization:

Simulate the behaviour of a group of particles

ex: Particle Swarm Optimization (PSO),

Ant Colony Algorithm (ACO)

Grey Wolf Optimizer (GWO)

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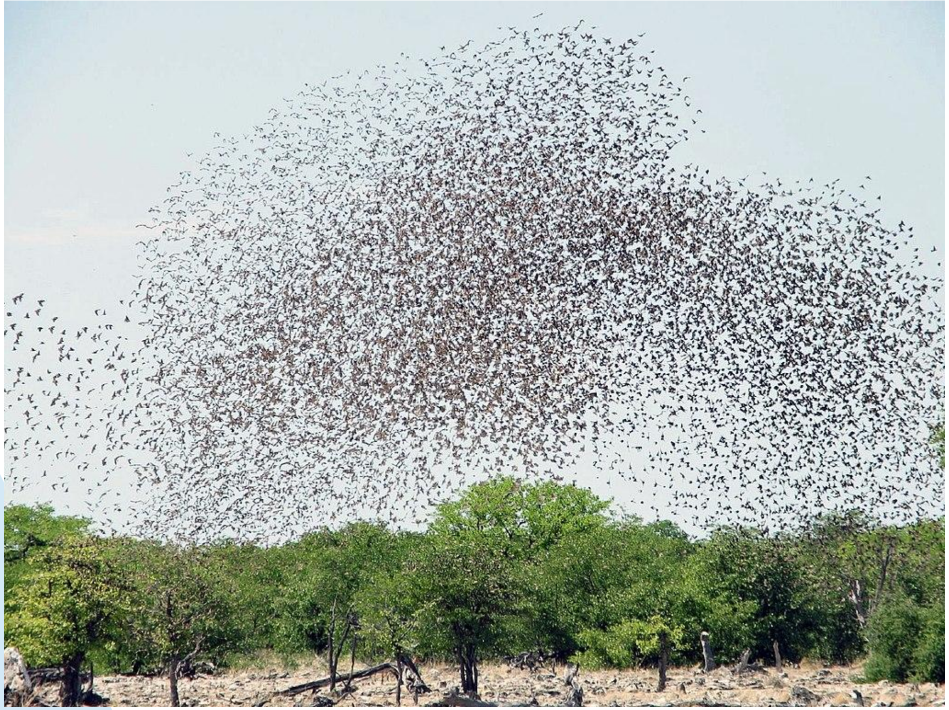
2. Particle Swarm Optimization

Swarm of birds



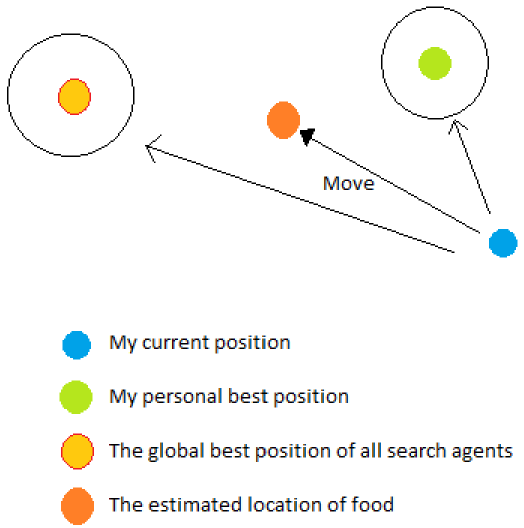
1. Gets inspired from social interaction of animals

2. Uses a number of agents (particles) that constitute a swarm moving around in the search space looking for the best solution

3. Each particle is treated as a point in a N-dimensional space which adjusts its "flying" according to its own flying experience as well as the flying experience of other particles
4. Each particles affords:
   1) Velocity
   2) Personal flying experience
   3) Global flying experience of the swarm

A particle has to decide where she should go

In what direction is the food ?



Update rules

x_q^k: *position of particle q*

p_q^k: *personal best*

G: *global best*

For each particle

V_q^k: *velocity of particle q*

r_{1q}, r_{2q} *random numbers*

Update the velocity

$$V_q^k(iter+1) = WV_q^k(iter)$$
$$\ldots + \gamma_1 r_{1q}(p_q^k - x_q^k(iter))$$
$$\ldots + \gamma_2 r_{2q}(G - x_q^k(iter))$$

My Previous velocity

My personal best position

The global best position of the whole swarm

Then move

$$x_q^k(iter+1) = x_q^k(iter) + V_q^k(iter+1)$$

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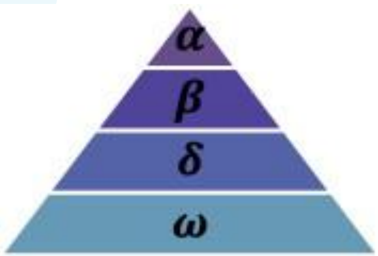
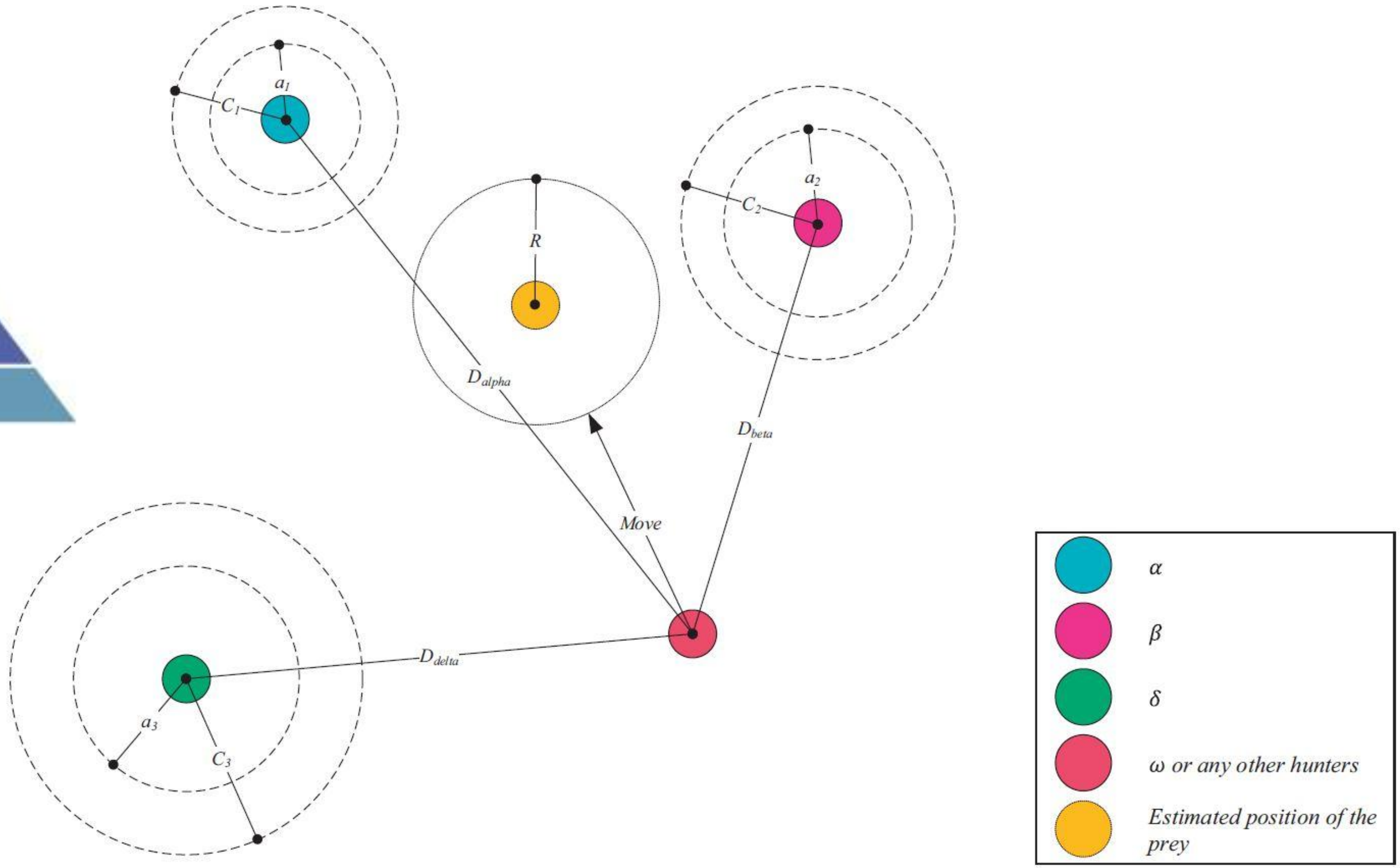
3. The classical Grey Wolf Optimizer (GWO)

Mimics the hunting behavior of a wolf pack

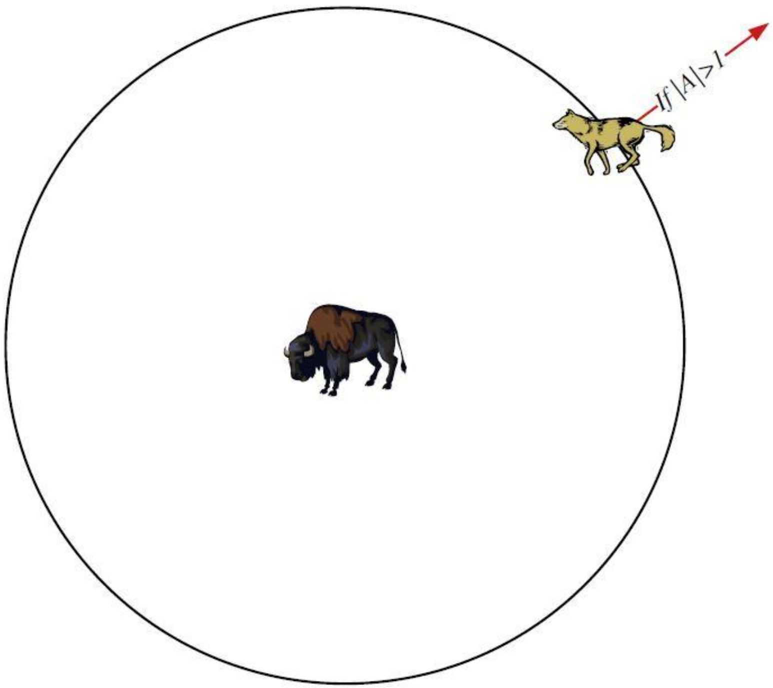
one wolf = one particle
prey = global minimum

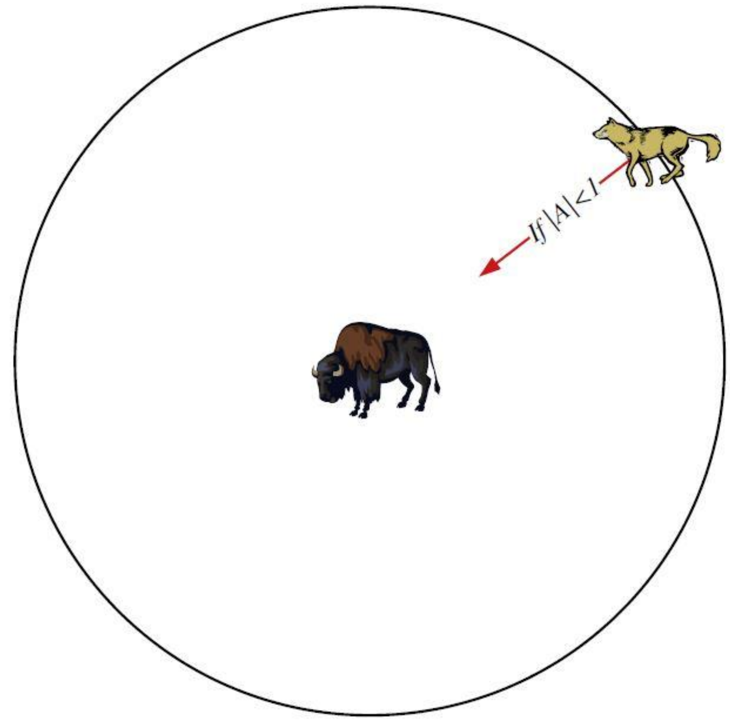
Wolf pack hierarchy



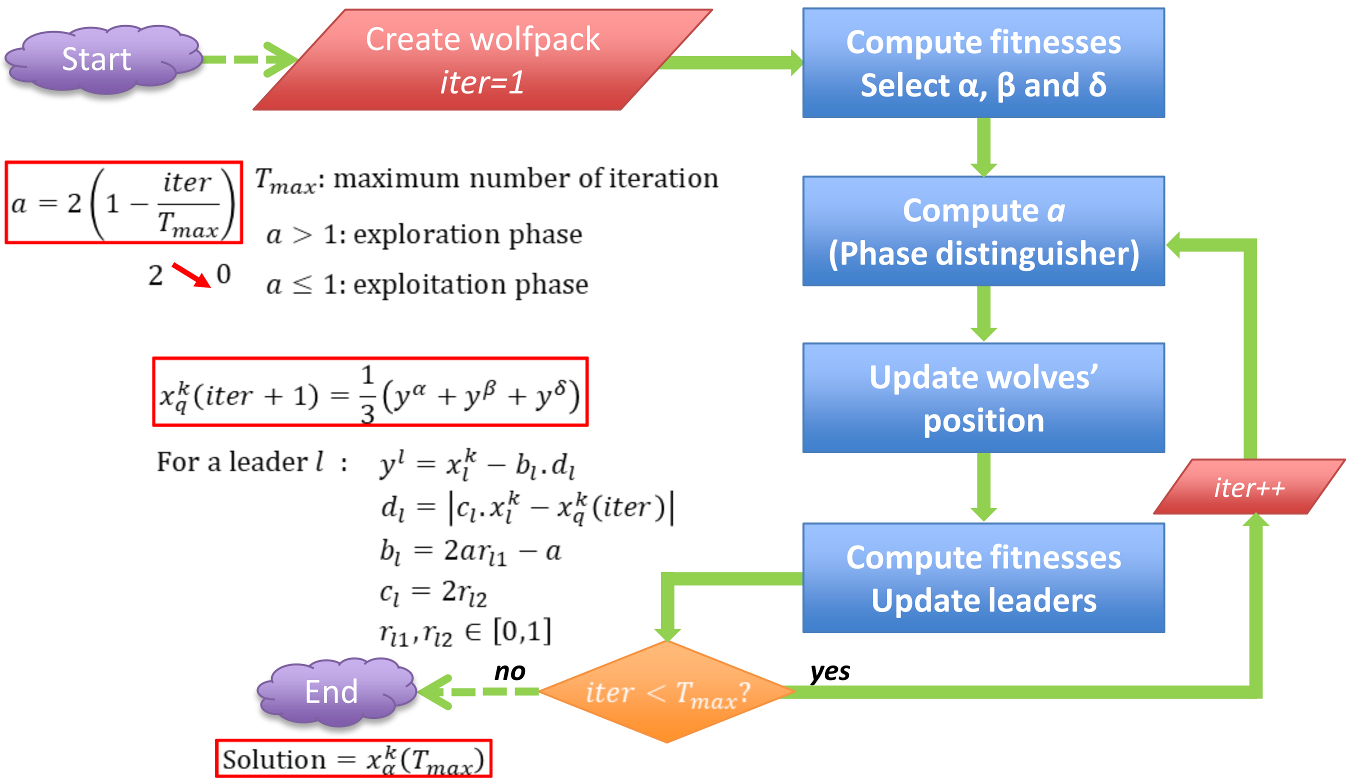


Exploration vs Exploitation





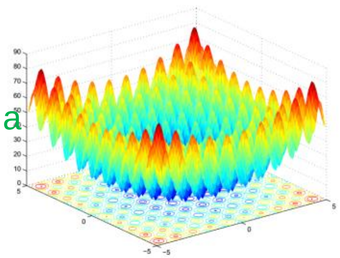
The algorithm's workflow



Bio-inspired optimization: the lock awaken

About the classical GWO:

- Designed to solve fully continuous problems Ex: $\widehat{K}_i \in [-10,10]$
- Robust to local minima



$$F_9(x) = \sum_{i=1}^{n} \left[x_i^2 - 10\cos(2\pi x_i) + 10\right]$$

About bio-inspired optimization:

- No method for non-binary or non-specific discrete problems

Ex: $\widehat{K}_i \in d_i^{val} = \{-10, -9, -8, \dots, 8, 9, 10\}$

- No method for mixed problems (continuous + discrete)

Create a bio-inspired optimization method based on GWO to tackle mixed problems

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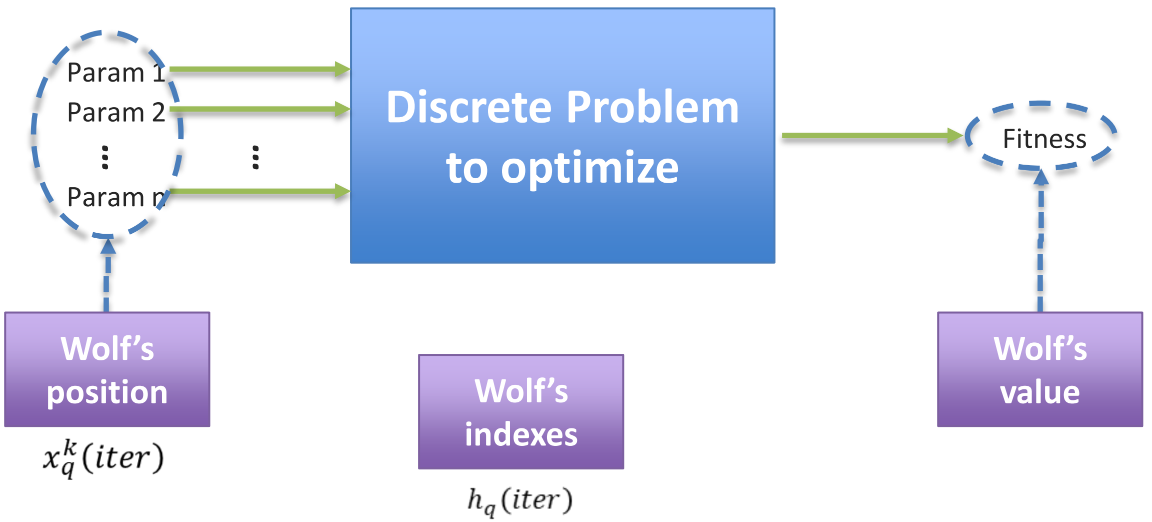
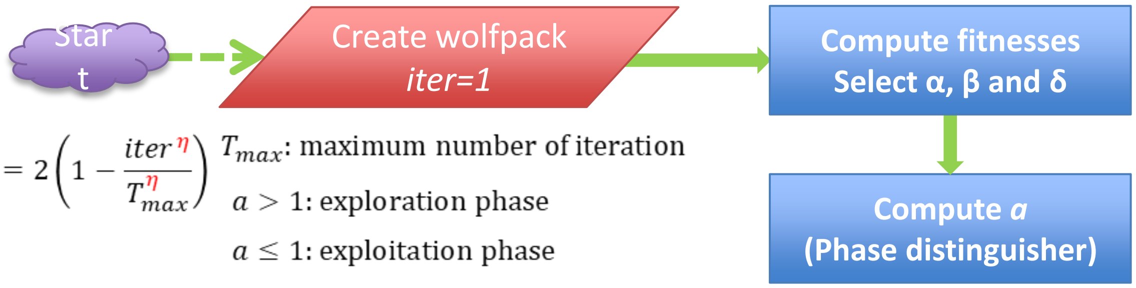
4. The mixedGWO

a. Improved Discrete GWO



Exemple: $d_i^{val} = \{2, 4, 8, 16, 32, \ldots, n-1, n\} \Rightarrow d_i^{ind} = \{1, 2, 3, 4, 5, \ldots, H_i - 1, H_i\}$

$\{\widehat{K_1}, \widehat{K_2}, \widehat{K_3}\} = \{8, 32, 2\} \Rightarrow h_q(iter) = \{3, 5, 1\}$



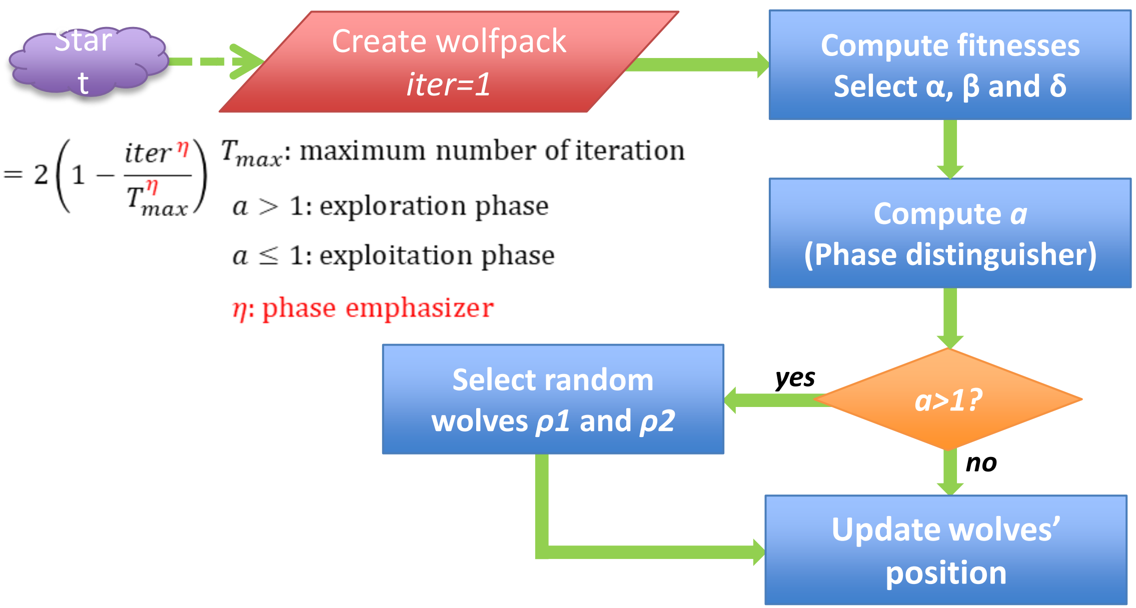
$$a = 2\left(1 - \frac{iter^{\eta}}{T_{max}^{\eta}}\right)$$

T_{max}: maximum number of iteration

$a > 1$: exploration phase

$a \leq 1$: exploitation phase

η: phase emphasizer



$$a = 2\left(1 - \frac{iter^{\eta}}{T_{max}^{\eta}}\right)$$

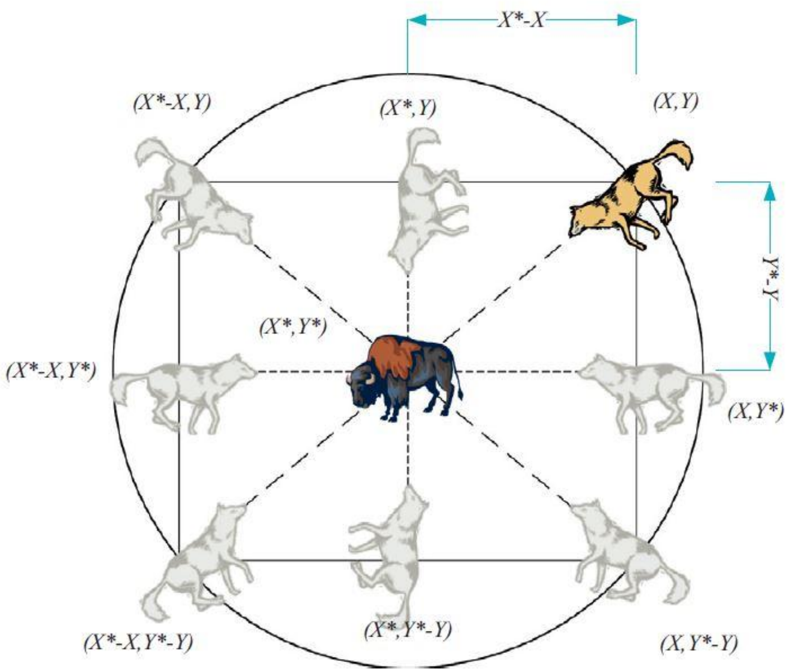
T_{max}: maximum number of iteration

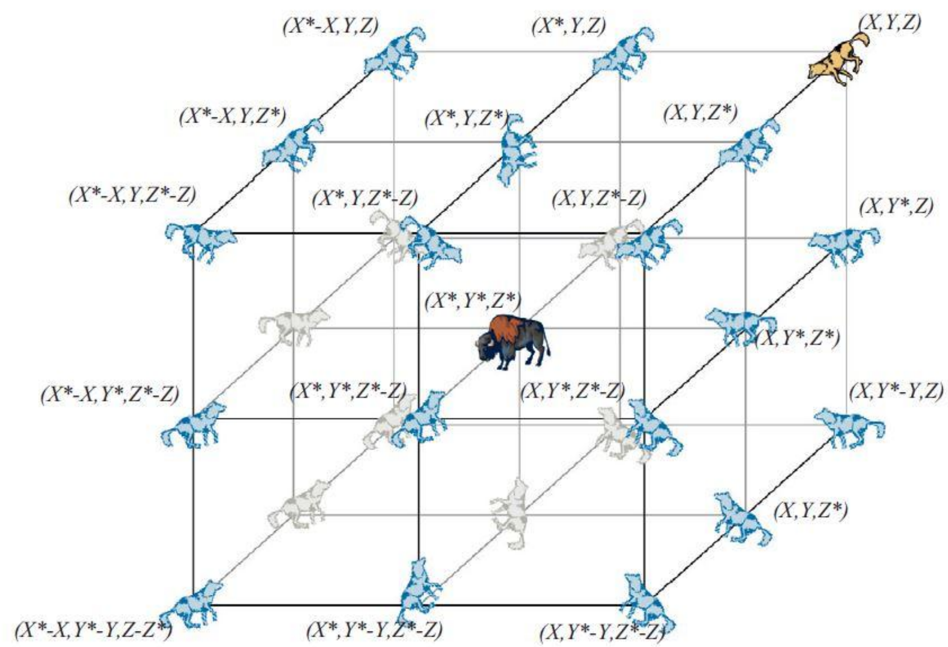
$a > 1$: exploration phase

$a \leq 1$: exploitation phase

η: phase emphasizer

Search space: continous vs discrete





For each wolf *q:*

$x_q^k(iter$ … $\frac{1}{3}$ … $+ y^{\delta})$

1. Select a leader *l* to follow:

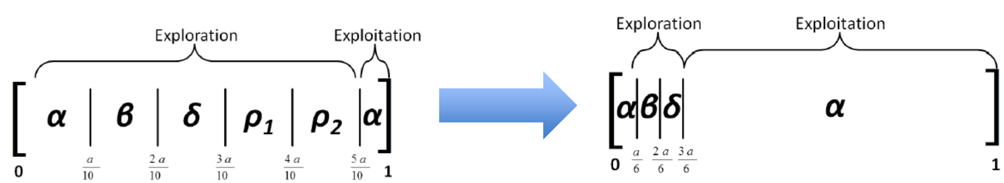
if $a > 1$:
(exploration)

$$x_l^k = \begin{cases} x_\alpha^k & if \quad r \le \frac{a}{10} \\ x_\beta^k & if \quad r > \frac{a}{10} \; and \; r \le \frac{2a}{10} \\ x_\delta^k & if \quad r > \frac{2a}{10} \; and \; r \le \frac{3a}{10} \\ x_{\rho_1}^k & if \quad r > \frac{3a}{10} \; and \; r \le \frac{4a}{10} \\ x_{\rho_2}^k & if \quad r > \frac{4a}{10} \; and \; r \le \frac{5a}{10} \\ x_\alpha^k & if \quad r > \frac{5a}{10} \end{cases}$$

if $a \le 1$:
(exploitation)

$$x_l^k = \begin{cases} x_\alpha^k & if \quad r \le \frac{a}{6} \\ x_\beta^k & if \quad r > \frac{a}{6} \; and \; r \le \frac{2a}{6} \\ x_\delta^k & if \quad r > \frac{2a}{6} \; and \; r \le \frac{3a}{6} \\ x_\alpha^k & if \quad r > \frac{3a}{6} \end{cases}$$

$r \in [0, 1]$



For each wolf *q:*

1. <u>Select a leader *l* to follow:</u>

2. <u>Select the displacement factor:</u>

If H is 'small':

$$\Delta = \begin{cases} 1 & \text{if} \quad \phi \leq \frac{a}{6} \\ 2 & \text{if} \quad \phi > \frac{a}{6} \quad \text{and} \quad \phi \leq \frac{2a}{6} \\ 4 & \text{if} \quad \phi > \frac{2a}{6} \quad \text{and} \quad \phi \leq \frac{3a}{6} \\ 1 & \text{if} \quad \phi > \frac{3a}{6} \end{cases}$$

For each wolf *q:*

1. Select a leader *l* to follow:

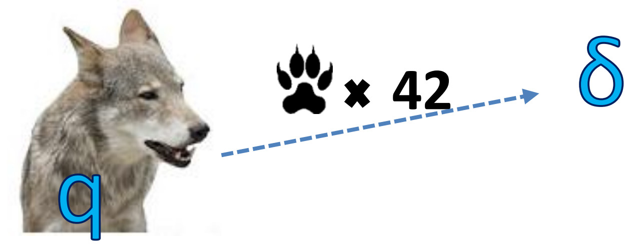
2. Select the displacement factor:

$$\Delta = \begin{cases} 1 & \text{if} \quad \phi \leq \frac{a}{2\Omega(H_i)} \\ 2 & \text{if} \quad \phi > \frac{a}{2\Omega(H_i)} \quad \text{and} \quad \phi \leq \frac{2a}{2\Omega(H_i)} \\ \vdots & \\ n_i & \text{if} \quad \phi > \frac{(n_i-1)a}{2\Omega(H_i)} \quad \text{and} \quad \phi \leq \frac{n_i a}{2\Omega(H_i)} \\ \vdots & \\ \Omega(H_i)-1 & \text{if} \quad \phi > \frac{(\Omega(H_i)-2)a}{2\Omega(H_i)} \quad \text{and} \quad \phi \leq \frac{(\Omega(H_i)-1)a}{2\Omega(H_i)} \\ \Omega(H_i) & \text{if} \quad \phi > \frac{(\Omega(H_i)-1)a}{2\Omega(H_i)} \quad \text{and} \quad \phi \leq \frac{\Omega(H_i)a}{2\Omega(H_i)} \\ 1 & \text{if} \quad \phi > \frac{\Omega(H_i)a}{2\Omega(H_i)} \end{cases}$$

$\phi \in [0,1]$

$$\Omega(H_i) = \begin{cases} \frac{H_i}{2} & \text{if } H_i \text{ is even} \\ \frac{H_i+1}{2} & \text{if } H_i \text{ is odd} \end{cases}$$

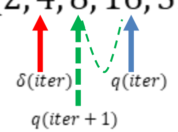
For each wolf *q:*



1. Select a leader *l* to follow:
2. Select the displacement factor:
3. Compute the new indexes: $h_i(iter+1) = \left(h_i(iter) + \Delta \operatorname{sgn}\left(h_i^l - h_i(iter)\right)\right) \bmod H_i$

$h_i^q(iter) = 4;\ h_i^\delta = 2$

$d_i^{val} = \{2, 4, 8, 16, 32, 64, 128\}$

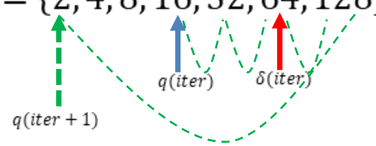


$\Delta = 1$

$h_i^q(iter+1) = 3$

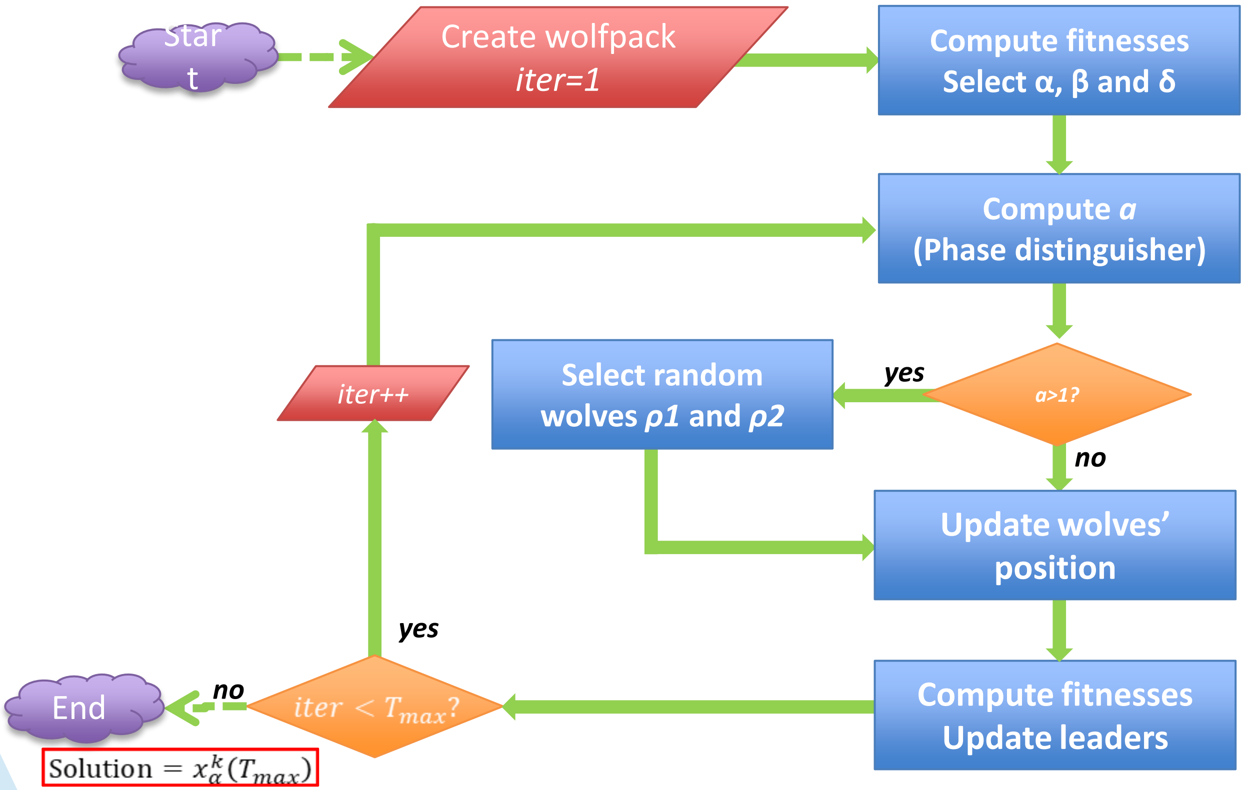
$h_i^q(iter) = 4;\ h_i^\delta = 6$

$d_i^{val} = \{2, 4, 8, 16, 32, 64, 128\}$



$\Delta = 4$

$h_i^q(iter+1) = 1$



Star
t
Create wolfpack
iter=1
Compute fitnesses
Select α, β and δ
Compute a
(Phase distinguisher)
a>1?
yes
Select random
wolves ρ1 and ρ2
no
Update wolves'
position
Compute fitnesses
Update leaders
iter < T_{max}?
yes
iter++
no
End
Solution = $x_\alpha^k(T_{max})$

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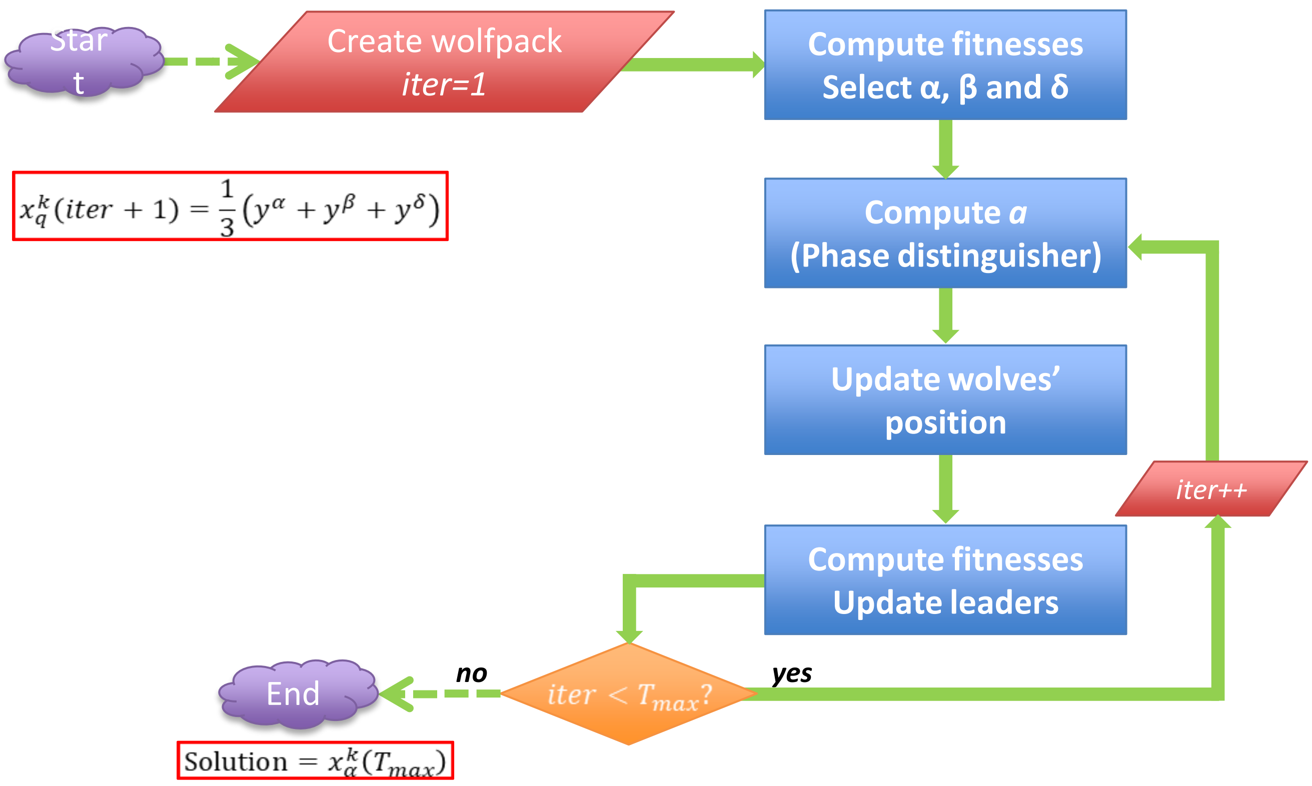
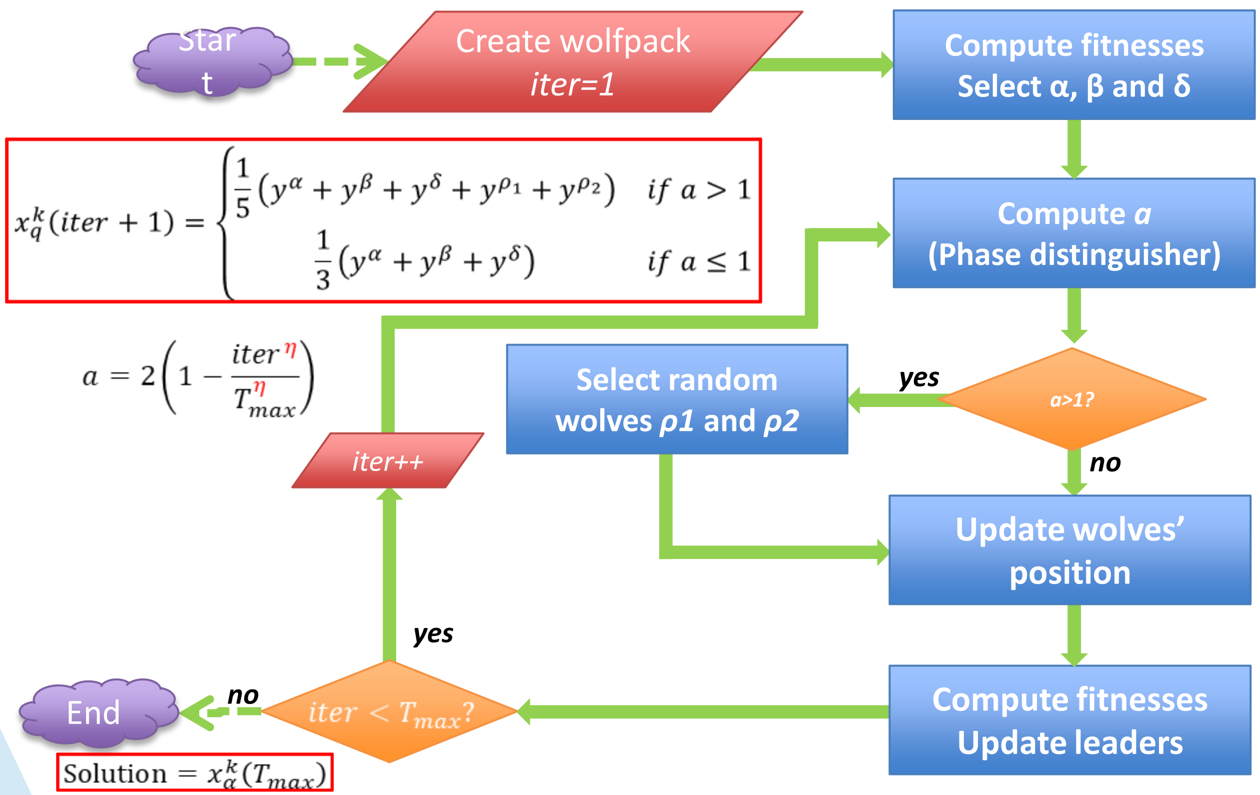
b. Global Continuous GWO




Star
t
Create wolfpack
iter=1
Compute fitnesses
Select α, β and δ
$x_q^k(iter+1) = \begin{cases} \frac{1}{5}(y^{\alpha}+y^{\beta}+y^{\delta}+y^{\rho_1}+y^{\rho_2}) & if\ a>1 \\ \frac{1}{3}(y^{\alpha}+y^{\beta}+y^{\delta}) & if\ a\leq 1 \end{cases}$
Compute a
(Phase distinguisher)
$a = 2\left(1-\frac{iter^{\eta}}{T_{max}^{\eta}}\right)$
Select random
wolves ρ1 and ρ2
yes
a>1?
no
iter++
Update wolves'
position
yes
Compute fitnesses
Update leaders
End
no
$iter < T_{max}?$
$\text{Solution} = x_{\alpha}^k(T_{max})$

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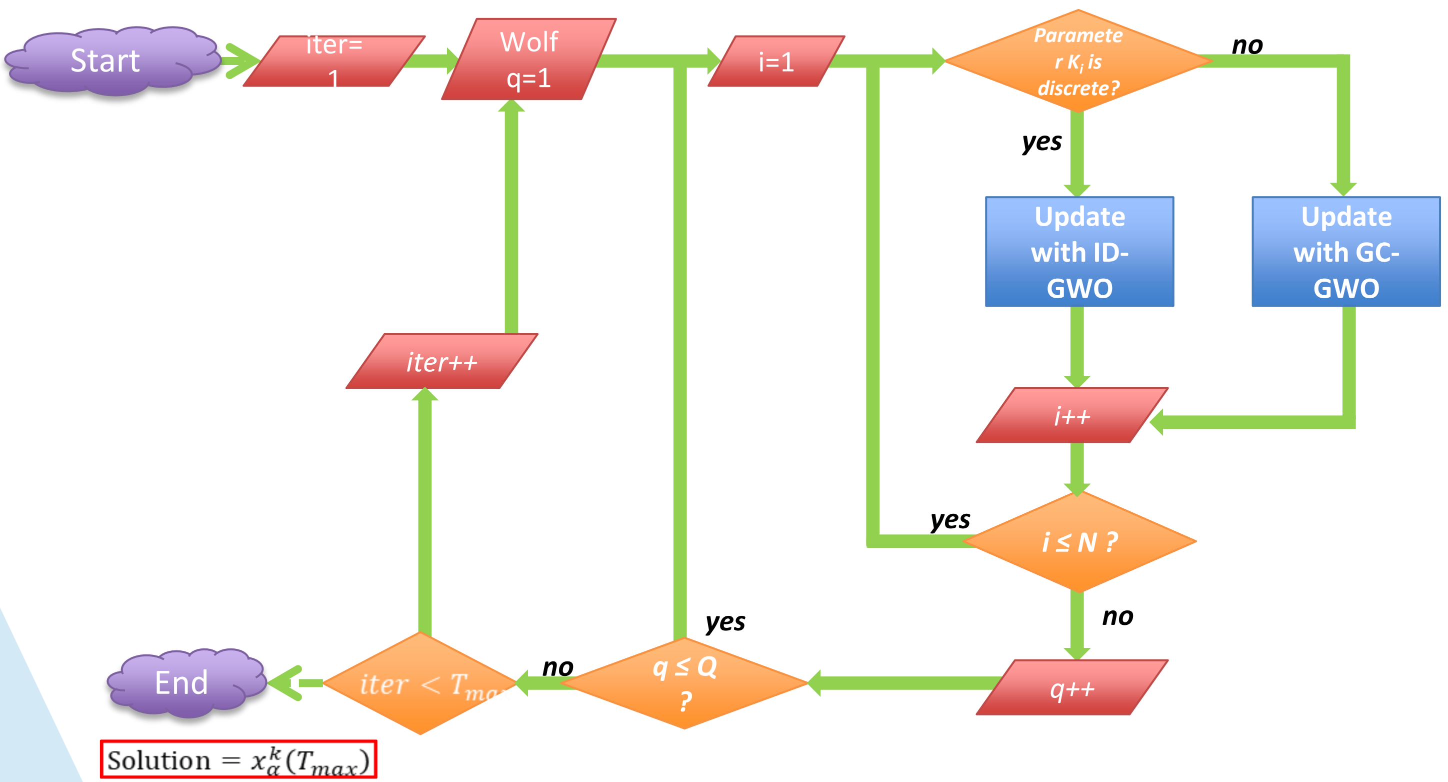
c. mixedGWO



Start
iter=
1
Wolf
q=1
i=1
Paramete
r K_i is
discrete?
no
yes
Update
with ID-
GWO
Update
with GC-
GWO
iter++
i++
yes
i ≤ N ?
no
yes
End
$iter < T_{ma}$
no
q ≤ Q
?
q++
Solution $= x_\alpha^k(T_{max})$

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d. The phase emphasizer and the adaptive GWO

$$\boldsymbol{a} = 2\left(1 - \frac{iter^{\eta}}{T_{max}^{\eta}}\right)$$

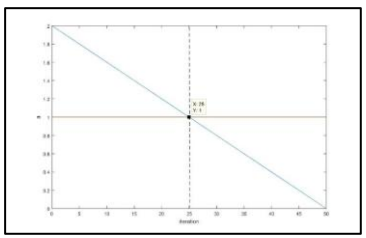
2 ↘ 0

T_{max}: maximum number of iteration

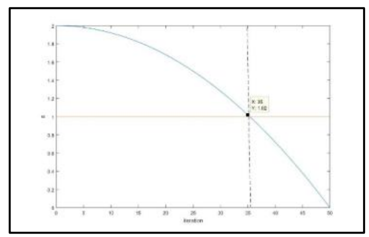
η: phase emphasizer

$a > 1$: exploration phase

$a \leq 1$: exploitation phase



$\eta = 1$



$\eta = 2$

Modified GWO (mGWO):

N. Mittal, U. Singh et B. Singh Sohi, “*Modified grey wolf optimizer for global engineering optimization*", 2016.

$$a = 2\left(1 - \frac{iter^{\eta}}{T_{max}^{\eta}}\right)$$

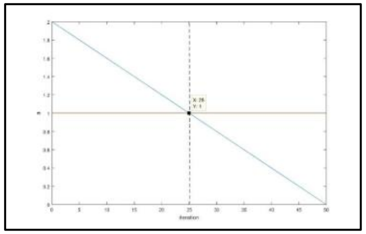
2 ➘ 0

T_{max}: maximum number of iteration

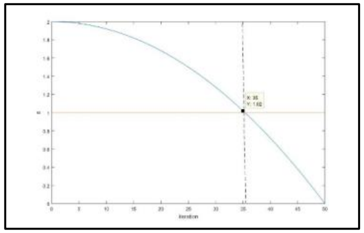
η: phase emphasizer

$a > 1$: exploration phase

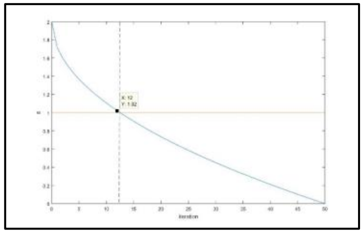
$a \leq 1$: exploitation phase



$\eta = 1$



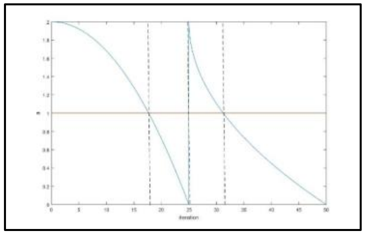
$\eta = 2$



$\eta = 0{,}5$

Adaptive mixed GWO (amixedGWO):

$$a = \begin{cases} 2\left(1 - \dfrac{iter^{\eta}}{\left(\frac{T_{max}}{2}\right)^{\eta}}\right) & if \quad iter \leq \dfrac{T_{max}}{2} \\ 2\left(1 - \dfrac{\left(iter - \frac{T_{max}}{2}\right)^{\frac{1}{\eta}}}{\left(\frac{T_{max}}{2}\right)^{\frac{1}{\eta}}}\right) & if \quad iter > \dfrac{T_{max}}{2} \end{cases}$$



$\eta = 2 \;\; then \;\; \dfrac{1}{\eta} = 0{,}5$

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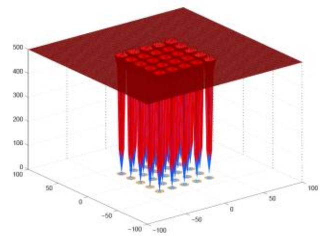
e. Performances

continuous problems

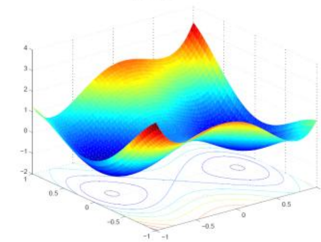
Tested on 9 fixed-dimension multimodal benchmark functions in continuous space

$$F_{14}(x) = \left(\frac{1}{500} + \sum_{j=1}^{25} \frac{1}{j + \sum_{i=1}^{2}(x_i - a_{ij})^6}\right)^{-1}$$



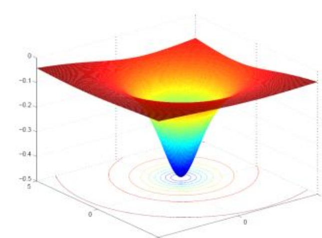
$f_{min} = 1$

$$F_{16}(x) = 4x_1^2 + 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1x_2 - 4x_2^2 + x_2^4$$



$f_{min} = -1{,}0316$

$$F_{21}(x) = -\sum_{i=1}^{5}[(X - a_i)(X - a_i)^T + c_i]^{-1}$$



$f_{min} = -10{,}1532$

$T_{max} = 3000$

Average values on 30 runs

Function		GWO	mGWO	mixedGWO	amixedGWO	f_{min}
F_{14}	*Avg.*	4,1333240	2,9999936	4,9999841	2,8362586	1
	Rank	3	2	4	1	
F_{15}	*Avg.*	3,09e-03	3,68e-03	4,40e-03	9,90e-04	3E-03
	Rank	2	3	4	1	
F_{16}	*Avg.*	-1,0316284522	-1,03162845227	-1,03162845241	-1,03162845331	-1,0316
	Rank	4	3	2	1	
F_{17}	*Avg.*	0,3979427	0,3979661	0,3979500	0,3979117	0,397887
	Rank	2	4	3	1	
F_{18}	*Avg.*	5,7000011	3,0000007	3,0000007	3,0000010	3
	Rank	4	1	1	3	
F_{20}	*Avg.*	-3,2563368	-3,2635423	-3,2980638	-3,2780500	-3,32
	Rank	4	3	1	2	
F_{21}	*Avg.*	-9,8163635	-9,2886724	-9,9847094	-9,9847752	-10,1532
	Rank	3	4	2	1	
F_{22}	*Avg.*	-10,2257553	-10,2257344	-10,4028815	-10,4029327	-10,4028
	Rank	3	4	2	1	
F_{23}	*Avg.*	-10,1758610	-10,0856251	-10,5363495	-10,5364004	-10,5363
	Rank	3	4	2	1	
Average Rank		3,11	3,11	2,33	1,33	
Overall rank		3	3	2	1	

Average minima obtained on continuous functions

Function	amixedGWO	PSO	GWO	ABC	TSA	GA
F_1^{CEC}	3070,2	462,8	1534,9	434,4	45,6	152,4
F_2^{CEC}	396,6	514,7	1126,2	63	25,5	508,5
F_3^{CEC}	542,5681	342,5489	684,1074	96,8305	21,4919	162,4
F_4^{CEC}	1,00e-04	0	4,6e-03	0	0	1,14e-13
F_5^{CEC}	9,6e-03	0	6,84e-01	0	0	1,7e-13
F_6^{CEC}	3,8e-03	0	8,2e-03	0	0	2,27e-13
F_7^{CEC}	4,2e-03	4,8e-03	9,5e-03	3,5e-03	0	8,4e-03
F_8^{CEC}	1,33e-01	0	7,18e-02	0	0	2,27e-13
F_9^{CEC}	1,99e-01	0	9,96e-02	0	0	3,41e-13
F_{10}^{CEC}	49,2603	1,69e-01	30,30	1,02e-01	0	6,24e-03
F_{11}^{CEC}	4,0561	2,39e-01	4,0174	9,89e-02	4,8e-02	3,5807
F_{12}^{CEC}	4,96e-01	1,12e-01	3,73e-01	4,49e-01	9,39e-02	5,12e-13
F_{13}^{CEC}	3,38e-02	3,81e-02	4,18e-02	4,78e-02	1,41e-02	2,75e-02
F_{14}^{CEC}	7,5e-03	1,32e-02	1,14e-02	2,49e-02	3,5e-03	1,67e-02
F_{15}^{CEC}	2,24e-02	0	1,04e-02	2,00e-04	0	1,38e-02
F_{16}^{CEC}	6,9e-03	1,09e-02	1,10e-02	1,15e-02	0	1,94e-02
F_{23}^{CEC}	99,0327	33,6887	112,9178	3915	7,56	160,4773
F_{24}^{CEC}	80,9750	89,8178	69,7263	57,1990	19,6872	100,6363
F_{25}^{CEC}	1,60e-02	1,00e-04	4,23e-02	4,20e-03	0	47,9145
F_{26}^{CEC}	2,47e-01	3,81e-01	1,53e-01	1,09e-01	0	1,1451
F_{27}^{CEC}	2,08e-01	5,37e-02	4,18e-01	1,02e-01	5,50e-03	5,96e-01
F_{28}^{CEC}	260,5198	28,8892	297,3808	107,5971	9,4956	176,3242

Average minima obtained on continuous functions from CEC2014

discrete problems

3 discrete variables

$T_{max} = 3000$

Average and medium values on 30 runs

Function	MODGWO		mixedGWO		amixedGWO		H_i
	Avg.	*Med.*	*Avg.*	*Med.*	*Avg.*	*Med.*	
F_1	8,100	6	0	0	0	0	201
F_2	0,183	0	0	0	0	0	41
F_3	7,100	5	0,067	0	0	0	201
F_4	2	2	0,067	0	0	0	201
F_5	23,217	18	4,650	0	2,583	0	61
F_6	8,692	7,5	0	0	0	0	401

Average and Medium values obtained on discrete functions

Table of contents

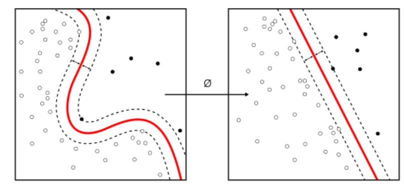
First Application: SVM+LBP training for gender recognition

Gender recognition:

automatically label an face as a woman's or a man's face

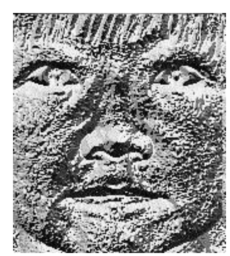
Needed:

- A machine learning algorithm (SVM)



- Training images (FERET)

- Image representation (LBP)



Problem to minimize:

False recognition rate (FRR) of a SVM trained for gender recognition

$$FRR = \frac{nb_{fr}}{nb\_testing\_images} \quad (fitness)$$

For non-separable data:

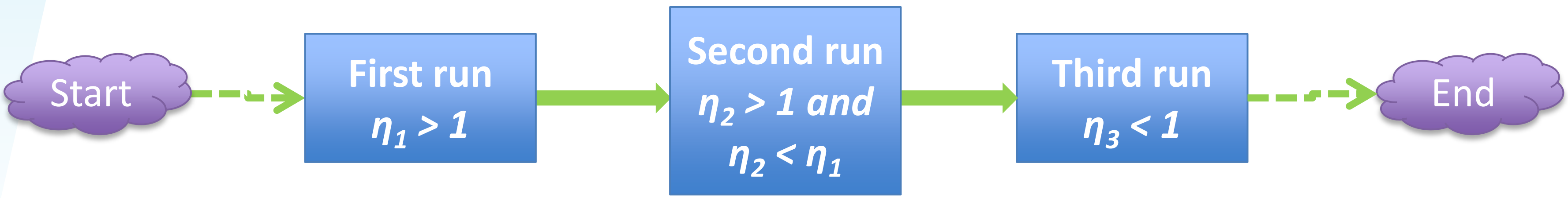
$$\min_{w,\varsigma_1,\ldots,\varsigma_l} \left[\frac{1}{2}\|w\|^2 + C \sum_{i=1}^{l} \varsigma_i \right] \quad with \quad C \in \mathbb{R}^{+*} \quad (K_1)$$

Gaussian (RBF) kernel:

$$K(x_i, x_j) = \exp\left(- \frac{\left\|\varphi(x_i) - \varphi(x_j)\right\|^2}{2\gamma^2} \right) \quad with \quad \gamma \in \mathbb{R}^{+*} \quad (K_2)$$

C	*γ*	*FRR*
100	1,00E-03	50,00%
100	1,00E-04	18,50%
1	1,00E-06	26,10%
10	1,00E-06	13,60%

Adaptive version of the classical GWO:



$$\eta_1 = 8 \qquad T_{max} = 15 = (3 \times 5)$$

$$\eta_2 = 2 \qquad Q = 20$$

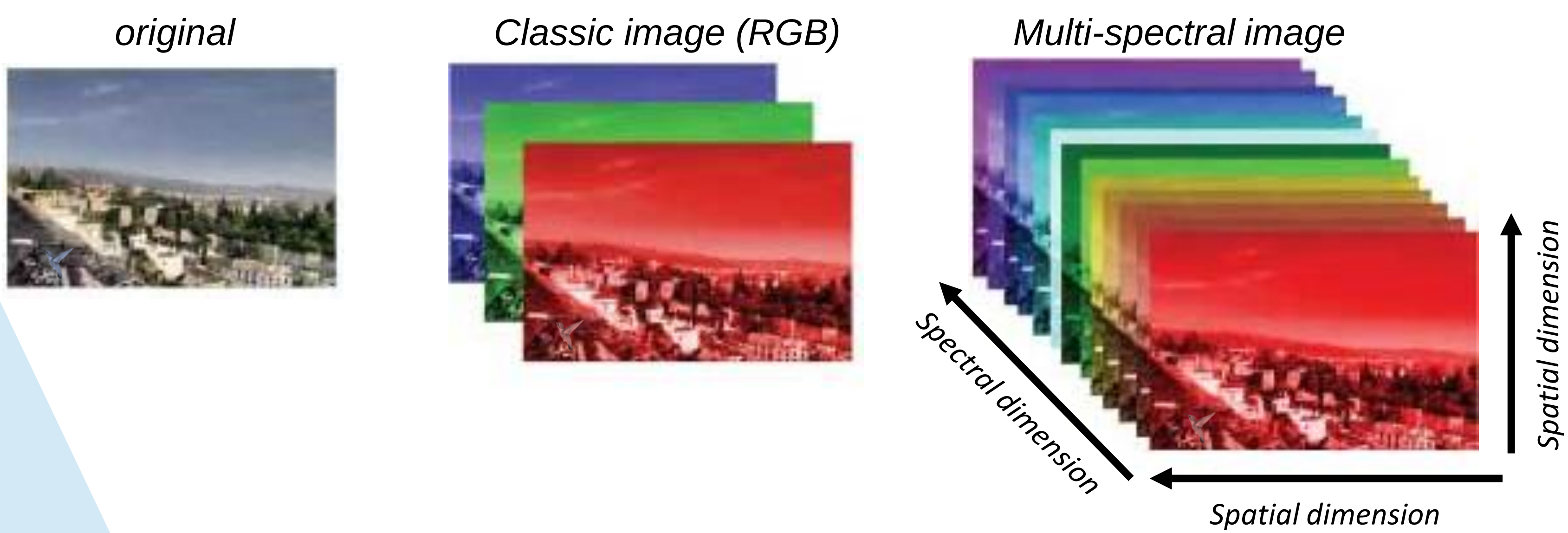
$$\eta_3 = 0{,}5$$

Method	*Best (γ,C)*	*100 - FRR*	*Time (sec.)*
PSO	(6,7E-07; 600)	91,40%	1970
GWO	(1,6E-07; 800)	91,50%	1970
mGWO	(5,0E-08, 83)	91,90%	1970
aGWO	(4,9e-08; 82)	91,90%	968

Problem's definition

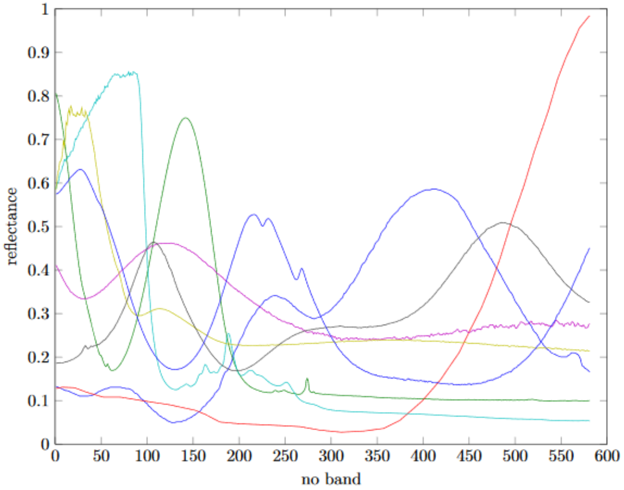
Multispectral image:

Image composed of more than 3 channels



Spectra unmixing:

Identification of endmembers (e.g. spectra of wood or ground)



$$y = \lambda_1 s_1 + \lambda_2 s_2 + n$$

Image denoising:

Process of removing noise ("cleansing") from an image

Goal:

Find the best balance between image denoising and spectra unmixing using the mixedGWO

Criteria to minimize for a multispectral image χ:

$$J^{LS}(K_1, K_2, K_3, f^{mix}, \lambda_1, \lambda_2) = \frac{1}{I_1 I_2 I_3} \|\chi_1 - \hat{\chi}(K_1, K_2, K_3)\|^2 + \frac{1}{I_3} \left\| y(f^{mix}, \lambda_1, \lambda_2) - \hat{y}(K_1, K_2, K_3) \right\|^2$$

χ_1: *reference image*

y: *reference spectrum*

I_i: *image size along mode i*

$\hat{\chi}$: *denoised image*

$\hat{y}$: *extracted spectrum*

Parameters to estimate:

For the denoising:

K_1, K_2, K_3: *rank values* $\in \left\{1, \frac{I_i}{8}, 2\frac{I_i}{8}, \ldots, I_i\right\}$

For the unmixing:

f^{mix}: *mixing model* $\in \{0; 1\}$

$\lambda_1, \lambda_2,$: *mixing coefficients* $\in [0; 1]$

School case

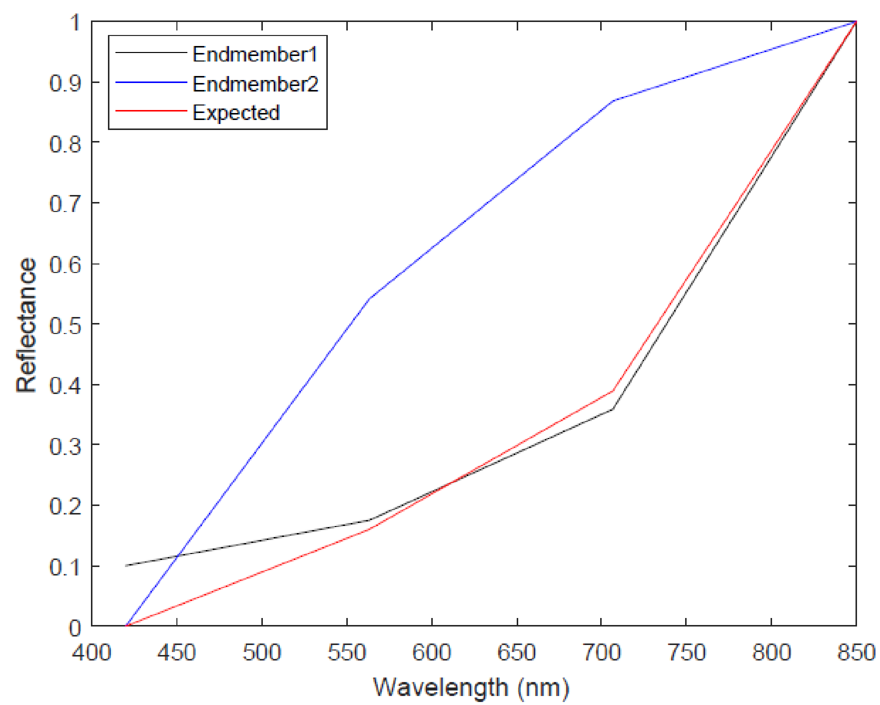
Input image χ *of size* $I_1 = I_2 = 32$ *and* $I_3 = 4$

Reference χ_1 *computed with Multiway Wiener Fourier (MWF) with* $K_1 = K_2 = 16$ *and* $K_3 = 4$

Expected spectrum y *obtained with :* $f^{mix} = 0$

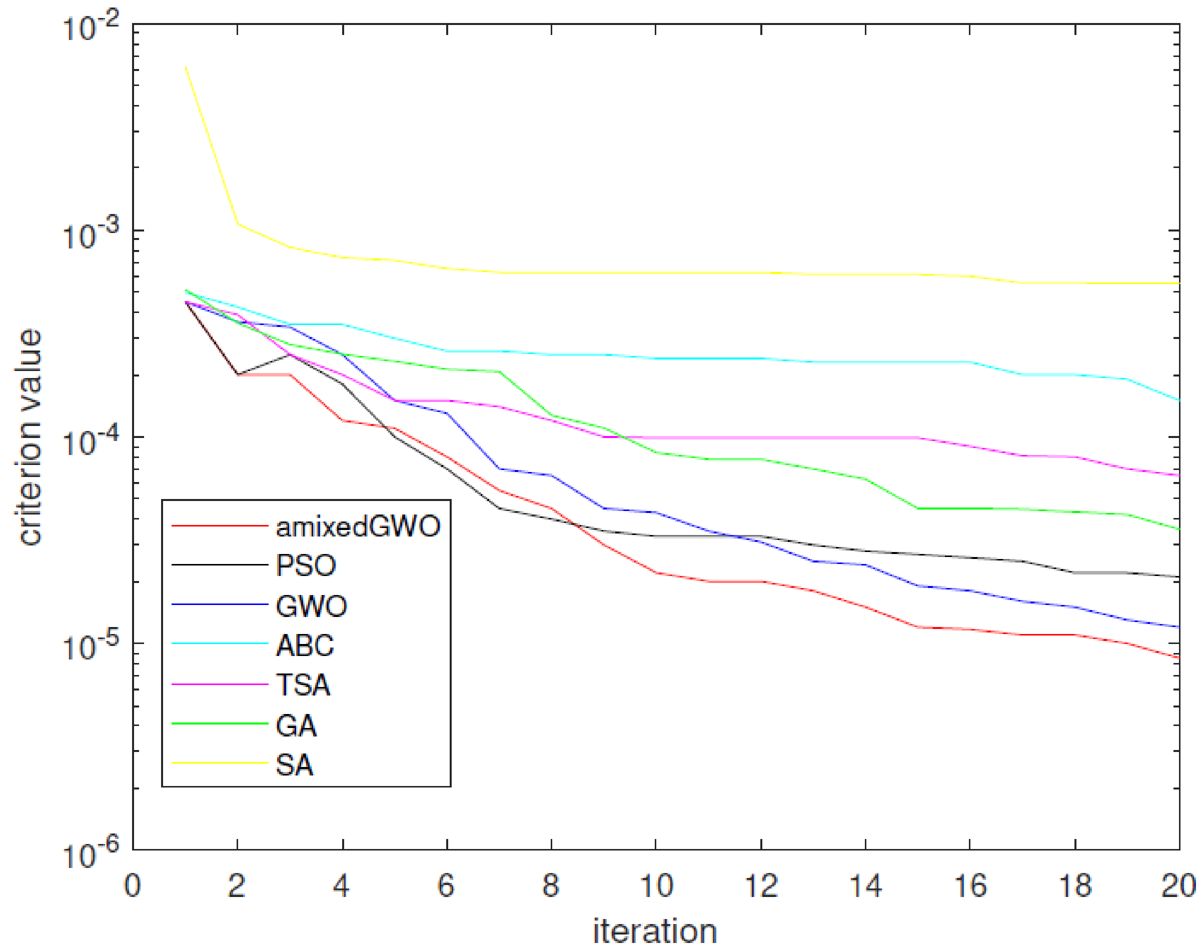
$\lambda_1 = 0{,}15$

$\lambda_2 = 0{,}41$



$T_{max} = 20$

$M = 10\ runs$



Parameters	*Expected Values*	*amixedGWO*	*PSO*	*GWO*	*ABC*	*TSA*	*GA*	*SA*
K_1	16	16,10	16,84	16,11	22,67	20,39	19,34	13,57
K_2	16	15,90	15,77	16,05	19,85	19,57	20,05	22,02
K_3	4	4	3,94	3,77	3,54	3,72	3,75	2,79
f^{mix}	0	0,2	0,45	0,22	0,40	0,28	0,46	0,57
λ_1	0,15	0,114	0,097	0,088	0,162	0,124	0,053	0,404
λ_2	0,41	0,534	0,610	0,456	0,649	0,537	0,495	0,9999

Estimated parameters

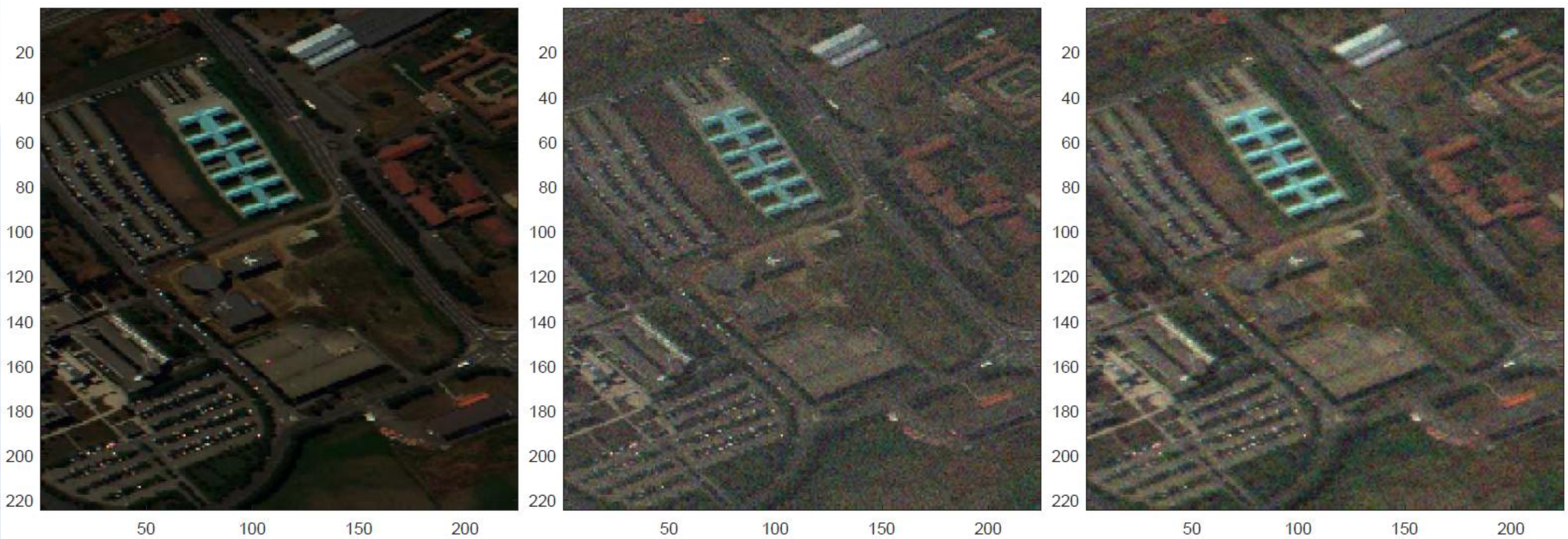
	amixedGWO	*PSO*	*GWO*	*ABC*	*TSA*	*GA*	*SA*
RE	7,20e-03	8,90e-03	7,748e-03	1,017e-02	8,022e-03	1,35e-02	1,20e-01
Rank	1	4	2	5	3	6	7

Obtained Spectrum Reconstructed Error (RE)

Realistic case

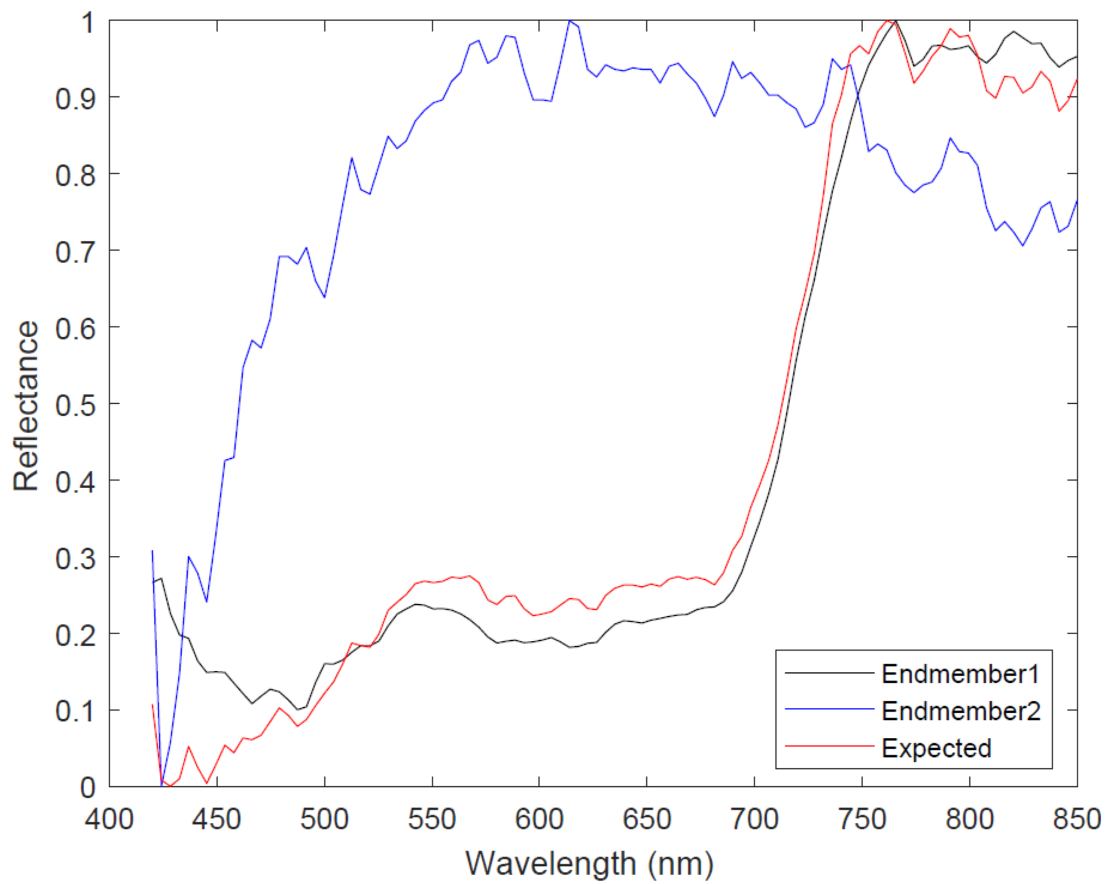
Input image χ *of size* $I_1 = I_2 = 256$ *and* $I_3 = 103$

Reference χ_1 *computed by applying a Wiener filtering process in Fourier domain to the impaired image* χ



Noise-free image χ | Impaired with $SNR_{in} = 10dB$ | Reference image χ_1

Expected spectrum y obtained with : $f^{mix} = 0 \quad \lambda_1 = 0{,}15 \quad \lambda_2 = 0{,}41$



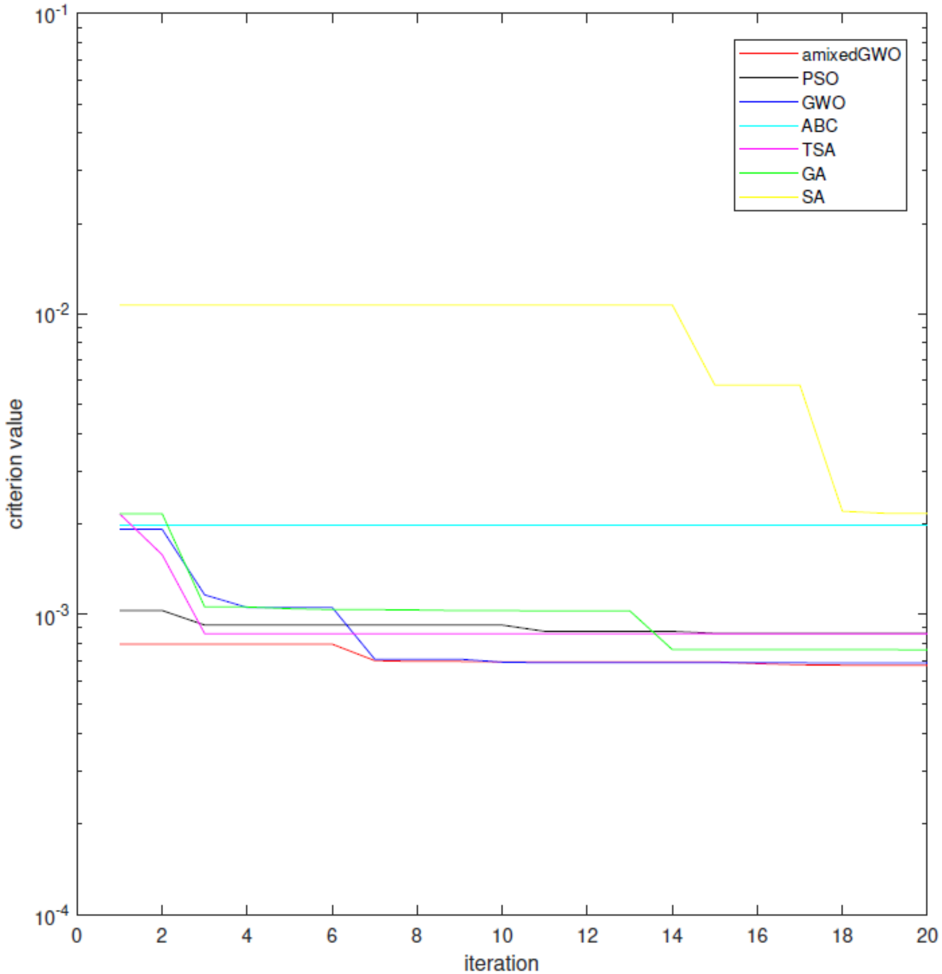
IV.2. Second Application: Joint denoising and unmixing of multispectral images

c. Realistic case

$SNR_{in} = 10dB$

$M = 3\ runs$

$T_{max} = 20$



SNR_{in}		Ref.	amixedGWO	PSO	GWO	ABC	TSA	GA	SA
$0dB$		3,99	9,333	7,659	7,745	5,560	8,429	8,45	6,42
	$Rank$	8	1	4	5	6	3	2	7
$5dB$		8,79	12,180	11,477	13,258	7,878	11,893	7,97	8,34
	$Rank$	5	2	4	1	7	3	8	6
$10dB$		13,24	17,777	14,159	17,108	7,719	15,310	8,88	2,49
	$Rank$	5	1	4	2	7	3	6	8
$15dB$		16,61	17,804	18,587	19,733	13,388	15,815	15,22	14,43
	$Rank$	4	3	2	1	8	5	6	7
$20dB$		18,26	22,664	20,869	22,653	17,417	18,504	15,37	13,21
	$Rank$	5	1	3	2	6	4	7	8
	$Avg. Rank$	5,4	1,6	3,4	2,2	6,8	3,6	5,8	7,2
	$Overall\ Rank$	5	1	3	2	7	4	6	8

Obtained SNR_{out}

SNR_{in}		amixedGWO	PSO	GWO	ABC	TSA	GA	SA
$0dB$		2,81e-03	4,44e-03	3,51e-03	5,30e-03	2,80e-03	4,00e-03	2,53e-02
	$Rank$	2	5	3	6	1	4	7
$5dB$		2,34e-03	4,41e-03	3,41e-03	5,45e-03	3,12e-03	5,63e-03	5,59e-03
	$Rank$	1	4	3	5	2	7	6
$10dB$		2,47e-03	3,12e-03	3,02e-03	3,68e-03	2,69e-03	2,79e-03	5,17e-03
	$Rank$	1	5	4	6	2	3	7
$15dB$		1,89e-03	2,82e-03	3,13e-03	4,40e-03	2,66e-03	2,83e-03	5,63e-03
	$Rank$	1	3	5	6	2	3	7
$20dB$		1,92e-03	2,82e-03	2,09e-03	2,75e-03	2,59e-03	4,26e-03	4,28e-03
	$Rank$	1	5	2	4	3	6	7
	$Avg.Rank$	1,2	4,4	3,4	5,4	2	4,6	6,8
	$Overall\ Rank$	1	4	3	6	2	5	7

Obtained Spectrum Reconstructed Error (RE)

Conclusion

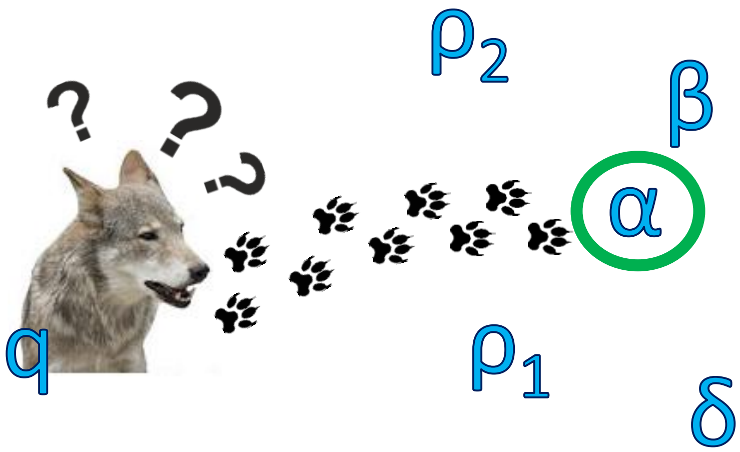
Pros

- *Good results obtained by the mixedGWO and better than the state-of-the-art*
- *Compatibility of the mixedGWO with a realistic problem*

Cons

- *Criterion to minimize dependent of a reference*

Thank you



For each wolf *q:*

$x_{\alpha}^{k}(iter$ … $\frac{1}{3}$ … $+ y^{\delta})$

1. Select a leader *l* to follow:

<u>*if* $a > 1$:</u>
(exploration)

$$x_l^k = \begin{cases} x_\alpha^k & if \quad r \le \frac{a}{10} \\ x_\beta^k & if \quad r > \frac{a}{10} \; and \; r \le \frac{2a}{10} \\ x_\delta^k & if \quad r > \frac{2a}{10} \; and \; r \le \frac{3a}{10} \\ x_{\rho_1}^k & if \quad r > \frac{3a}{10} \; and \; r \le \frac{4a}{10} \\ x_{\rho_2}^k & if \quad r > \frac{4a}{10} \; and \; r \le \frac{5a}{10} \\ x_\alpha^k & if \quad r > \frac{5a}{10} \end{cases}$$

<u>*if* $a \le 1$:</u>
(exploitation)

$$x_l^k = \begin{cases} x_\alpha^k & if \quad r \le \frac{a}{6} \\ x_\beta^k & if \quad r > \frac{a}{6} \; and \; r \le \frac{2a}{6} \\ x_\delta^k & if \quad r > \frac{2a}{6} \; and \; r \le \frac{3a}{6} \\ x_\alpha^k & if \quad r > \frac{3a}{6} \end{cases}$$

$r \in [0, 1]$