Stochastic excitation and delayed oscillation of a micro-oscillator

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Stochastic processes in the form of the classical Brownian motion or internal phonon fluctuations may be studied by the use of a microcantilever. We present certain dynamic aspects of the behavior of the noise and the delayed fluctuations exhibited by a microcantilever. In particular, we present an analytic solution describing the transient response of the oscillator and discuss how self-excitation may determine the presence of small delays and, conversely, how delay-induced excitation may provide sensory information. As an application of the analysis, we present dynamic measurement of surface displacement with a preliminary sensitivity of 2 Hz/μm in the chosen frequency window, which may effectively be extended to the nanometer regime.

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The implications of uncorrelated thermal agitations dominating the microscopic realm are of paramount importance in both equilibrium and nonequilibrium systems.[1,2] In a medium of viscosity η, a Brownian particle undergoing diffusion performs a random walk[2] with an averaged squared displacement growing linearly with both time t and temperature T, i.e., $\langle |u|^2\rangle(t,T)=k_BTt/\gamma$, where k_B is the Boltzmann constant and γ is the drag coefficient (see investigations by Einstein[3] and a discussion of the Einstein-Smoluchowski diffusion by Islam[4]). Picturing a microcantilever oscillator (MCO) as a giant molecule of an effective mass m at a position u and bound to a harmonic potential U with a force constant k, i.e., $U(r)=ku^2/2$, then in this medium, the Langevin-like equation $m\ddot{u}(t)+\gamma\dot{u}(t)+ku(t)+\xi(t)=0$, with ξ representing the random forces,[5] describes the dynamics of the oscillator. Under equilibrium, an application of the fluctuation-dissipation theorem (FDT) as generalized by Callen and Welton[6] yields, in the Fourier space, for the uncorrelated variations in the position of this point mass, a power spectral density (PSD), which near its single resonance frequency ω_0 is given by $\langle |u|^2\rangle(\omega_0,T)=\alpha Q_0^{-2}$, where $\alpha=2m^2k_BT\Delta\omega/\pi\gamma^3$, and $Q_0=m\omega_0/\gamma$ determines the spectral linewidth while $\Delta\omega$ is the measurement bandwidth. Representing the MCO as an extended elastic body,[7] one arrives at a mode n dependent PSD of the (resonant) form $\langle |u_n|^2\rangle \times(\omega_n,T)=\alpha Q_n^{-2}$, where $\langle |u_n|^2\rangle$ is the Fourier transform of the autocorrelation function $\langle u_n(1,t),u_n(1,0)\rangle$ for a MCO of normalized length, and Q_n is proportional to the eigenfrequencies ω_n.

The Brownian fluctuation resulting in a detectable excitation may act as a sensor exhibiting a shift in the excitation frequency and amplitude as a result of a MCO-molecule encounter. The resulting memoryless dynamics described by the stochastic differential equations (SDEs) and under equilibrium by the FDT above, that is, oscillation in the presence of noise,[5,8] has been studied extensively.[9] A higher spectral resolution may be achieved by the engagement of a state feedback; however, prevailing retardation quickly renders the above results challenging such that the dynamics of fluctuation in the presence of noise and delays, governed by the stochastic delay differential equations (SDDEs), remains an active area of research due to the non-Markovian position of the MCO. A posing problem is then whether the MCO described by a SDDE partially or completely violates the FDT, and if so whether this violation is mode dependent. Attempts to approximate solutions of certain SDDE, such as in the limit of small delay,[10] stationary state,[11] and numerical,[12] continue to be reported and applications as diverse as gene regulation[13] and Brownian motor[14] have been investigated. Recently, Budini and Cáceres developed a functional approach to characterize the linear delay Langevin equation for arbitrary noise.[15] To date, the SDDE formulation of a MCO has not been treated, and as a result, an understanding of the mode-dependent interplay between delays and noise and stability criteria will depend mainly on experimental observations. Experimental work on MCO, from early studies largely concerned with Q control in atomic force microscopy (AFM) in ambient and fluidic media to most recent basic research on sub-Kelvin cooling of oscillation, continues to improve various dynamic aspects of imaging and sensing.[16–29]

Within the Euler-Bernoulli approximation, with a proper scaling, the nondimensionalized form of the partial differential equation, describing the position u of the MCO at point $0\leqslant x\leqslant 1$ and time t, reads $\tilde{\mathcal{A}}u(x,t)=w(x,t)$, with the operator $\tilde{\mathcal{A}}\equiv\partial_x^4+\partial_t^2+\beta\partial_t$, where the dimensionless constant $\beta=\epsilon\gamma$ with ϵ containing the material properties of the MCO, and w is a general forcing.[23] In this Brief Report, we consider two functional forms for w, an impulsive one and one leading to a SDDE.

First, the stochastic parameters may be studied via the transient behavior of the MCO. The dissipation mechanisms due to the classical Brownian molecular bombardment, and to the internal losses (collectively including Zener,[30] dislocation, and impurity) involved, may be distinguished by the application of an impulse. Figure 1 shows an instance of such a measurement, where a 1 μs, 1 V pulse was applied to the piezobimorph of an AFM holding the MCO. Most of the power appears to have been lost through the lowest-frequency mode, but several higher-frequency modes are also involved that due to the low Nyquist frequency are displaced by aliasing. Furthermore, in all experiments, apart from decaying, a complicated frequency modulation appears to modify the amplitude. A numerical fit to the envelope of

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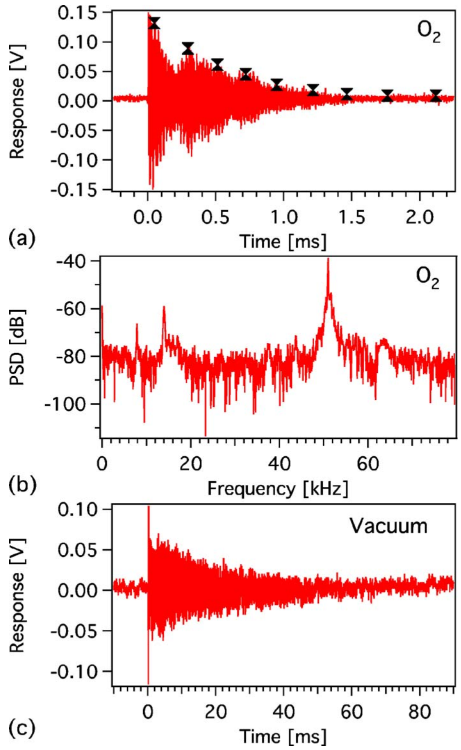

FIG. 1. (Color online) Mode interference in transient response of the MCO and decay back to the noise level in (a) oxygen and (c) vacuum environments. (b) Constructed power spectral density, displaying a major loss channel centered at the fundamental frequency F_0, also reveals several smaller peaks as aliases due to a Nyquist frequency of $F_N = 80$ kHz$<F_n$, $n>0$. The symbols in (a) mark the loci to which an exponential function was fitted.

the signal [Fig. 1(a)] provides a direct measure of the dissipation. The origin of the observed frequency beat pattern may be explored on the basis of an eigenmode interference as follows. For $w(x,t)=0$, separation of variables yields $u(x,t)=\Phi(x)\Theta(t)$. Denoting the Laplace transform by $\mathcal{L}\{u(x,t)\}=\hat{u}(x,s)$, we obtain the solution for the case $w\neq 0$ by a transformation into Laplace space and the eigenfunction expansions

$$\hat{u}(x,s)=\sum_{k=1}^{\infty} a_k(s)\Phi_k(x), \quad \hat{w}(x,s)=\sum_{k=1}^{\infty} b_k(s)\Phi_k(x). \tag{1}$$

Since $\{\Phi_k(x)\}_1^\infty$ form an orthonormal set, the expansion coefficients can be shown to be

$$b_k=\int_0^1 \hat{w}(u,s)\Phi_k(u)du, \quad a_k=\frac{b_k}{(s+\eta)^2+\omega_k^2}, \tag{2}$$

where $\omega_k=\sqrt{\lambda_k^4-\eta^2}$ with the eigenvalues of the spatial solution denoted by λ_k, $k=1,2,\ldots$, and $\eta=\beta/2$. Thus,

$$\hat{u}(x,s)=\sum_{k=1}^{\infty}\frac{\Phi_k(x)}{(s+\eta)^2+\omega_k^2}\int_0^1 \hat{w}(u,s)\Phi_k(u)du. \tag{3}$$

We apply Borel's theorem to obtain the formal solution

$$u(x,t)=\sum_{k=1}^{\infty}\frac{\Phi_k(x)}{\omega_k}\int_0^1 \Phi_k(u)du\int_0^1 \Gamma_k(\tau,t,u)d\tau, \tag{4}$$

where $\Gamma_k=w(u,\tau)e^{-\eta(t-\tau)}\sin\omega_k(t-\tau)$. Based on the experiment of Fig. 1, we assume here the following excitation $w(x,t)=w_a\delta(t-t^*)\delta(x,x^*)$, where w_a denotes the magnitude of the impulse (in scaled variables), and $0\leqslant x^*\leqslant 1$ and $0\leqslant t^*$ are dimensionless space and time impulse parameters denoting the position along the MCO and the time at which the driving impulse is applied. Thus, the kth solution reduces to

$$u_k(x,t)=w_a e^{-\eta(t-t^*)}\Phi_k(x)\Phi_k(x^*)\frac{\sin[\omega_k(t-t^*)]}{\omega_k}, \tag{5}$$

such that the general solution $u(x,t)=\sum_{k=1}^{\infty}u_k(x,t)$ for $t\geqslant t^*$ and $u(x,t)=0$ for $t<t^*$. It is now evident from the last term, weighted with the eigenfrequencies, that an (eigenfunction) interference results in the observed beat pattern of Fig. 1. The contribution from the higher modes declines as ω_k^{-1} as supported by the PSD in Fig. 1. Measured from the first experimental frequency, Eq. (5) predicts that the contributions of the higher modes scale as 0.16 and 0.06 for the second and the third respectively. It is noteworthy that (from the FDT) the stochastic noise of the MCO $\langle|u_n|^2\rangle\propto\omega_n^{-2}$ is severely suppressed for higher modes. From the definition of ω_k and λ_k, one may propose that such a superposition of the modes itself may provide sensory information due to the position of the beats.

Although in view of the FDT the stochastic noise is severely reduced for higher modes, we show that a proper amount of induced delay may excite these fast modes under a state-feedback condition, allowing for measurements at higher frequencies. To examine one such condition, we take $w(x,t)$ to be the retarded position of the MCO, then $u(x,\tau)$ satisfies the SDDE $\tilde{\mathcal{A}}u(x,t)=\xi(t)+\zeta u(x,t-\tau)$, with the parameter ζ characterizing the coupling strength (for $\zeta=0$, the stochastic process u is a Markov chain due to ξ). This is a pointwise reinjection, whereas experimentally, the reinjection only carries information about the retarded position of the laser-illuminated segment of the MCO and not all x on [0, 1]. For very small delays, the SDDE reduces to the SDE $\tilde{\mathcal{A}}u(x,t)=\xi(t)+\zeta\sum_{n=0}^{N}(-\tau\partial_t)^n u(x,t)/n!+\mathcal{O}(\tau^{N+1})$; however, the applicability of the FDT remains questionable in either case. Since $\langle\xi(t)\rangle=0$, the formal solution reads[23] $\langle u(x,t)\rangle=\sum_{k,l}\Phi_k(x)p_{kl}(t)\exp(\rho_{kl}+i\omega_{kl})t$, where the polynomials p_{kl} are determined by the prefunctions on $[-\tau,0]$, and for each spatial eigenmode k, there is an infinite number l of eigenvalues (ρ_{kl},ω_{kl}) due to the delay τ. In the absence of the delay, the solutions (the prefunctions to the case with nonzero delay) are given by $\langle u(x,t)\rangle=\sum_k\Phi_k(x)p\exp(\rho+i\omega_k)t$, where $\omega_k^2=(\omega_k^0)^2-(\rho/4)^2$, with ω_k^0 representing the eigenfrequencies in the absence of any dissipation and $\rho=-2\gamma/m$ [Hz] measures the relaxation time of the oscillator. With a nonzero delay, the eigenvalues multiply and, for each k and l, satisfy

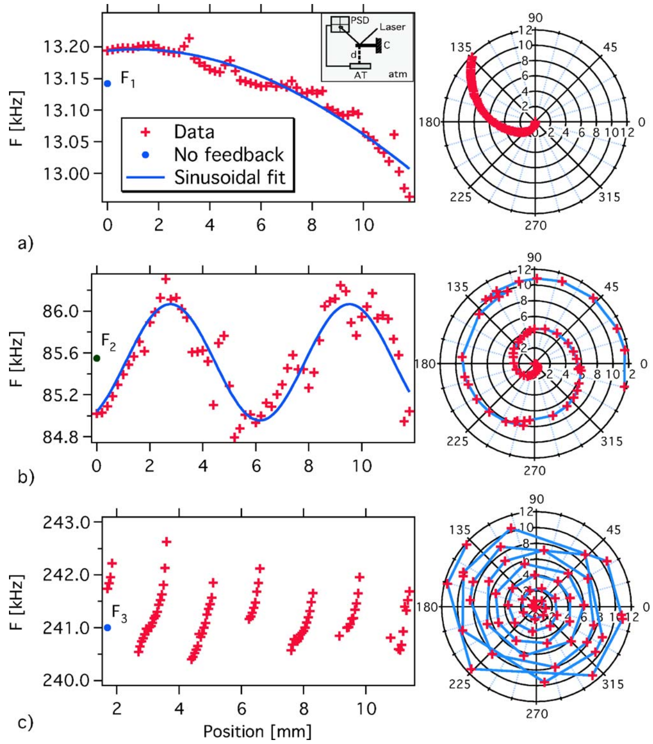

FIG. 2. (Color online) Delay-induced loci of the eigenfrequencies. The modes disappear for certain delays and reappear periodically due to the induced suppression and amplification. Left: delayed frequency F response of the resonances F_1, F_2, and F_3 of the MCO. The acoustically coupled feedback system depicted in the inset in (a) is excited by the stochastic noise. Right: the phase variation versus distance for the measured peaks. The distance (represented by the radius) d is given in mm, while the phase is measured in degrees. (a) corresponds to F_1=13 kHz, (b) to F_2 =85 kHz, and (c) to F_3=241 kHz.

$$\tau = \arctan\left[\frac{\omega_{kl}(\rho/2 - 2\rho_{kl})}{\rho_{kl}^2 - \omega_{kl}^2 - \rho\rho_{kl}/2 + (\omega_k^0)^2}\right]\omega_{kl}^{-1}. \qquad (6)$$

This equation can be derived from the characteristic equation of the delay partial differential equation of the MCO.[23] The sign of ρ_{kl} conditions the quotient $\tau_{kl}=\tau/T_{kl}$. The criterion $(m+j)<\tau_{kl}<(m+1/2)$, with $j=1/4$ (3/4) for positive (negative) coupling strength and $m \geqslant 0$ an integer, implies for a fix ζ that when τ is varied, ω_{kl} and ρ_{kl} fluctuate such that a mode k may be amplified only for τ in a band dictated by m. Thus, within the same band m, scanning τ may enforce the system to excite higher modes in satisfying the criterion. We propose that both ζ and τ provide sensory information as exemplified below. To this end, the analysis in Eq. (6) is the first reportage.

An example of how this result may be utilized is the remote detection of nanometer solid or liquid surface (and potentially subsurface) displacements. Briefly, prior to engaging the feedback at time $t=t_0$, the MCO is maintaining a frequency spectrum consistent with the distribution of its normal modes, as prescribed by the eigenfunctions of the partial differential equation representing the motion of the MCO. Engaging a feedback, the eigenvalue spectrum of the

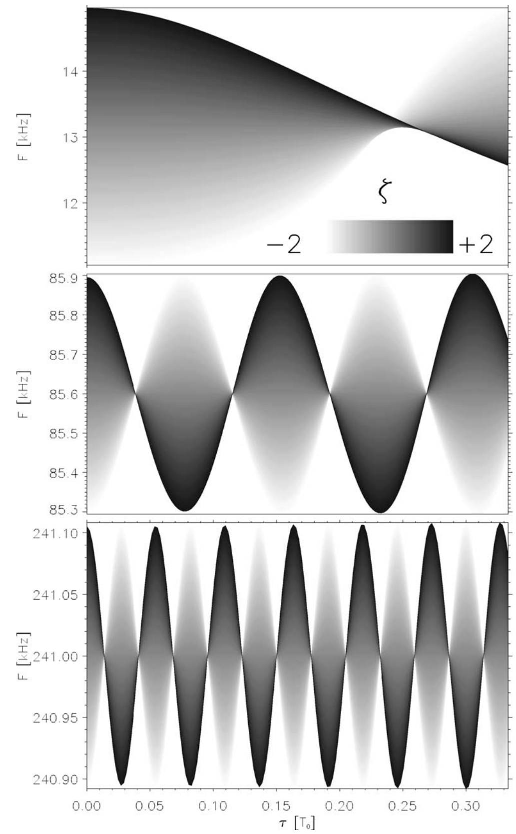

FIG. 3. Simulated delay-induced loci of the eigenfrequencies corresponding to the experimental values, in good agreement with Fig. 2. The gray scale measures the coupling strength.

system will be modified. The thermomechanical noise of the MCO is detected by the optical readout system of an AFM, amplified by a controller unit using a signal access module,[31] and is finally reinjected. The retarded position of the MCO may be reinforced either explicitly via a piezobimorph or implicitly via acoustic coupling through a gas, in which case, the internal delay of the feedback system is varied when working in different media [see inset of Fig. 2(a)]. In the former case, an analog delay generator was incorporated so as to compensate for any delay-induced frequency shift, maintain a fix frequency, or tune the frequency.

For a MCO with the measured resonances at 13.14, 85.6, and 241.0 kHz, this delay corresponds to phase shifts of 0.69°, 4.49°, and 12.75°, respectively. The measured total resonance frequency and phase variations are shown in Fig. 2 for F_i, $i=1,2,3$, as a function of d, the distance between the MCO and the acoustic excitation source, where the effects of a continuous variation in delay are shown to shift all frequencies due to the feedback mechanism. While all peaks underwent periodical shifts with distance, F_1 only experienced one quarter of a cycle, and F_2 and F_3 exhibited two and seven cycles, respectively, consistent with the phase information in the right column in Fig. 2. As can be seen, for the peak with a period of T_1=76.9 μs, the phase traverses only a quarter of the plane, whereas the higher peaks T_2 =11.76 μs and T_3=4.15 μs allow two and seven zero-phase crossings. Such clear frequency shifts, observed for the monitored peaks, give these modes excellent probing capabilities. While the maximum frequency shifts are $\Delta F_1 = 250$ Hz and $\Delta F_2 = 1.6$ kHz for the first two peaks, the third mode undergoes a shift of $\Delta F_3 = 100$ Hz per 50 μm, making this peak the most sensitive probe. This enables the motion detection of an acoustic source by a fraction of a micrometer. In light of Eq. (6), the measured periodic dependence is in remarkable agreement with the transcendental dependence of the eigenvalues on the delay and the frequency. A simulation of the observed variation in Fig. 2 is shown in Fig. 3 for the three frequencies investigated. Thus, the ability to reach a desired frequency as a result of a change in the time delay is summarized in these figures for the first three frequencies. Figure 3 theoretically shows the variation of the experimentally examined frequencies with delay, whereas Fig. 2 shows the experimental data. Thus, in the case of resonance, for a given coupling strength, a domain of frequencies is available to choose from by varying the time delay.

In conclusion, we have investigated the noise induced self-oscillation dynamics, transient response, and an acoustically coupled feedback for a MCO. For MCOs of high Young's modulus, megahertz windows are readily accessible from the fundamental modes, or from higher resonance of typical AFM MCOs. Under delayed oscillations, say, at 1 MHz, a striking sensitivity of 1 nm surface displacement will be plausible. Nanosecond delay detection has the potential of environmental sensing. The results show great potential for applications in fields such as remote microscale surface movement measurements, gas composition detection, and acoustic wave detection to provide sensory information of a kind not previously reported. Elastic versus viscous effects may be separated by the presented impulse excitation and formation of the beats. Under otherwise identical conditions, the geometry dependence of the frictional (drag) force may be determined, while for MCOs with the same geometry but different material, the material dependence may be determined. To discern the internal losses from those due to the drag force, the same can be repeated in vacuum.

Many problems related to the nanoscale thermal and stochastic processes remain open. Straightforward experiments were presented that will stimulate further discussions; in particular, we propose that the implications of FDT be explored within the context of delayed coupling. Stochastic processes in the equilibrium or in steady-state limit provide a direct window into the nanoscale realm of adsorption and diffusion. The stochastic excitation of the system was utilized in a delayed feedback to amplify or cool the oscillations at various modes of oscillation. As an application of the mode-dependent dynamics, we proposed and tested an experiment that readily allows surface displacement with potential for nanometer displacement using pure time delay via an acoustic coupling. A time-dependent delay may be induced by fluctuating d, resulting in parametric dynamics. Energy dissipation in form of elastic and viscous damping may be studied by impulse excitation of a silicon MCO. The typical decay predicted by a simple point mass, that is, an exponentially modified oscillation amplitude, does not correctly predict the experimental results. In view of a beam model, the experimentally observed frequency beats were explained. The beat frequency was observed to shift with the viscous damping in various environment. The presented feedback mechanism, facilitated by an acoustic coupling, conveniently provides pure time delay generation that is necessary for the excitation of the higher-frequency modes of the MCO.

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