# Angular tolerant resonant grating filters under oblique incidence

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Resonant grating filters have been proposed as a promising alternative to multilayer stacks for narrowband free-space filtering. The efficiency of such filters under normal incidence has been demonstrated. Unfortunately, under oblique incidence, the limited angular tolerance of the resonance forbids any filtering applications with use of standard collimated incident beams. Using a multimode planar waveguide and a bi-atom grating, we show how to increase the angular tolerance up to the divergence of standard beams (0.2 deg) without modifying the spectral bandwidth (0.1 nm), under any oblique angle of incidence. © 2005 Optical Society of America

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### 1. INTRODUCTION

A resonant grating filter (RGF) is basically a periodically perturbed planar waveguide that reflects light specularly. It is well known that the reflectivity of such a structure possesses peaks for some incident wavelengths that are generated by the coupling and outcoupling of a mode of the waveguide. These peaks can be tailored to create free-space narrow-bandpass filters.<sup>2,3</sup> It has been shown experimentally that an RGF can achieve spectral linewidths of the order of angstroms, wide rejection bands, and near 100% diffraction efficiency at the center wavelength.<sup>4,5</sup> On the other hand, these resonances have been shown to display extreme sensitivity to the angular orientation of the incident wave.<sup>6</sup> It has been observed that under oblique incidence, the angular linewidth is proportional to the spectral linewidth, <sup>4,7</sup> typically  $\Delta \theta / \Delta \lambda = 0.1 \text{ deg nm}^{-1}$ . This angular sensitivity prohibits de facto the use of this device for narrowband filtering. Increasing the angular tolerance while keeping a narrow spectral peak is thus a crucial issue. It has been shown that under normal incidence a better angular tolerance can be obtained<sup>6,8</sup> with some modifications to the structure. In this configuration, two counterpropagating modes are excited by the incident beam. The interaction between these two modes broadens the angular response, while the spectral peak is not affected.<sup>9</sup> Several grating designs have been proposed to take advantage of this property: "doubly periodic" gratings,9 very deep gratings, 10 and gratings on top of a multimode waveguide. 11 Each of these was shown to display disproportionately broad angular selectivity under normal incidence.

To our knowledge, no work has hitherto addressed the issue of increasing the angular tolerance under oblique incidence. Yet most free-space filters are required to work under oblique incidence to avoid the use of a beam separator in collecting the filtered signal. Hence, high

angular tolerance is desirable for dense wavelength division multiplexing filters especially because they are usually cascaded to filter many different wavelengths. In the following, we first describe the principle underlying the broadening of the angular response at normal incidence, and then we show how to extend it to oblique incidence.

### 2. INCREASING ANGULAR TOLERANCE UNDER NORMAL INCIDENCE

In this section we present a simple intuitive explanation of the broadening of the angular response under normal incidence. Consider a waveguide supporting a guided mode described by the dispersion relation linking the guided mode's wave vector to its wavelength  $\alpha_m(\lambda)$  or  $\lambda_m(\alpha)$ . The guided wave is by definition nonleaky, so that  $\alpha_m(\lambda) > \max(n_{\mathrm{sub}}, \, n_{\mathrm{sup}}) 2 \pi / \lambda$ , where  $n_{\mathrm{sub}}, \, n_{\mathrm{sup}}$  are the substrate and superstrate refraction indices, respectively. In all of the examples below, the mode is TE polarized, but similar conclusions apply to TM-polarized The waveguide is perturbed by a onedimensional lamellar grating coupler with period d and height h; see Fig. 1(a). Hereafter, the grating is considered to be a small perturbation of the reference planar waveguide system, so that  $h \leq \lambda$ . We introduce the relative permittivity expansion coefficient of the grating  $\epsilon_a$ defined by

$$\epsilon(x) = \sum_{q=-\infty}^{\infty} \epsilon_q \exp(iqKx),$$
 (1)

where  $K=2\pi/d$  and  $\epsilon(x)$  is the relative permittivity of the structure restricted to the grating region. The incident plane wave illuminating the structure is described by its angle of incidence  $\theta$  and its wavelength  $\lambda$ , which varies about  $\lambda_0$ , the wavelength to be filtered. The spatial incident pulsation  $(2\pi \text{ times the spatial frequency})$  is  $\alpha(\lambda)=2\pi\sin\theta/\lambda$ , and the incidence angle and grating

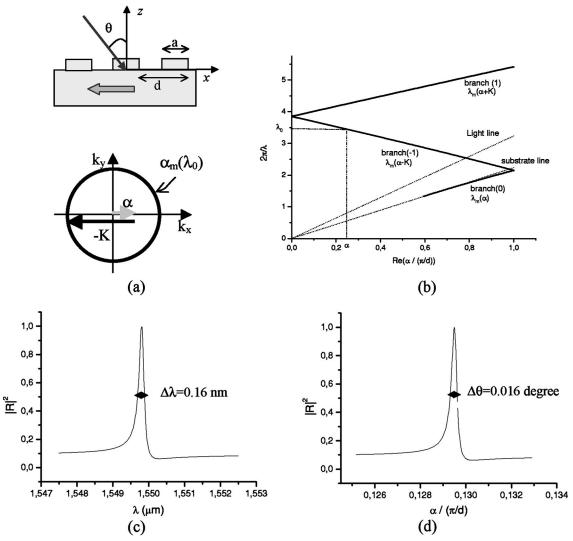


Fig. 1. (a) Illumination configuration and representation in the Fourier space of the phase-matching condition at a given wavelength. (b) Dispersion relation of the mode in the perturbed waveguide. (c), (d) Reflectivity of the grating versus the wavelength and incident angle.  $n_{\rm sub}=1.448,\,n_{\rm sup}=1,\,n_{\rm layer}=2.07,\,$  thickness  $e=300\,{\rm nm},\,d=971\,{\rm nm},\,h=20\,{\rm nm},\,a=721\,{\rm nm}.$ 

period are chosen so that  $|\alpha-K|\approx \alpha_m(\lambda_0)$ . In this case, a spectral resonance peak occurs close to  $\lambda_m(\alpha-K)\approx \lambda_0$ , and both its spectral and angular linewidth are proportional<sup>6,7</sup> to  $|h\,\epsilon_1|^2$ ; see Figs. 1(c) and 1(d). When the incident spatial pulsation is changed slightly (by altering the angle of incidence) to  $\alpha+\Delta\alpha$ , the spectral resonance peak will be shifted toward  $\lambda_m(\alpha+\Delta\alpha-K)^{10}$ ; see Fig. 1(b). Hence the guiding idea for increasing the angular tolerance without modifying the spectral linewidth is to design a reference system supporting a dispersionless mode in the vicinity of  $\lambda_0$ .

One possibility for obtaining a locally dispersionless mode is to transform the planar waveguide into a mode bandgap (MBG) waveguide and work in the vicinity of the stop band. This amounts to introducing a periodic perturbation of the structure, thus forming a Bragg grating, with period d' such that  $\pi/d' = K'/2 \approx \alpha_m(\lambda_0)$ . This modulation opens up a mode-frequency bandgap (or stop band) close to  $\lambda_0$  (Ref. 12) owing to the coupling of the mode with wave vector  $\alpha_m$  and the Bragg reflected mode with wave vector  $-\alpha_m$  through the reciprocal space lattice

vector K' satisfying  $\alpha - K' \approx -\alpha$ . The coupling strength and gap width depend on the Fourier coefficient of the periodic relative permittivity of the MBG waveguide associated with K'.  $^{6,7,13}$  At the frequency edge of the gap, the mode is a stationary nonleaky wave word whose dispersion relation becomes flatter with increasing gap width. Once the MBG waveguide is designed, it suffices to add a grating coupler to couple and outcouple the locally dispersionless mode and obtain an angular tolerant reflectivity resonance. The spectral linewidth depends only on the parameters of the grating coupler, and the angular behavior is linked to that of the MBG waveguide: In this sense, they are totally independent.

Now, if one wants to obtain a device that transmits and reflects light in the specular direction only, the structure formed by the superposition of the MBG waveguide with the grating coupler should be periodic with a period L that satisfies  $\alpha + 2q \pi/L > \max(n_{\text{sub}}, n_{\text{sup}}) 2\pi/\lambda_0$ , with  $q \neq 0$ . In this case, the sole possibility is to take  $\alpha$  null and d equal to a multiple of d'; hereafter, we choose d = 2d' = L. The solutions with  $\alpha \neq 0$ , i.e., oblique inci-

dence, correspond to Littrow mounting, in which case at least two orders will be reflected. Thus the final structure is a *d*-periodic grating whose pattern is optimized to assume the roles of both the grating coupler and the MBG waveguide; see Fig. 2(a). The permittivity coefficient  $\epsilon_1$ is responsible for the coupling of the incident beam into the counterpropagating modes and the spectral linewidth, while  $\epsilon_2$  (which is associated with the MBG wave vector 2K = K') is responsible for the mutual interaction of the modes and the angular tolerance, as summarized in Fig. 2(a). Hence we have established the fact that under normal incidence, a resonant grating may possess angularly tolerant resonances due to the naturally flat dispersion relation of the mode in the vicinity of the second-order stop band, as seen in Fig. 2(b). 1,6,10,11 A possible design for an angularly tolerant device is the bi-atom grating depicted in Fig. 2. The pattern of the grating has been chosen such that the permittivity Fourier coefficients satisfy  $|\epsilon_2| > |\epsilon_1|$ . The spectral and angular reflectivity curves plotted in Figs. 2(c) and 2(d) of the bi-atom grating, described in Fig. 3, show that the angular tolerance has been increased by a factor of 30 compared with that of the simple lamellar grating studied in Fig. 1. We now show how this approach can be used to design angularly tolerant RGF functioning under oblique incidence.

## 3. ANGULARLY TOLERANT RESONANT GRATING FILTERS UNDER OBLIQUE INCIDENCE

We have seen that a solution for obtaining angularly tolerant resonances is to excite two counterpropagating modes that strongly interact with each other so that the resulting mode exhibits a flat dispersion relation. This happens naturally under normal incidence, where the same mode is excited along two opposite directions. Now, planar waveguides can also support several modes. Thus it is possible to excite two counterpropagating modes under oblique incidence if the grating period and angle of incidence satisfy [see Fig. 3(a)]

$$\alpha + K \approx \alpha_{m2}(\lambda_0), \quad \alpha - K \approx -\alpha_{m1}(\lambda_0),$$
 (2)

where  $\alpha_m 1(\lambda)$  and  $\alpha_m 2(\lambda)$  are the wave vectors of two different modes of the planar waveguide. The angle of incidence depends solely on the difference between the effective index of the modes  $\alpha_{mj} = 2 \pi n_{\text{eff}} / \lambda (j = 1, 2)$ :

$$\sin(\theta) = [n_{\text{eff1}}(\lambda_0) - n_{\text{eff2}}(\lambda_0)]/2. \tag{3}$$

When the two modes are TE and TM polarized, this configuration allows the existence at the angle of incidence given by Eq. (3) of a resonance peak about  $\lambda_0$  that is

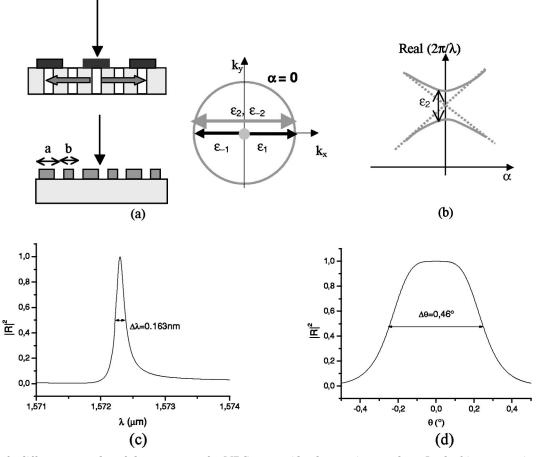


Fig. 2. (a) Left, different examples of the geometry of a MBG waveguide plus grating coupler. In the bi-atom grating, the pattern consists of two ridges with different widths centered about one fourth and three fourths of the period. Right, representation in the Fourier space of the phase-matching condition when two modes are excited under normal incidence. (b) Dispersion relation in the vicinity of the second-order stop band. (c) Reflectivity of the bi-atom grating as a function of the wavelength at normal incidence. (d) Reflectivity of the bi-atom as a function of the incident angle for a wavelength corresponding to the maximum of reflectivity ( $\lambda = 1572.3 \text{ nm}$ ).

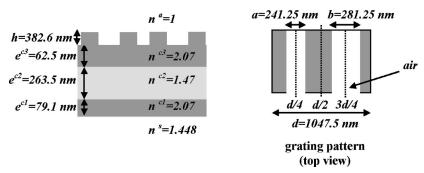


Fig. 3. Geometry of the bi-atom grating whose reflectivity spectra are plotted in Figs. 2(c) and 2(d). The substrate and superstrate are the same as in Fig. 1; the planar waveguide is a multilayer with refractive indices 2.07, 1.47, 2.07 and thicknesses 79.1, 263.5, 62.5 nm from bottom to top. The bi-atom grating is defined by a=241.25 nm, b=281.25 nm, d=1047.5 nm, h=382.6 nm, n=2.07. The Fourier coefficients of the relative permittivity of the grating are  $\epsilon_1=0.093$  and  $\epsilon_2=1.037$ .

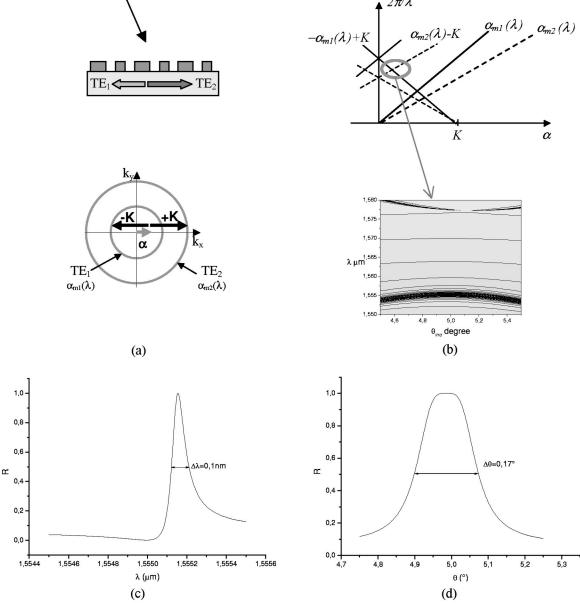


Fig. 4. (a) Top, illumination configuration and bottom, representation in the Fourier space of the phase-matching condition when two different modes are excited at a given wavelength under oblique incidence. (b) Top, dispersion relation when the waveguide supports two TE modes in the limit h tends to 0. Bottom, zoom-in of the mini-stop band when the TE<sub>1</sub> branch intersects the TE<sub>2</sub> branch obtained by calculating the reflectivity of the grating as a function of the angle of incidence and wavelength. (c) Reflectivity of the grating as a function of the wavelength for  $\theta = 5$  deg. (d) Reflectivity of the grating as a function of the angle of incidence for a wavelength corresponding to the maximum of reflectivity ( $\lambda = 1555$  nm).

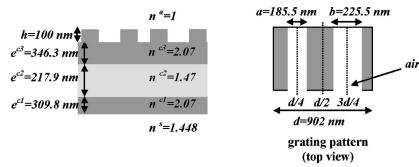


Fig. 5. Geometry of the bi-atom grating whose reflectivity spectra are plotted in Figs. 4(c) and 4(d). The substrate and superstrate are the same as in Fig. 1; the planar waveguide is a multilayer with refraction indices 2.07, 1.47, 2.07 and thicknesses 346.3, 217.9, 309.8 nm from top to bottom. The bi-atom grating is defined by a=225.5 nm, b=185.5 nm, d=902 nm, h=100 nm, n=2.07. The effective indices of the TE<sub>1</sub> and TE<sub>2</sub> modes of the imperturbed structure are, respectively, 1.6236 and 1.8120.

independent of the incident polarization.<sup>15</sup> In our case, on the contrary, the modes must have the same polarization to be able to interact mutually. As in the normalincidence case,  $\epsilon_1$  is responsible for the coupling of the incident beam into the modes, while  $\epsilon_2$  determines the mutual interaction of the modes. Hence a structure similar to that presented in Fig. 2 except that the waveguide now supports two TE modes will have an angularly tolerant resonance peak under oblique incidence. In Fig. 4(a) we consider a bi-atom RGF supporting two TE modes (indicated by mode 1 and mode 2 on the plot) such that the mutual excitation of the modes along two opposite directions occurs for an incident angle close to 5 deg. In Fig. 4(b) we present a contour map of the reflection as a function of the wavelength and incident angle, which gives a good representation of the relation dispersion. As expected, we observe the opening of a gap and the flattening of the dispersion relation, outside the edge of the Brillouin zone, about the intersection point of the branches of the reference modes. Contrary to what happens in normal incidence, there exists a resonance peak at both band-edge frequencies. Indeed, when the system functions under oblique incidence, the modes and the incident wave do not have any symmetry mismatch that would forbid their coupling.<sup>9</sup> Yet, in our configuration, the chosen angle of incidence being close to zero, one of the resonances remains much narrower than the other. In Fig. 4(c) we plot the reflectivity of the RGF as a function of the wavelength and the incident angle about the edge point of the dispersion relation.

The optogeometrical characteristics of the bi-atom grating are plotted on Fig. 5. The shape of the resonance with respect to the wavelength is similar to that obtained with a classic single-mode configuration [Fig. 1(c)], while that with respect to the incident angle is similar to that obtained under normal incidence [Fig. 2(d)]. The ratio  $\Delta\theta/\Delta\lambda$  of the double-mode structure has increased by a factor of 17 compared with that of the single-mode structure [Figs. 1(c) and 1(d)]. We have thus designed a RGF that functions under oblique incidence with an angular tolerance that is compatible with the use of standard collimated beams. It is worth noting that our solution does not resort to extreme optimization of the periodic structure. If small errors on the structure parameters are committed, the angular-tolerance resonance will still exist, though at a different wavelength and angle of incidence. An error of 1 nm on the period of the structure, the etching depth, and the ridge width, will shift the central wavelength of the filter approximately 1, 0.05, and 0.07 nm, respectively. The incident angle remains remarkably stable near 5 deg with a variation of 0.01, and the widths of the reflectivity peak versus the wavelength and incident angle are not affected. Thus fabrication errors, especially on the period, have consequences essentially on the central wavelength of the filter. In cases where several filters with the same bandpass and different wavelengths are required, this problem is not insurmountable.

### 4. CONCLUSION

Resonant gratings are a powerful tool for filtering applications, but the resonance peak obtained by exciting one single mode does not possess all of the properties required for a useful device. In this paper we have shown that under oblique incidence, the excitation of two different interacting counterpropagating modes produces an angularly tolerant resonance. We have given an explanation of this behavior by studying the formation of local mode bandgaps outside the edge of the Brillouin zone in a periodically perturbed multimode waveguide. In our opinion, the full potential of resonant gratings when several modes are excited remains to be discovered.

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