Phenomenological theory of filtering by resonant dielectric gratings

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Using a phenomenological theory of diffraction gratings made by perturbing a planar waveguide allows us to deduce important properties of the sharp filtering phenomena generated by this kind of structure when the incident light excites a guided wave. It is shown that the resonance phenomenon occurring in these conditions acts on one of the two eigenvalues of the Hermitian reflection matrix only. As a consequence, we deduce a mathematical expression of the reflectivity and demonstrate that high-efficiency filtering of unpolarized light requires the simultaneous excitation of two uncoupled guided waves. Numerical examples are given.

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1. INTRODUCTION

The purpose of this paper is to present a phenomenological theory of filtering properties of one-dimensional or two-dimensional gratings deposited on planar waveguides. It is well known that the efficiencies of such structures as a function of the wavelength may present peaks or anomalies generated by the excitation of guided waves propagating inside the layers. These anomalies have been widely studied in the case of classical one-dimensional gratings illuminated with in-plane mountings, in particular when only one order is reflected or transmitted by the grating.[1,2] For shallow gratings, the reflectivity is in general close to that of the planar structure except for a sharp peak culminating at 100% that corresponds to the excitation of a guided mode. The width and location of the peak depend on the grating parameters. The resonant behavior of the reflectivity of the grating may be valuable for the purpose of light filtering; but, unfortunately, in that case it is limited to polarized light.[3,4] This limitation prevents the use of this property in many technological applications of filtering, for example for the purpose of dense wavelength-division multiplexing for optical communications. Thus the use of off-plane (conical) mounting or two-dimensional gratings has been suggested to design filters for unpolarized light.[5–7] However, the behavior of the reflectivity as a function of the wavelength, angle of incidence, and incident polarization in the general vectorial case is still little understood.

The aim of the paper is to describe and predict, using as few parameters as possible, the reflectivity and transmittivity of resonant gratings when the spatial frequencies and temporal frequencies imposed by the incident beam are close to those of an eigenmode of the structure. As in Refs. 1 and 2, the study makes use of the notion of analytic continuation of complex functions of a real variable in the complex plane. In addition, the use of a general scattering matrix of size 4×4 and reflection or transmission scattering submatrices is needed. The definition of poles and zeros of the eigenvalues of Hermitian matrices derived from scattering matrices allows us to predict the performances of the structures for filtering of unpolarized light. The result that emerges is that high-efficiency filtering properties for unpolarized light are quite impossible to obtain if the incident light cannot excite several modes at the same wavelength. Rigorous numerical results will confirm these theoretical predictions.

2. PRESENTATION OF THE STRUCTURE AND NOTATION

In Fig. 1 we consider a Cartesian coordinate system of axes xyz. The periodic guiding structure limited on top ($z = 0$) by air and at the bottom by a substrate of real relative permittivity ε_s has a relative permittivity $\varepsilon(x, y, z)$ that is real and periodic along two different, possibly nonorthogonal directions (however, hereafter, we assume for simplicity that $\varepsilon(x, y, z)$ is periodic in x and y). The permittivity $\varepsilon(x, y, z)$ is obtained by perturbing slightly a permittivity $\varepsilon'(z)$ in a periodic manner. The nonperturbed structure is assumed to be a waveguide [for example, $\varepsilon'(z)$ constant and greater than ε_s]. Figure 2 shows examples of such structures.

The incident plane wave with wave vector $\mathbf{k}^{i+}$ (with $|\mathbf{k}^{i+}| = k = 2\pi/\lambda$, λ wavelength in vacuum) illuminates the grating with an incidence characterized by angles ϕ (angle between the x axis and the projection of $\mathbf{k}^{i+}$ on the xy plane) and θ (angle between the z axis and $\mathbf{k}^{i+}$). To define the polarization of the incident wave, the amplitude of the incident electric field is projected on two unit vectors $\hat{s}^{i+}$ and $\hat{p}^{i+}$ orthogonal to $\mathbf{k}^{i+}$ and orthogonal to each other, $\hat{s}^{i+}$ being perpendicular to the z axis and $\hat{p}^{i+}$ parallel to the plane of incidence,

$$\hat{s}^{i+} = \frac{\mathbf{k}^{i+} \times \hat{z}}{|\mathbf{k}^{i+} \times \hat{z}|}, \qquad \hat{p}^{i+} = \frac{\hat{s}^{i+} \times \mathbf{k}^{i+}}{|\hat{s}^{i+} \times \mathbf{k}^{i+}|} \tag{1}$$

where the components $(\alpha,\ \beta,\ -\gamma^{+})$ of $\mathbf{k}^{i+}$ are given by

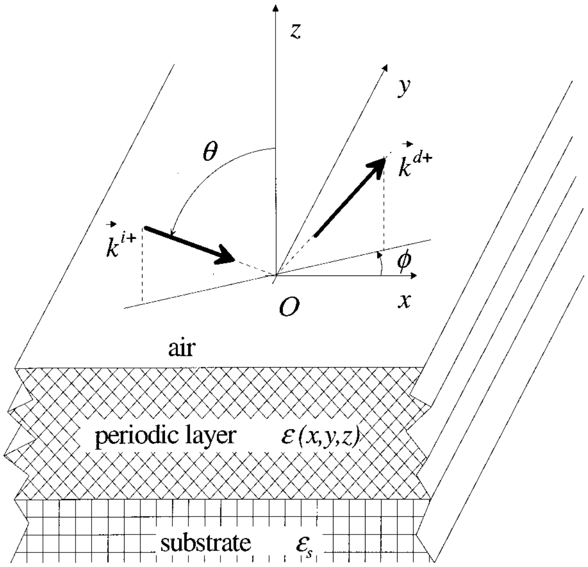

Fig. 1. Periodic guiding structure.

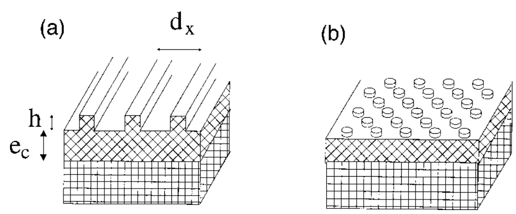

Fig. 2. Examples of periodic guiding structures. (a) Classical lamellar grating that can be used in conical mounting, (b) crossed grating with circular bumps and hexagonal symmetry.

$$\alpha = k\sin(\theta)\cos(\phi), \qquad \beta = k\sin(\theta)\sin(\phi),$$
$$\gamma^+ = (k^2 - \alpha^2 - \beta^2)^{1/2}. \tag{2}$$

Using a time dependence in $\exp(-i\omega t)$, the electric field of the incident wave can be written

$$\mathbf{E}^{i+} = \mathbf{P}^{i+}\exp(i\alpha x + i\beta y - i\gamma^+ z), \tag{3}$$

with

$$\mathbf{P}^{i+} = P^{i+,s}\hat{s}^{i+} + P^{i+,p}\hat{p}^{i+}. \tag{4}$$

The grating formula shows that the scattered field $\mathbf{E}^s$ can be expressed in the form of Rayleigh expansions outside the periodic guiding structure:

$$\mathbf{E}^s = \sum_{n=-\infty}^{+\infty}\sum_{m=-\infty}^{+\infty}\mathbf{P}^+_{n,m}\exp(i\alpha_n x + i\beta_m y + i\gamma^+_{n,m}z) \quad \text{if } z > 0, \tag{5}$$

$$\mathbf{E}^s = \sum_{n=-\infty}^{+\infty}\sum_{m=-\infty}^{+\infty}\mathbf{P}^-_{n,m}\exp(i\alpha_n x + i\beta_m y - i\gamma^-_{n,m}z) \quad \text{if } z < -e, \tag{6}$$

with

$$\alpha_n = \alpha + nK_x, \qquad K_x = 2\pi/d_x, \tag{7}$$

$$\beta_m = \beta + mK_y, \qquad K_y = 2\pi/d_y, \tag{8}$$

$$\gamma^+_{n,m} = (k^2 - \alpha_n^2 - \beta_m^2)^{1/2}, \quad \mathrm{Re}(\gamma^+_{n,m}) + \mathrm{Im}(\gamma^+_{n,m}) > 0, \tag{9}$$

$$\gamma^-_{n,m} = (k^2\varepsilon_s - \alpha_n^2 - \beta_m^2)^{1/2}, \quad \mathrm{Re}(\gamma^-_{n,m}) + \mathrm{Im}(\gamma^-_{n,m}) > 0, \tag{10}$$

d_x and d_y being the periods of the grating along the x and y axes.

We assume that the only reflected and transmitted waves are the (0, 0) orders, the other orders being evanescent. In these conditions, the asymptotic value of the field at infinity reduces to the sum of the incident field and the (0, 0) orders

$$\mathbf{E} \approx \mathbf{P}^{i+}\exp(i\alpha x + i\beta y - i\gamma^+ z) + \mathbf{P}^+_{0,0}\exp(i\alpha x + i\beta y + i\gamma^+ z) \quad \text{if } z \to \infty, \tag{11}$$

$$\mathbf{E} \approx \mathbf{P}^-_{0,0}\exp(i\alpha x + i\beta y - i\gamma^- z) \quad \text{if } z \to -\infty, \tag{12}$$

with $\gamma^- = \gamma^-_{0,0} = (k^2\varepsilon_s - \alpha^2 - \beta^2)^{1/2}$.

The polarization of the reflected and transmitted (0, 0) orders can be projected on two unit vectors

$$\hat{s}^{d+} = -\frac{\mathbf{k}^{d+}\times\hat{z}}{|\mathbf{k}^{d+}\times\hat{z}|}, \qquad \hat{p}^{d+} = -\frac{\hat{s}^{d+}\times\mathbf{k}^{d+}}{|\hat{s}^{d+}\times\mathbf{k}^{d+}|}, \tag{13}$$

$$\mathbf{P}^+_{0,0} = P^{d+,s}\hat{s}^{d+} + P^{d+,p}\hat{p}^{d+}, \tag{14}$$

$$\hat{s}^{d-} = \frac{\mathbf{k}^{d-}\times\hat{z}}{|\mathbf{k}^{d-}\times\hat{z}|}, \qquad \hat{p}^{d-} = \frac{\hat{s}^{d-}\times\mathbf{k}^{d-}}{|\hat{s}^{d-}\times\mathbf{k}^{d-}|}, \tag{15}$$

$$\mathbf{P}^-_{0,0} = p^{d-,s}\hat{s}^{d-} + P^{d-,p}\hat{p}^{d-}, \tag{16}$$

$(\alpha, \beta, \gamma^+)$ and $(\alpha, \beta, -\gamma^-)$ being the components of $\mathbf{k}^{d+}$ and $\mathbf{k}^{d-}$, respectively.

To define a scattering matrix for this structure, let us notice that a plane wave illuminating the structure from the substrate with a wave vector $\mathbf{k}^{i-}$ of components $(\alpha, \beta, \gamma^-)$ generates transmitted and reflected waves having wave vectors $\mathbf{k}^{d+}$ and $\mathbf{k}^{d-}$. If we define defining unit vectors orthogonal to $\mathbf{k}^{i-}$,

$$\hat{s}^{i-} = -\frac{\mathbf{k}^{i-}\times\hat{z}}{|\mathbf{k}^{i-}\times\hat{z}|}, \qquad \hat{p}^{i-} = -\frac{\hat{s}^{i-}\times\mathbf{k}^{i-}}{|\hat{s}^{i-}\times\mathbf{k}^{i-}|}, \tag{17}$$

this second incident wave can be written

$$\mathbf{E}^{i-} = \mathbf{P}^{i-}\exp(i\alpha x + i\beta y + i\gamma^- z), \tag{18}$$

with

$$\mathbf{P}^{i-} = P^{i-,s}\hat{s}^{i-} + P^{i-,p}\hat{p}^{i-}, \tag{19}$$

and thus the asymptotic expression of the field in the substrate becomes

$$\mathbf{E} \approx \mathbf{P}^i_-\exp(i\alpha x + i\beta y + i\gamma^- z) + \mathbf{P}^-_{0,0}\exp(i\alpha x + i\beta y - i\gamma^- z) \quad \text{if } z \to -\infty. \tag{20}$$

3. SCATTERING MATRICES: DEFINITION AND PROPERTIES

First, we define four incident and diffracted column matrices of two elements by

$$I^{\pm} = (P^{i\pm,s}\sqrt{\gamma^{\pm}},\ P^{i\pm,p}\sqrt{\gamma^{\pm}}),$$

$$D^{\pm} = (P^{d\pm,s}\sqrt{\gamma^{\pm}},\ P^{d\pm,p}\sqrt{\gamma^{\pm}}), \tag{21}$$

and from the linearity of Maxwell equations, the diffracted column matrices can be expressed linearly from the incident ones through the definition of square reflection matrices R_1 and R_2 and transmission matrices T_1 and T_2 of size 2×2:

$$D^{+} = R_1 I^{+} + T_2 I^{-}, \tag{22}$$

$$D^{-} = T_1 I^{+} + R_2 I^{-}. \tag{23}$$

By defining the incident and the diffracted column matrices I and D with four components $(P^{i+,s}\sqrt{\gamma^{+}}, P^{i+,p}\sqrt{\gamma^{+}}, P^{i-,s}\sqrt{\gamma^{-}}, P^{i-,p}\sqrt{\gamma^{-}})$ and $(P^{d+,s}\sqrt{\gamma^{+}}, P^{d+,p}\sqrt{\gamma^{+}}, P^{d-,s}\sqrt{\gamma^{-}}, P^{d-,p}\sqrt{\gamma^{-}})$, respectively, we can condense Eqs. (22) and (23) into a single one,

$$D = SI, \tag{24}$$

the square scattering matrix S of dimension 4×4 being obtained from the reflection and transmission matrices by

$$S = \begin{bmatrix} R_1 & T_2 \\ T_1 & R_2 \end{bmatrix}. \tag{25}$$

Now let us demonstrate two important properties of the S matrix. First, since the materials are lossless, the energy balance can be written $|D| = |I|$. This property shows that the S matrix is unitary, which entails

$$S^{*}S = 1, \tag{26}$$

1 denoting here the unit diagonal matrix of size 4 and S^{*} denoting the adjoint of S.

Second, to show a symmetry property of the S matrix, we use the reciprocity theorem.[8] With this aim, we associate to the mounting depicted previously a second one, where the incident-wave parameters α and β take opposite values. Let us call I', D', and S' the corresponding incident, diffracted, and scattering matrices. The reciprocity theorem can be written as follows:

in reflection,

$$\langle I^{+}, D'^{+}\rangle = \langle I'^{+}, D^{+}\rangle \qquad \text{if } I^{-} = I'^{-} = 0, \tag{27}$$

$$\langle I^{-}, D'^{-}\rangle = \langle I'^{-}, D^{-}\rangle \qquad \text{if } I^{+} = I'^{+} = 0, \tag{28}$$

and in transmission,

$$\langle I^{+}, D'^{+}\rangle = \langle I'^{-}, D^{-}\rangle \qquad \text{if } I^{-} = I'^{+} = 0, \tag{29}$$

$$\langle I^{-}, D'^{-}\rangle = \langle I'^{+}, D^{+}\rangle, \qquad \text{if } I^{+} = I'^{-} = 0, \tag{30}$$

with $\langle V, U\rangle = V_1U_1 + V_2U_2$, where $V = (V_1, V_2)$ and $U = (U_1, U_2)$.

Thus, from Eqs. (22) and (27),

$$\langle I^{+}, R_1 I'^{+}\rangle = \langle I'^{+}, R_1 I^{+}\rangle, \tag{31}$$

and since $\langle I'^{+}, R_1 I^{+}\rangle = \langle t(R_1) I'^{+}, I^{+}\rangle = \langle I^{+}, t(R_1) I'^{+}\rangle$, where $t(R_1)$ is the transpose of R_1,

$$\langle I^{+}, R_1 I'^{+}\rangle = \langle I^{+}, t(R_1) I'^{+}\rangle, \tag{32}$$

and we deduce that

$$t(R_1) = R_1'. \tag{33}$$

In the same way, with use of Eqs. (22), (23), and (27)–(30), it turns out that

$$t(R_2) = R_2', \tag{34}$$

$$t(T_2) = T_1', \tag{35}$$

$$t(T_1) = T_2'. \tag{36}$$

Finally, from equations (33)–(36), we find that

$$S' = t(S). \tag{37}$$

Now let us consider the case in which the diffracting structure is symmetrical with respect to the z axis. If we notice that the case in which the constants of propagation are $(-\alpha, -\beta)$ can be deduced from the original case by making the same symmetry of the incident and diffracted waves with respect to the z axis, we deduce that $S' = S$, and thus from Eq. (37), S is symmetrical. Other properties of the S matrix can be derived from other symmetries of the structure, especially if the xy plane is a plane of symmetry. Since this symmetry is difficult to realize in practice, this case is not developed in the paper.

4. POLES AND ZEROS OF THE SCATTERING MATRICES

We have assumed that the nonperturbed structure was a waveguide. A guided wave propagating in an arbitrary direction of the xy plane can be written, for $z > 0$,

$$\mathbf{E} = \mathbf{P}^{g+}\exp(i\alpha^{g}x + i\beta^{g}y + i\gamma^{g+}z), \tag{38}$$

with

$$\gamma^{g+} = i[(\alpha^{g})^2 + (\beta^{g})^2 - (k^{g,\ \mathrm{plan}})^2]^{1/2}.$$

$k^{g,\ \mathrm{plan}}$ is the wave number, and in the substrate,

$$\mathbf{E} = \mathbf{P}^{g-}\exp(i\alpha^{g}x + i\beta^{g}y - i\gamma^{g-}z), \tag{39}$$

with

$$\gamma^{g-} = i[(\alpha^{g})^2 + (\beta^{g})^2 - (k^{g,\ \mathrm{plan}})^2\varepsilon_s]^{1/2},$$

in such a way that its amplitude exponentially decreases as $|z| \to \infty$. This mode is TE or TM polarized.

Now we suppose that a perturbation is introduced into the structure. For the same propagation constants (α^{g}, β^{g}), the wave number becomes equal to $k^{g,\ \mathrm{perturb}}$, and the Floquet–Bloch theorem allows us to state that the guided wave takes the form of a series,

$$\mathbf{E} = \sum_{n=-\infty}^{+\infty}\sum_{m=-\infty}^{+\infty}\mathbf{P}^{g+}_{n,m}\exp(i\alpha^{g}_{n}x + i\beta^{g}_{m}y + i\gamma^{g+}_{n,m}z) \quad \text{if } z > 0, \tag{40}$$

$$\mathbf{E} = \sum_{n=-\infty}^{+\infty}\sum_{m=-\infty}^{+\infty}\mathbf{P}^{g-}_{n,m}\exp(i\alpha^{g}_{n}x + i\beta^{g}_{m}y - i\gamma^{g-}_{n,m}z) \quad \text{if } z < -e, \tag{41}$$

with

$$\alpha^{g}_{n} = \alpha^{g} + nK_x \tag{42}$$

$$\beta^{g}_{m} = \beta^{g} + mK_y \tag{43}$$

$$\gamma_{n,m}^{g+} = \sqrt{(k^{g,\text{ perturb}})^2 - (\alpha_n^g)^2 - (\beta_m^g)^2},$$
$$\text{Re}(\gamma_{n,m}^{g+}) + \text{Im}(\gamma_{n,m}^{g+}) > 0, \tag{44}$$

$$\gamma_{n,m}^{g-} = \sqrt{(k^{g,\text{ perturb}})^2 \varepsilon_s - (\alpha_n^g)^2 - (\beta_m^g)^2},$$
$$\text{Re}(\gamma_{n,m}^{g-}) + \text{Im}(\gamma_{n,m}^{g-}) > 0. \tag{45}$$

It is worth noticing that in Eqs. (38) and (39), $\gamma^{g\pm}$ is a pure imaginary number, whereas in Eqs. (40) and (41), $\gamma_{n,m}^{g\pm}$ are complex numbers with real and imaginary parts different from 0. Indeed, in the series contained in the right-hand terms of Eqs. (40) and (41), some values of the couple (α_n^g, β_m^g) can be such that

$$(\alpha_n^g)^2 + (\beta_m^g)^2 < (|k^{g,\text{perturb}}|)^2, \tag{46}$$

and if we assume by continuity that $k^{g,\text{perturb}}$ is close to $k^{g,\text{plan}}$, the corresponding term in the series on the right-hand side of Eq. (40) is close to a plane wave going from the grating surface to $z = +\infty$. The same remark can be made for the field in the substrate, and thus we are led to the conclusion that, in contrast to the guided wave propagating in the nonperturbed structure, the wave propagating in the perturbed structure presents losses. This seems to be in contradiction to the fact that the propagation constants α^g and β^g are assumed to be real. In fact, there is no contradiction if the wave number $k^{g,\text{perturb}}$ is allowed to be complex, since in that case, the corresponding frequency $\omega^{g,\text{perturb}} = ck^{g,\text{perturb}}$ (c is the speed of light) is complex too, and if the imaginary parts of $\omega^{g,\text{perturb}}$ and $k^{g,\text{perturb}}$ are negative, the field, which behaves in $\exp(-i\omega^{g,\text{perturb}}t)$, decreases exponentially with time, a direct consequence of losses. In the following, we will assume that there is only one term in Eq. (40) and only one term in Eq. (41) that represent lossy terms, in other words, such that relation (46) or the equivalent relation in the substrate is satisfied. By convention, we will affix to this lossy term the subscripts (p, q). When $|z| \to \infty$, the field tends to this term asymptotically:

$$\mathbf{E} \approx \mathbf{P}_{p,q}^{g+} \exp(i\alpha_p^g x + i\beta_q^g y + i\gamma_{p,q}^{g+} z) \qquad \text{if } z \to +\infty, \tag{47}$$

$$\mathbf{E} \approx \mathbf{P}_{p,q}^{g-} \exp(i\alpha_p^g x + i\beta_q^g y - i\gamma_{p,q}^{g-} z) \qquad \text{if } z \to -\infty. \tag{48}$$

It is to be noticed that relation (46) is a necessary but not sufficient condition for the existence of the lossy terms expressed in approximations (47) and (48). In some special cases the amplitudes $\mathbf{P}_{p,q}^{g-}$, $\mathbf{P}_{p,q}^{g+}$ can vanish for reasons of symmetry. If the field components of the modes are odd along the x or the y axis, for example, they cannot be scattered along the normal to the structure. In other terms, in this case the period is chosen such that $\alpha_p^g = \beta_p^g = 0$ but $\mathbf{P}_{p,q}^{g+} = \mathbf{P}_{p,q}^{g-} = 0$, owing to the symmetry properties of the mode.

We now compare the asymptotic expressions of the mode given by approximations (47) and (48) with the asymptotic expansion of the total field generated by the same structure illuminated by plane waves whose wave vector is given by $\alpha = \alpha_p^g$ and $\beta = \beta_p^g$ [approximations (11) and (20)]: It turns out that the mode is identical to the total field, provided that the amplitudes $\mathbf{p}^{i+}$ and $\mathbf{p}^{i-}$ of the incident waves are taken equal to zero. In other words, the mode has the same expression as a scattered field that would exist without any incident field for spatial frequencies (α_p^g, β_p^g). This property is not surprising, since like a guided wave, a scattered field satisfies the outgoing wave condition at infinity. The important consequence of this observation is that in the conditions where the guided wave exists, the diffracted column matrix D of Eq. (24) is different from 0 while the incident column matrix I vanishes. In other words, the scattering matrix S has a pole. In general, this property means not only that the determinant of S has a pole but also that all the coefficients of the S matrix have the same pole, as shown in the study of grating anomalies.[1,2] From a mathematical point of view, if the propagation constants α and β of the incident waves are fixed, the S matrix is a function of the real wave number k. This function has a unique analytic continuation in the complex plane of k, at least in the vicinity of the real axis. The complex wave number $k^{g,\text{perturb}}$ is the pole of this analytic continuation. As a consequence, l_1^S, one of the four eigenvalues l_i^S of the S matrix, has the same pole, the corresponding eigenvector V_1^S being associated with the projections of the amplitudes $\mathbf{P}_{p,q}^{g+}$ and $\mathbf{P}_{p,q}^{g-}$ on the polarization vectors for the scattered fields defined by Eqs. (13) and (15):

$$V_1^S = (P^{g+,s}, P^{g+,p}, P^{g-,s}, P^{g-,p}). \tag{49}$$

In the general case, for $k = k^{g,\text{perturb}}$, any incident wave with propagation constants α_0^g and β_0^g generates an infinite scattered field proportional to V_1^S, whatever its polarization or propagation medium (air or substrate). It can be deduced from this remark that for $k = k^{g,\text{perturb}}$, one eigenvalue of each of the submatrices R_1, R_2, T_1, T_2 has a pole, and the corresponding eigenvector of R_1 and T_1 is equal to $V_1^{S+} = (P^{g+,s}, P^{g+,p})$ while the corresponding eigenvector of R_2 and T_2 is equal to $V_1^{S-} = (P^{g-,s}, P^{g-,p})$.

Finally, the continuity between the fields in the planar and the perturbed waveguide allows us to establish the existence of roots of the determinant of the submatrices R_1, R_2, T_1, T_2. With this aim, let us define a perturbation parameter p such that $p = 0$ for the planar waveguide, increasing values of p corresponding to greater perturbations. Obviously, when the propagation constants (α^g, β^g) are fixed,

$$k^{g,\text{perturb}}(p) \to k^{g,\text{plan}} \quad \text{if } p \to 0. \tag{50}$$

Furthermore, let us express mathematically that each of the submatrices R_1, R_2, T_1, T_2 has a pole for $k = k^{g,\text{perturb}}(p)$. For instance, for T_1, let us call $l_1^{T_1}$ the singular eigenvalue associated with the eigenvector $V_1^{T_1} = V_1^{S+}$:

$$l_1^{T_1}(k, p) = \frac{l_1^{\sim T_1}(k, p)}{k - k^{g,\text{perturb}}(p)}, \tag{51}$$

with $l_1^{\sim T_1}(k^{g,\text{perturb}}(p), p) \neq 0$. When p tends to 0 (planar waveguide), $T_1(k)$ tends to a diagonal matrix that can be expressed in closed form and that is bounded, even when $k = k^{g,\text{perturb}}(0) = k^{g,\text{plan}}$, where the eigenvectors (1, 0) and (0, 1) represent the s and p polarizations, respectively. From relations (51) and (50), the expression of the eigenvalue for $p = 0$ is given by

$$l_1^{T_1}(k, 0) = \frac{l_1^{\sim T_1}(k, 0)}{k - k^{g,\text{plan}}}, \tag{52}$$

and thus

$$l_1^{\sim T_1}(k, 0) = (k - k^{g,\text{plan}}) l_1^{T_1}(k, 0) \tag{53}$$

It emerges from Eq. (53) that $l_1^{\sim T_1}(k, 0)$ has a root for $k = k^{g,\text{plan}}$. By reason of continuity, it can be conjectured that $l_1^{T_1}(k, p)$ has a root for $k = k_1^{T_1,\text{root}}(p)$ such that

$$k_1^{T_1,\text{root}}(0) = k^{g,\text{plan}} = k^{g,\text{rperturb}}(0). \tag{54}$$

Thus the expression of the eigenvalue becomes

$$l_1^{T_1}(k, p) = u(k, p) \frac{k - k_1^{T_1,\text{root}}(p)}{k - k^{g,\text{perturb}}(p)}. \tag{55}$$

Since the pole of the T_1 matrix is unique, at least at the vicinity of $k^{g,\text{plan}}$, the root of $l_1^{T_1}(k, p)$ is unique, too. Indeed, when p tends to 0, the existence of a second root would entail the existence of a root of the eigenvalue $u(k, 0)$ of the transmission matrix of the planar waveguide. For the same reason, the second eigenvalue $l_2^{T_1}(k, p)$ has no pole and zero at the vicinity of $k^{g,\text{plan}}$. In conclusion, $u(k, p)$ and $l_2^{T_1}(k, p)$ are analytic functions with no pole and no root in the vicinity of $k^{g,\text{plan}}$.

The same reasoning can be used for the other submatrices that have the same pole $k^{g,\text{perturb}}(p)$ for the first eigenvalue as T_1 but other roots $k_1^{R_1,\text{root}}$, $k_1^{R_2,\text{root}}$, $k_1^{T_2,\text{root}}$. When p tends to 0, the pole and the roots of the eigenvalues tend to the same point $k^{g,\text{plan}}$ of the complex plane, and thus the eigenvalues take a bounded value.

5. PROPERTY OF SELECTIVE FILTERING OF SYMMETRICAL STRUCTURES

Now we will show that adequate symmetries of the structure entail very important properties of the roots of the eigenvalues, which become real. First, we will examine some consequences of the existence of a root of T_1. With this aim, we recall two mathematical properties of analytic functions of the complex variable:

- The analyticity of the complex function of the complex variable $f(z)$ entail the analyticity of $\bar{f}(\bar{z})$.
- An analytic function that vanishes on a segment of the real axis vanishes in a domain of analyticity containing this segment.

By using these two properties, the unitarity of the S matrix expressed by Eq. (26) for real values of k can be extended to the complex plane,

$$S^*(\bar{k}) S(k) = 1, \tag{56}$$

and, by using the reflection and transmission submatrices,

$$R_1^*(\bar{k}) R_1(k) + T_1^*(\bar{k}) T_1(k) = 1, \tag{57}$$

$$T_2^*(\bar{k}) T_2(k) + R_2^*(\bar{k}) R_2(k) = 1, \tag{58}$$

$$R_1^*(\bar{k}) T_2(k) + T_1^*(\bar{k}) R_2(k) = 0, \tag{59}$$

$$T_2^*(\bar{k}) R_1(k) + R_2^*(\bar{k}) T_1(k) = 0. \tag{60}$$

Since

$$T_1(k_1^{T_1,\text{root}}) V_1^{T_1,\text{root}} = 0, \tag{61}$$

it emerges from Eq. (60) that

$$T_2^*(\bar{k}_1^{T_1,\text{root}}) R_1(k_1^{T_1,\text{root}}) V_1^{T_1,\text{root}} = 0, \tag{62}$$

and since Eq. (57) shows that $T_1(k_1^{T_1,\text{root}}) V_1^{T_1,\text{root}}$ and $R_1(k_1^{T_1,\text{root}}) V_1^{T_1,\text{root}}$ cannot vanish together, it turns out that the conjugate of the root $k_1^{T_1,\text{root}}$ of T_1 is the root of T_2^*, the associated eigenvector being equal to $R_1(k_1^{T_1,\text{root}}) V_1^{T_1,\text{root}}$. In the case in which the diffractive structure is symmetrical with respect to the z axis, it has been established from Eq. (37) that S is symmetrical and thus that

$$T_2 = t(T_1). \tag{63}$$

Using Eqs. (62) and (63), it comes out that $\bar{k}_1^{T_1,\text{root}}$ is a zero of T_1, with an associated eigenvector $\bar{R}_1(k_1^{T_1,\text{root}}) \bar{V}_1^{T_1,\text{root}}$. Since we have shown that the root of T_1 is unique in the vicinity of $k^{g,\text{plan}}$, we are led to the conclusion that $k_1^{T_1,\text{root}}$ is real and that

$$R_1(k_1^{T_1,\text{root}}) V_1^{T_1,\text{root}} = \bar{V}_1^{T_1,\text{root}}. \tag{64}$$

This interesting result can be expressed in the following way: When the diffractive structure is symmetrical with respect to the z axis, there exists a real wavelength $\lambda = 2\pi/k_1^{T_1,\text{root}}$ and a polarization of the incident wave propagating in the air such that the reflected energy is equal to the incident one, the transmitted energy being rigorously equal to zero. The polarization of this incident wave is obtained by identifying the incident column matrix given by Eqs. (21) with the eigenvector $V_1^{T_1,\text{root}}$ of the T_1 matrix associated with the eigenvalue that is equal to zero. In these conditions, the diffracted column matrix D^+ is equal to $\bar{V}_1^{T_1,\text{root}}$. In the following, these conditions will be called total-reflection configuration.

Now, let us study what happens when, starting from the total-reflection configuration, the polarization or the wavelength of the incident wave is changed. We assume that the polarization is linear and that the incident energy $\langle I^+ | I^+ \rangle$ is equal to unity (in this paper, $\langle V|U\rangle = \bar{V}_1 U_1 + \bar{V}_1 U_1$ is the Hermitian scalar product and should not be confused with the Euclidian product $\langle V, U\rangle = V_1 U_1 + V_2 U_2$ defined in Section 3). Then the reflected energy ρ is given by

$$\rho = \langle D^+ | D^+ \rangle = \langle R_1 I^+ | R_1 I^+ \rangle = \langle R_1^* R_1 I^+ | I^+ \rangle. \tag{65}$$

Thus we are led to the study of the Hermitian reflection matrix $R_1^* R_1$, which has two real and positive eigenvalues $l_1^{R_1^*R_1}(k,p)$ and $l_2^{R_1^*R_1}(k,p)$ associated with two orthogonal eigenvectors $V_1^{R_1^*R_1}(k,p)$ and $V_2^{R_1^*R_1}(k,p)$. First, we show that when $k = k_1^{T_1,\text{root}}(p)$, one of the eigenvectors [$V_1^{R_1^*R_1}(k,p)$ by convention] is identical to $V_1^{T_1,\text{root}}$, the associated eigenvalue $l_1^{R_1^*R_1}(k,p)$ being equal to 1. It suffices to remember that when the diffractive structure is symmetrical with respect to the z axis, the S matrix (and thus the R_1 matrix) is symmetrical, which entails that $R_1^* = \bar{R}_1$. Using Eq. (64) and then the conjugate equation, we get, for $k = k_1^{T_1,\mathrm{root}}(p)$,

$$R_1^* R_1 V_1^{T_1,\mathrm{root}} = V_1^{T_1,\mathrm{root}}. \quad (66)$$

The $R_1^* R_1$ matrix has two poles [$k = k^{g,\mathrm{perturb}}(p)$ due to R_1 and $k = \bar{k}^{g,\mathrm{perturb}}(p)$ due to R_1^*] and two conjugate and complex roots. These poles and roots are present in the first eigenvalue $l_1^{R_1^* R_1}(k, p)$, while the second one, $l_2^{R_1^* R_1}(k, p)$, which has no pole and no root, is not sensitive to the excitation of the guided mode when k is close to $k^{g,\mathrm{perturb}}(p)$.

Mathematically, the reflected energy can be written in the form

$$\rho = l_1^{R_1^* R_1}(k, p) |\langle I^+ | V_1^{R_1^* R_1}(k, p)\rangle|^2 + l_2^{R_1^* R_1}(k, p) |\langle I^+ | V_2^{R_1^* R_1}(k, p)\rangle|^2. \quad (67)$$

Using the following notation

$$V_1^{R_1^* R_1}(k, p) = [\cos q, \sin q \exp(i\phi)], \quad (68)$$

taking into account the orthogonality of $V_1^{R_1^* R_1}(k, p)$ and $V_2^{R_1^* R_1}(k, p)$, and bearing in mind that the incident wave is unitary and linearly polarized,

$$I^+ = [\cos(\delta), \sin(\delta)], \quad (69)$$

we can make a straightforward calculation showing from Eq. (67) that

$$\rho = \frac{l_1^{R_1^* R_1}(k, p) + l_2^{R_1^* R_1}(k, p)}{2} + \frac{l_1^{R_1^* R_1}(k, p) - l_2^{R_1^* R_1}(k, p)}{2} \times \tau \cos(2\delta - \psi), \quad (70)$$

$$\tau = [\cos(2q)^2 + \sin(2q)^2 \cos(\phi)^2]^{1/2}, \quad (71)$$

$$\tan(\psi) = \tan(2q)\cos(\phi). \quad (72)$$

When $k = k_1^{T_1,\mathrm{root}}(p)$, the reflected energy oscillates sinusoidally as the polarization angle δ is varied, and its maximum value

$$1 - \frac{1 - l_2^{R_1^* R_1}(k, p)}{2}(1 - \tau)$$

and minimum value

$$1 - \frac{1 - l_2^{R_1^* R_1}(k, p)}{2}(1 + \tau)$$

are obtained when $\delta = \psi/2$ and $\delta = (\psi + \pi)/2$, respectively. We point out that Eq. (70) is a general and interesting result that is not restricted to resonant gratings.

Now, if p is small, we can conjecture that the eigenvalues of $R_1^* R_1$ (and therefore the reflectivity) are close to that of the planar structure when k is taken far enough from $k_1^{T_1,\mathrm{root}}(p)$. In the vicinity of $k_1^{T_1,\mathrm{root}}(p)$, the eigenvalue $l_1^{R_1^* R_1}(k, p)$ has two poles and two roots. Hence its value increases from that of the planar waveguide to unity when k tends to $k_1^{T_1,\mathrm{root}}(p)$. It is important to understand that if the incident wave has the same polarization as the eigenvector of $R_1^* R_1$, the reflectivity of the structure is identical to the corresponding eigenvalue. Thus it varies from the reflectivity of the planar waveguide to unity. And if the planar waveguide is a poor reflector, the perturbed waveguide will constitute a high-efficiency filter for a given polarized light. On the other hand, for the orthogonal polarization, the structure will reflect the incident light as the planar waveguide.

It can be deduced from these considerations that it is quite impossible to use the grating structure as a selective frequency filter with an efficiency close to unity for unpolarized light if only one mode is excited.

6. NUMERICAL VERIFICATION AND APPLICATIONS

To illustrate the conclusions of the phenomenological approach, we consider a two-dimensional resonant grating that supports one pseudoperiodic eigenmode for the real spatial frequencies (α, β) [and subsequent harmonics (α_n, β_m)] and complex wave number k_g. In this example, the grating is shallow and both one harmonic of the spatial eigenfrequencies and the wave number of the mode are close to those of the TE guided wave that would exist in the unperturbed planar layer.

We study the reflectivity of the structure when the spatial frequencies of the incident beam are fixed to (α, β) and when the wave number is varied in the vicinity of the real part of k_g for various incident polarizations. The grating is symmetrical with respect to the z axis with square cell and bumps. We observe in Fig. 3(a) that the reflectivity in both s and p polarization presents sharp peaks with different maximum values. In Fig. 3(b) we study the eigenvalues of the $R_1^* R_1$ matrix as a function of the wave number. As expected, one eigenvalue $l_1^{R_1^* R_1}$ peaks at unity for $\lambda = \lambda_0$ while the other one, $l_2^{R_1^* R_1}$, is not modified in the vicinity of λ_0. The eigenvector corresponding to the first eigenvalue is linearly polarized with $\delta = 68°$ at λ_0. As a result, the reflectivity of the system at λ_0 oscillates between one and $l_2^{R_1^* R_1}$ when the incident linear polarization vector is rotated in the $(\hat{s}^{i+}, \hat{p}^{i+})$ plane, as shown in Fig. 3(c). Note that the eigenvector of the R_1 matrix at a frequency close to the resonance condition describes the polarization state of the electric field scattered in free space by the periodic defects of the waveguide. These defects are, in first approximation, illuminated by the eigenmode. When the eigenmode is close to a TE or a TM guided wave, the electric field of the mode is linearly polarized. It can be shown with a perturbation method that the scattered field is then also linearly polarized, a fact that is confirmed by our numerical results. Hence the eigenvector of the R_1 matrix is real, is equal to that of the $R_1^* R_1$ matrix, and corresponds to a linear polarization.

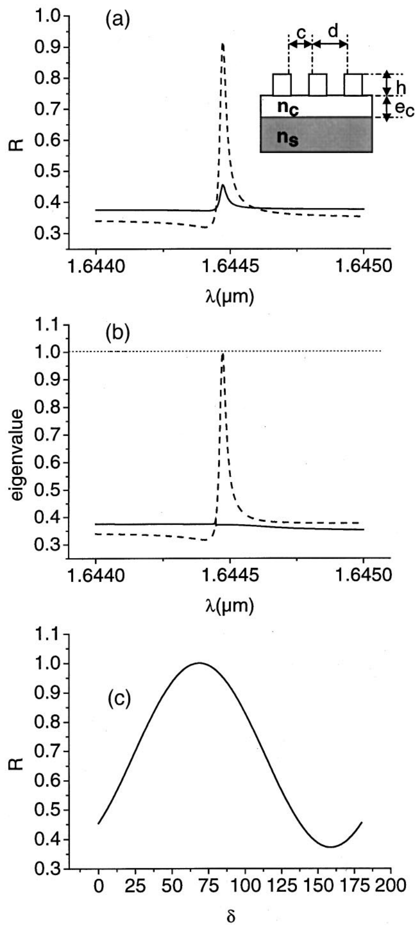

Fig. 3. Two-dimensional grating with square bumps, refractive indices $n_s = 1.5$, $n_c = 2.5$, thickness $e_c = 133$ nm, bump height $h = 7$ nm, periods $d_x = d_y = d = 930$ nm, and bump width $d - c = 465$ nm. The index of the bumps is equal to n_c. The incident parameters are $\theta = 15°$ and $\phi = 28°$. (a) Reflection factor versus wavelength for both s (solid curve) and p (dashed curve) polarizations. (b) First (solid curve) and second (dashed curve) eigenvalues of the Hermitian reflection matrix versus wavelength. (c) Reflection factor versus angle of polarization δ.

The eigenmode of the periodic structure is not always close to one propagating guided wave of the planar system; it can also stem from a combination of guided waves. We consider as an example a one-dimensional lamellar grating illuminated under purely conical mounting (β, $\alpha = 0$). The period d_x of the grating is chosen such that at the given wave number k_g^{plan}, two TE guided waves propagating along different directions with wave vectors $(\beta, -2\pi/d)$ $(\beta, 2\pi/d)$ can be excited in the planar waveguide [see Fig. 4(a)]. In other words, when the periodic perturbation disappears, the homogeneous problem has two solutions for the same spatial (β, $\alpha = 0$) and temporal frequencies. As soon as a perturbation is introduced, these two guided waves are coupled to each other, and as a result, one obtains two eigenmodes whose field components are either odd or even with respect to *Oy* (due to the symmetry of the structure) and whose complex wave number k_g^s and k_g^p are in general different (since the field repartition of the modes and their electromagnetic energy are not the same in general[9]). In this example, the eigenmodes of the resonant grating resemble standing waves with different symmetry properties. When the x component of the electric field of the mode is odd [upper display in Fig. 4(a)], at the wave number k_g^p its scattered field in the $(\beta, 0)$ direction is p polarized, whereas it is s polarized when the y component of the field is odd [lower display in Fig. 4(a) corresponding to k_g^s]. From the phenomenological analysis, one expects to get a sharp peak of reflectivity culminating at 100% for an s-polarized incident beam when the wave number gets close to k_g^s and another one for a p-polarized incident beam when the wave number nears k_g^p. In Fig. 4(b) we plot the reflectivity as a function of the wave number in s and p polarization. In this example, the eigenvectors of the $R_1^* R_1$ matrix are (1, 0) and (0, 1) whatever the wavelength; they represent the linear s and p polarization states. Hence the eigenvalues of the $R_1^* R_1$ matrix are identical to the reflectivity for these two polarizations. In other words, there is no depolarization. Because the peaks of reflectivity appear for different wave numbers in s and p polarization, this structure cannot be used as a filter for unpolarized light, as seen in Fig. 4(c). Moreover, the widths of the peaks, cor-

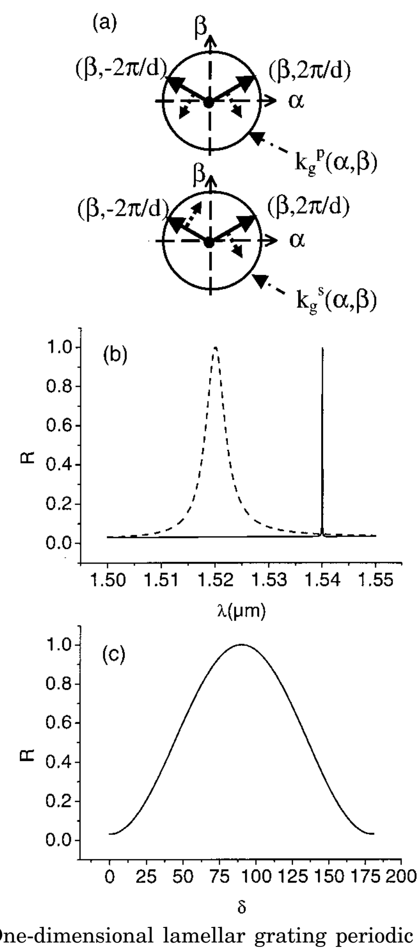

Fig. 4. One-dimensional lamellar grating periodic along the x axis, with refractive indices $n_s = 1.5$, $n_c = 2.07$, thickness $e_c = 300$ nm, groove depth $h = 87.5$ nm, period $d_x = 904$ nm, and groove width $c = 226$ nm. The incident parameters are $\theta = 7.85°$, $\phi = 90°$. (a) Solid arrows, scheme of the wave vectors of the guided waves of the planar waveguide; dashed arrows, direction of their associated electric field. (b) Reflection factor versus wavelength for both s (solid curve) and p (dashed curve) polarizations. (c) Reflection factor versus the angle of polarization δ at $\lambda = 1.52$ μm.

responding to the imaginary part of the wave number of the modes, are generally very different. However, one can modify the real and imaginary parts of the wave numbers by the parameters of the grating and bring together these two resonances.[6] Indeed, in this particular case the modes can exist together for the same spatial and temporal frequencies, since the coupling between them is impossible for symmetry reasons. In other words, the reluctance of resonances to merge is avoided.

We have shown that as long as only one eigenmode exists for given spatial frequencies (α, β) in a certain domain of wavelength, one can always find an incident polarization state that will not be modified by the presence of the mode. Hence, for creation of a filter for unpolarized light, the structure should support two eigenmodes at the same frequency. This degeneracy is possible only if the modes cannot couple to each other (i.e., are orthogonal). It is possible to obtain a degeneracy, for example, with one-dimensional or two-dimensional gratings with symmetry properties that present odd and even modes at given spatial frequencies (α, β).[6,7] Yet obtaining the same complex wave number for these two orthogonal modes is quite difficult and requires one to adjust the parameters of the grating precisely.

On the other hand, it is not hard to obtain an unpolarized filter with a two-dimensional grating with square cell and bump illuminated under normal incidence.[5] Symmetry considerations can explain easily why this kind of structure yields the same reflectivity whatever the polarization under normal incidence. The two-dimensional grating is invariant under a rotation of $\pi/2$. As a result, the reflectivity of a linear polarized beam, at normal incidence, is the same for an angle of polarization δ and $\delta + \pi/2$. To reconcile this property with Eq. (70), we see that the reflectivity must be a constant whatever the polarization state. Moreover, since the grating is symmetrical with respect to the z axis, there exists a particular wavelength for which the reflectivity reaches 100%. Similar reasoning can be done with any two-dimensional grating whose cell is invariant under a rotation of π/n, for example, a hexagonal cell. Among the properties coming from symmetry considerations lies the fact that in the frequency domain close to the maximum of reflectivity, the structure supports two orthogonal eigenmodes with the same spatial frequencies (0, 0) and the same complex wave number. When the grating is shallow, one expects these two modes to stem from even standing waves that are a symmetric combination of guided waves that propagate in opposite directions (with wave vectors directed either along Ox or Oy). Note that the antisymmetric combination, which leads to odd modes, cannot appear as eigenvectors of the S matrix calculated for ($\alpha = 0, \beta = 0$) since the only propagating field component in their plane-wave expansion is null. Yet odd modes appear when one departs slightly from normal incidence. In Fig. 5 we plot the eigenvalues of $R_1^* R_1$ as a function of the wavelength. As expected, the two eigenvalues are identical, and they present a peak that reaches unity. The eigenvectors are (1, 0) and (0, 1) so that the structure conserves the polarization.

Up to now, we have considered gratings that are symmetric with respect to the z axis. As a result, the peak of reflectivity reaches 100% for a given polarization state. If asymmetric gratings are used, this property is not always verified (note, however, that the symmetry condition is sufficient but not necessary). In all previous examples, the eigenvectors of the $R_1^* R_1$ matrix correspond to a linear polarization (i.e., they are real). This is not always the case. We have studied a shallow one-dimensional grating that is asymmetric with respect to the z axis and illuminated under purely conical mounting. We have found that the eigenvectors of the $R_1^* R_1$ matrix, which are generally real and correspond to s or p polarization, become complex in the vicinity of a resonance. This means that the eigenmodes of the structure are elliptically polarized. By modifying the asymmetry of the grating, we can change the eigenvectors so that they correspond to a circular polarization. In this case, the reflectivity of the system is roughly a constant whatever the incident *linear* polarization state. However, since only one mode is excited, this reflectivity cannot be close to unity.

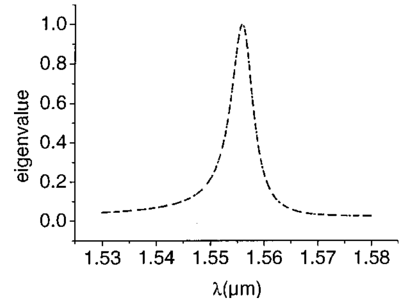

Fig. 5. First (solid curve) and second (dashed curve) eigenvalues versus wavelength for a two-dimensional grating with square bumps, illuminated in normal incidence, with refractive indices $n_s = 1.448$, $n_c = 2.07$, thickness $e_c = 300$ nm, bump height $h = 87.5$ nm, period $d = 900$ nm, and bump width $d - c = 600$ nm. The two curves are identical.

7. CONCLUSION

We have presented a phenomenological study of resonant gratings that permits one to describe the behavior of the reflectivity as a function of the wavelength and the incident polarization state in the vicinity of a resonance. We have considered the Hermitian reflection matrix of order 2 that links the s and p amplitudes of the reflected order to the s and p amplitudes of the incident wave.

We have shown that when only one eigenmode exists in the structure close to the spatial and temporal frequencies of the incident beam, one (and only one) eigenvalue of the reflection matrix presents a complex pole and a complex root in the complex plane of the wave number. Thus in this case there exists an incident polarization for which the reflectivity is not modified by the mode and presents no anomaly. Furthermore, we have found that if the grating is symmetrical with respect to the z axis, there exists one incident polarization for which the reflectivity presents a peak that reaches 100% as a function of the wavelength, while the reflectivity for the orthogonal polarization remains a gently varying function (that is close to that of the planar structure if the periodic modulation is small). Hence, in order to design a high-efficiency filter for unpolarized light, one must consider gratings that support two eigenmodes for the same spatial and temporal frequencies.

Our approach was supported by various numerical experiments with one-dimensional and two-dimensional gratings. The approach permits one to retrieve the behavior of the reflectivity of the resonant grating as a function of the wavelength and the incident polarization state from the knowledge of very few parameters such as the complex pole, the transmission root, the eigenvectors, and the Fresnel factors of the planar structure. A perturbative approach could be used to derive analytical expressions for these various parameters. Our work is in progress in that direction.

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