Electromagnetic forces on a discrete spherical invisibility cloak under time-harmonic illumination

Patrick C. Chaumet,[1] Adel Rahmani,[2] Frédéric Zolla,[1] and André Nicolet[1]
[1]*Institut Fresnel, CNRS, Aix-Marseille Université, Campus de St-Jérôme 13013 Marseille, France*
[2]*School of Mathematical Sciences, University of Technology, Sydney, Broadway NSW 2007, Australia*

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We study electromagnetic forces and torques on a discrete spherical invisibility cloak under time-harmonic illumination. We consider the influence of material absorption and losses, and we show that while the impact of absorption on the optical force remains confined to frequencies near the absorption peak, its impact on the electromagnetic torque experienced by the cloak is spectrally broader and follows the spectrum of the absorption cross section of the cloak. We also investigate the mechanical shielding of a test particle within the cloak. We find that even an imperfect cloak can reduce the radiation pressure on the particle significantly; however, under certain conditions the force on the particle can be stronger than it would be in the absence of the cloak.

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I. INTRODUCTION

Recent studies have focused on the electromagnetic forces experienced by an electromagnetic cloak in either the frequency domain (time-harmonic regime) or the time domain. For the time-harmonic case, it has been shown that the optical force on an *ideal* electromagnetic cloak is exactly zero [1,2] and that the cloak remains at rest. On the other hand, in the time domain, an electromagnetic cloak, initially at rest and illuminated by an electromagnetic pulse, would experience a nonzero electromagnetic force and hence move while it interacts with the pulse. By the time the pulse has died out, however, the cloak would have acquired a zero net momentum and experienced a zero net displacement [3].

In this article we deal with the time-harmonic regime, and we study how a nonideal cloak interacts optomechanically with an electromagnetic wave. Any actual cloaking device, even in the absence of fabrication imperfections, can achieve only partial cloaking due to two main factors: material dispersion and absorption, and the composite (i.e., discrete) geometry of the metamaterial forming the cloak. Although the effects of dispersion and absorption are sometimes taken into account in the definition of the permittivity and permeability of the cloak, the discrete geometry is more difficult to account for and therefore seldom considered. Our goal is to study the effect of the composite geometry of the cloak on optical forces, without having to worry about the specifics of the metamaterial implementation used to describe the cloak. To this end we use a generic model of a discrete (i.e., composite) cloak based on the discrete dipole approximation (DDA) [4–6]. In the DDA formalism, the computation of the electromagnetic force on a discrete cloak amounts to computing the force on a collection of anisotropic magnetodielectric particles. Recent studies have addressed optical forces on isotropic magnetodielectric particles [7–9]. In this article we start by briefly describing, in Sec. II, how particles with tensorial permittivity and permeability can be handled in optical force calculations. In Sec. III we present the results obtained for a discretized cloak. We also discuss the influence of material dispersion and absorption on the electromagnetic force experienced by the cloak, and we investigate how well a particle inside the cloak is protected from optical forces. In Sec. IV we present our conclusion.

II. OPTICAL FORCES AND TORQUES ON AN ANISOTROPIC, MAGNETODIELECTRIC OBJECT

Consider an object whose electromagnetic properties are described by position-dependent permittivity and permeability tensors. We discretize the object into a set of N polarizable subunits over a cubic lattice with period d [10,11]; i.e., d is the mesh size for the discretization. Each subunit is characterized by an electric polarizability tensor $\boldsymbol{\alpha}^{\rm e}$ and a magnetic polarizability tensor $\boldsymbol{\alpha}^{\rm m}$. When the object is illuminated by an incident electromagnetic field $(\mathbf{E}^{\rm inc},\mathbf{H}^{\rm inc})$ the local fields at a subunit located at $\mathbf{r}_i$ can then be written as

$$\mathbf{E}(\mathbf{r}_i,\omega) = \mathbf{E}^{\rm inc}(\mathbf{r}_i,\omega) + \sum_{j=1}^{N}[\mathbf{T}^{\rm ee}(\mathbf{r}_i,\mathbf{r}_j,\omega)\boldsymbol{\alpha}^{\rm e}(\mathbf{r}_j,\omega)\mathbf{E}(\mathbf{r}_j,\omega) + \mathbf{T}^{\rm em}(\mathbf{r}_i,\mathbf{r}_j,\omega)\boldsymbol{\alpha}^{\rm m}(\mathbf{r}_j,\omega)\mathbf{H}(\mathbf{r}_j,\omega)], \quad (1)$$

$$\mathbf{H}(\mathbf{r}_i,\omega) = \mathbf{H}^{\rm inc}(\mathbf{r}_i,\omega) + \sum_{j=1}^{N}[\mathbf{T}^{\rm me}(\mathbf{r}_i,\mathbf{r}_j,\omega)\boldsymbol{\alpha}^{\rm e}(\mathbf{r}_j,\omega)\mathbf{E}(\mathbf{r}_j,\omega) + \mathbf{T}^{\rm mm}(\mathbf{r}_i,\mathbf{r}_j,\omega)\alpha^{\rm m}(\mathbf{r}_j,\omega)\mathbf{H}(\mathbf{r}_j,\omega)], \quad (2)$$

where ω is the angular frequency, and the quantities labeled $\mathbf{T}$ are free-space field susceptibility tensors [7,12]. For the sake of brevity we will omit the dependence on ω henceforth. The polarizabilities of the subunits, in the anisotropic case, are defined from the Clausius-Mossotti relation:

$$\boldsymbol{\alpha}_0^{\rm e}(\mathbf{r}_i) = \frac{3d^3}{4\pi}[\boldsymbol{\varepsilon}(\mathbf{r}_i)-\mathbf{I}][\boldsymbol{\varepsilon}(\mathbf{r}_i)+2\mathbf{I}]^{-1}, \quad (3)$$

$$\boldsymbol{\alpha}_0^{\rm m}(\mathbf{r}_i) = \frac{3d^3}{4\pi}[\boldsymbol{\mu}(\mathbf{r}_i)-\mathbf{I}][\boldsymbol{\mu}(\mathbf{r}_i)+2\mathbf{I}]^{-1}. \quad (4)$$

Taking radiation reaction [5] into account and introducing the wave number k_0, the polarizabilities tensors in Eqs. (1) and (2) can be written as

$$\boldsymbol{\alpha}^{\rm e}(\mathbf{r}_j) = \left[\mathbf{I}-(2/3)ik_0^3\boldsymbol{\alpha}_0^{\rm e}(\mathbf{r}_i)\right]^{-1}\boldsymbol{\alpha}_0^{\rm e}(\mathbf{r}_i), \quad (5)$$

$$\boldsymbol{\alpha}^{\rm m}(\mathbf{r}_j) = \left[\mathbf{I}-(2/3)ik_0^3\boldsymbol{\alpha}_0^{\rm m}(\mathbf{r}_i)\right]^{-1}\boldsymbol{\alpha}_0^{\rm m}(\mathbf{r}_i). \quad (6)$$

Ultimately, the scattering problem is cast as a $6N\times 6N$ linear system, which can be solved for the electric and magnetic fields inside the object. For large objects (with respect to the wavelength), the linear system should be solved iteratively using a solver adapted to the magnetodielectric nature of the object [13]. Once the fields inside the object are known, the fields anywhere outside the object can be calculated by adding the contributions of all the subunits to the scattered field. Since we are not merely interested in the scattered fields but in the electromagnetic force and torque as well, we also need the spatial derivatives of the fields. These spatial derivatives at subunit i are obtained as [14]

$$\nabla\mathbf{E}(\mathbf{r}_i) = \nabla\mathbf{E}^{\mathrm{inc}}(\mathbf{r}_i) + \sum_{j=1}^{N}[\nabla\mathbf{T}^{\mathrm{ee}}(\mathbf{r}_i,\mathbf{r}_j)\boldsymbol{\alpha}^{\mathrm{e}}(\mathbf{r}_j)\mathbf{E}(\mathbf{r}_j) + \nabla\mathbf{T}^{\mathrm{em}}(\mathbf{r}_i,\mathbf{r}_j)\boldsymbol{\alpha}^{\mathrm{m}}(\mathbf{r}_j)\mathbf{H}(\mathbf{r}_j)], \tag{7}$$

$$\nabla\mathbf{H}(\mathbf{r}_i) = \nabla\mathbf{H}^{\mathrm{inc}}(\mathbf{r}_i) + \sum_{j=1}^{N}[\nabla\mathbf{T}^{\mathrm{me}}(\mathbf{r}_i,\mathbf{r}_j)\boldsymbol{\alpha}^{\mathrm{e}}(\mathbf{r}_i)\mathbf{E}(\mathbf{r}_i) + \nabla\mathbf{T}^{\mathrm{mm}}(\mathbf{r}_i,\mathbf{r}_j)\boldsymbol{\alpha}^{\mathrm{m}}(\mathbf{r}_j)\mathbf{H}(\mathbf{r}_j)]. \tag{8}$$

Once the local field and their spatial derivatives are known at each subunit, the kth Cartesian component of the optical force experienced by the ith subunit of the discretized object can be written as (repeated indices are summed over) [7]

$$F^k(\mathbf{r}_i) = \frac{1}{2}\mathrm{Re}\left\{p^l(\mathbf{r}_i)\partial^k[E^l(\mathbf{r}_i)]^* + m^l\partial^k[H^l(\mathbf{r}_i)]^* - \frac{2k_0^4}{3}\epsilon^{kln}p^l(\mathbf{r}_i)[m^n(\mathbf{r}_i)]^*\right\}, \tag{9}$$

where ϵ^{kln} is the Levi-Civita tensor, k, l, or n stands for either x, y, or z, and $*$ denotes the complex conjugate of a complex variable. As discussed in Ref. [7] the third term in Eq. (9) becomes negligible if the object under study is discretized in small enough subunits.

To compute the optical torque we must add up the *intrinsic* and *extrinsic* torques experienced by each subunits. However, as emphasized in Refs. [15–17], to satisfy the conservation of angular momentum one should include the radiation reaction term in the intrinsic part of the optical torque. Hence the optical torque, on an anisotropic subunit, can be written as

$$\boldsymbol{\Gamma}(\mathbf{r}_i) = \mathbf{r}_i \times \mathbf{F}(\mathbf{r}_i) + \tfrac{1}{2}\mathrm{Re}\{\mathbf{p}(\mathbf{r}_i) \times [\mathbf{E}(\mathbf{r}_i) + \tfrac{2}{3}ik_0^3\mathbf{p}(\mathbf{r}_i)]^* + \mathbf{m}(\mathbf{r}_i) \times [\mathbf{H}(\mathbf{r}_i) + \tfrac{2}{3}ik_0^3\mathbf{m}(\mathbf{r}_i)]^*\}, \tag{10}$$

where $\frac{2}{3}ik_0^3\mathbf{p}(\mathbf{r}_i)$ and $\frac{2}{3}ik_0^3\mathbf{m}(\mathbf{r}_i)$ are the electric and magnetic reaction fields for a small polarizable particle, respectively [12].

III. RESULTS

Throughout this article we assume that the cloak is illuminated by a plane wave, traveling in the positive z direction with linear or left circular polarization; see Fig. 1. In this section the optical force and torque are normalized to to $4\pi\epsilon_0|E^{\mathrm{inc}}|^2$. Hence optical forces and torques are given in squared meter and cubic meters, respectively.

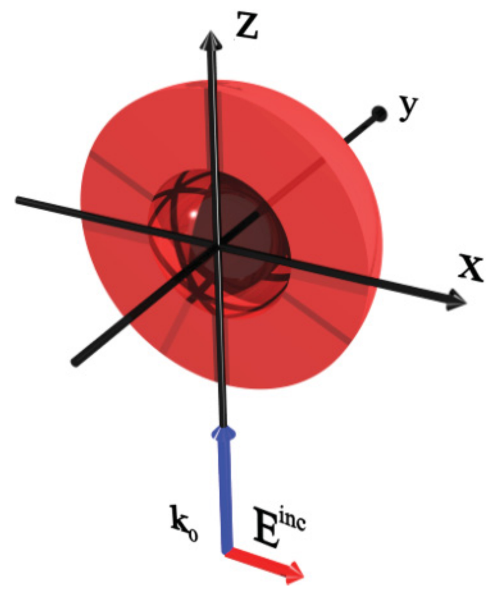

FIG. 1. (Color online) Sketch of the geometry. A cloak with outer radius twice the inner radius. Illumination by a plane wave traveling in the positive z direction and with either linear polarization (electric field along the x axis) or left circular polarization.

A. Optical force and torque on a cloak without material losses

To get a sense of the influence of the discrete geometry of the cloak on optical forces, we first study the optical force on a lossless spherical invisibility cloak illuminated by a plane wave at wavelength λ. For this type of cloak the relative permittivity and permeability can be written as [18]

$$\boldsymbol{\varepsilon}(\mathbf{r}) = \boldsymbol{\mu}(\mathbf{r}) = \frac{a}{a-b}\left(\mathbf{I} - \frac{2br - b^2}{r^4}\mathbf{r}\otimes\mathbf{r}\right), \tag{11}$$

where the cloak is centered on the origin with inner radius b and outer radius a chosen such that $a/b = 2$. We recall that for an ideal cloak (made of continuous, nondispersive, and nonlossy materials) the incident wave experiences no scattering, and there would obviously be no net optical force on the cloak, as was shown recently in Ref. [1].

1. *Influence of the discretization on the force experienced by a discrete cloak*

We now use the DDA to study the influence of the size of the discretization, or equivalently of the number of subunits, on the optical force experienced by the cloak. In Fig. 2(a) the net optical force on the cloak is plotted versus the number of layers of discretization of the cloak ($N_l = 2a/d$, where d is the size of the discretization mesh) for three different radii of the cloak. As N_l increases the net optical force decreases to zero; i.e., the discrete cloak tends toward an idealized, continuous cloak. However, it is interesting to note that even with a coarse discretization the optical cloaking effect is noticeable. To better appreciate this and quantify the cloaking effect, consider the optical force that a homogeneous dielectric sphere of the same size as the cloak but with $\varepsilon = 2.25$ and $\mu = 1$ would experience (see Table I; notice that the computation of the optical force experienced by the sphere is done exactly using a Mie series). For the two smallest radii, a discrete spherical cloak with only $N_l = 16$

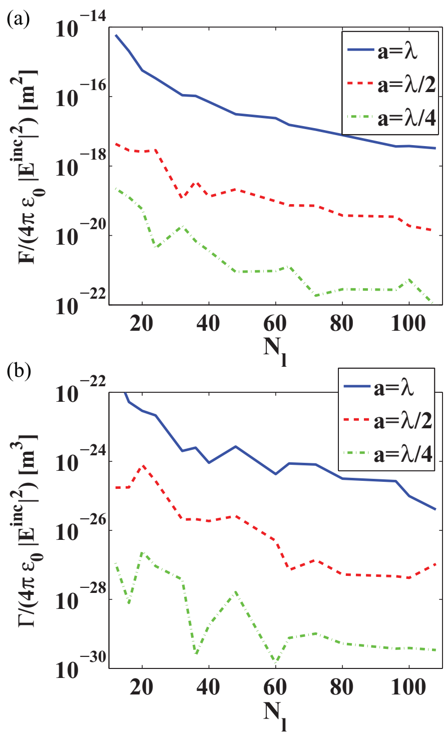

FIG. 2. (Color online) Optical force and torque on a discrete cloak of outer radius a (inner radius $b = a/2$). Solid line $a = \lambda$, dashed line $a = \lambda/2$ and in dash-dotted line $a = \lambda/4$. (a) Optical force on the cloak due to a linearly polarized plane wave versus the number of layer N_l. (b) Optical torque on the cloak due to a left circularly polarized plane wave versus the number of discretization layer N_l.

discretization layers can reduce the optical force by more than three orders of magnitude compared to the homogeneous dielectric sphere. This strong decreases of the electromagnetic force shows that a discrete cloak with this level of discretization is quite effective as a shield against electromagnetic forces. For $a = \lambda$ the larger radius means that we must discretize the cloak more finely ($N_l = 40$) in order to observe the same attenuation of the force compared to the homogeneous sphere (notice that $N_l = 16$ and $N_l = 40$ correspond to 1896 and 29 328 subunits to represent the cloak, respectively).

For a linearly polarized incident plane wave the invisibility cloak experiences a zero optical torque, but this is no longer true for a circularly polarized incident wave [see Fig. 2(b)]. In that case we see that for small values of N_l the cloak experiences an optical torque resulting from the anisotropy of the permittivity [19] and permeability of the cloak. When N_l increases the optical torque vanishes as, once again, the discretized cloak tends toward an ideal (continuous) cloak.

TABLE I. Comparison between the optical force experienced by a homogeneous dielectric sphere ($\varepsilon = 2.25$, $\mu = 1$) and a spherical cloak for different radii and level of discretization for the cloak.

Radius	$a = \lambda/4$	$a = \lambda/2$	$a = \lambda$
Homogeneous sphere	1.1×10^{-15}	1.1×10^{-14}	4.4×10^{-14}
Cloak $N_l = 16$	1.2×10^{-19}	2.8×10^{-18}	2.0×10^{-15}
Cloak $N_l = 40$	3.6×10^{-21}	1.3×10^{-19}	7.0×10^{-17}

2. *Density of optical force and torque inside the cloak*

It is interesting to look at the optical force (torque) on each element of the composite structure as it gives us an insight in the mechanical stress inside the cloak. Moreover, if the optical force (torque) experienced by each element forming the cloak is normalized to its volume, we obtain the density of force (torque) inside the cloak. In Figs. 3(a)–3(c) [Figs. 3(d)–3(f)] we plot the density of force (torque) as a vector field with $N_l = 23$ for $a = \lambda/4$ and $a = \lambda/2$ and $N_l = 41$ for $a = \lambda$. On the same graphs [Figs. 3(a)–3(f)] the color scale represents the density of electromagnetic energy. Figure 3 highlights the fact that the density of energy is uniform outside the cloak and negligible within the cloak. Notice that the energy density is largest near the outer boundary of the cloak. Thus, even if the net optical force experienced by the cloak vanishes as the metamaterial elements become smaller, there remains nonetheless a nonzero density of optical force (DOF), which means that the cloak is subject to mechanical stress, a result in agreement with the study of an ideal, continuous cloak presented in Ref. [1]. Figures 3(a)–3(c) show that for all three sizes of cloaks, the mechanical stress "stretches" the cloak in the z direction and "compresses" it in the x direction. Note that the amount of stress decreases with the radius of the cloak, which can be understood intuitively in terms of the curvature imposed by the cloak on the incident wave.

For the three cloaks the maximum of DOF is more or less the same, i.e., about 2×10^{7} Nm^{-3}. One can see that the stress is stronger close to the interior edge (particularly for the larger cloak) where the density of energy vanishes. Thus, although the density of energy and the density of force are both quadratic forms in the fields, they behave differently. Regarding the density of optical torque, Figs. 3(d)–3(f) shows that it vanishes overall for the cloak, but there remains a nonzero density of optical torque inside the cloak. Notice that one cannot obtain the density of optical torque directly from Figs. 3(a)–3(c) with $\mathbf{r} \times \mathbf{F}$ as one should add the intrinsic part of the optical torque Eq. (10).

3. *Optical force on a particle inside the cloak*

In this section we study the optical force experienced by a minute test particle of radius $\lambda/72$ located within the discrete cloak. We emphasize that the presence of the test particle is self-consistently taken into account. This means that the electromagnetic coupling between the test particle and the cloak is taken into account to compute the optical force experienced by the test particle. Figure 4(a) shows the z component of the optical force experienced by the particle (either lossless or absorbing) versus its position along the z axis inside the cloak ($x = y = 0$). For both particles, we see that when the particle is close to the inner boundary of the cloak it experiences a repulsive force that pushes it away from the boundary. Note, however, that this is not a general behavior. If we look at the optical force experienced by a particle as it moves inside the cloak (not represented) we observe one common feature: The magnitude of the optical force is weak when the particle is far from the internal edge and increases dramatically near the edge. As for the specific spatial profile of the force on the particle, it will depend on several parameters such as the polarizability of the particle and the size of the

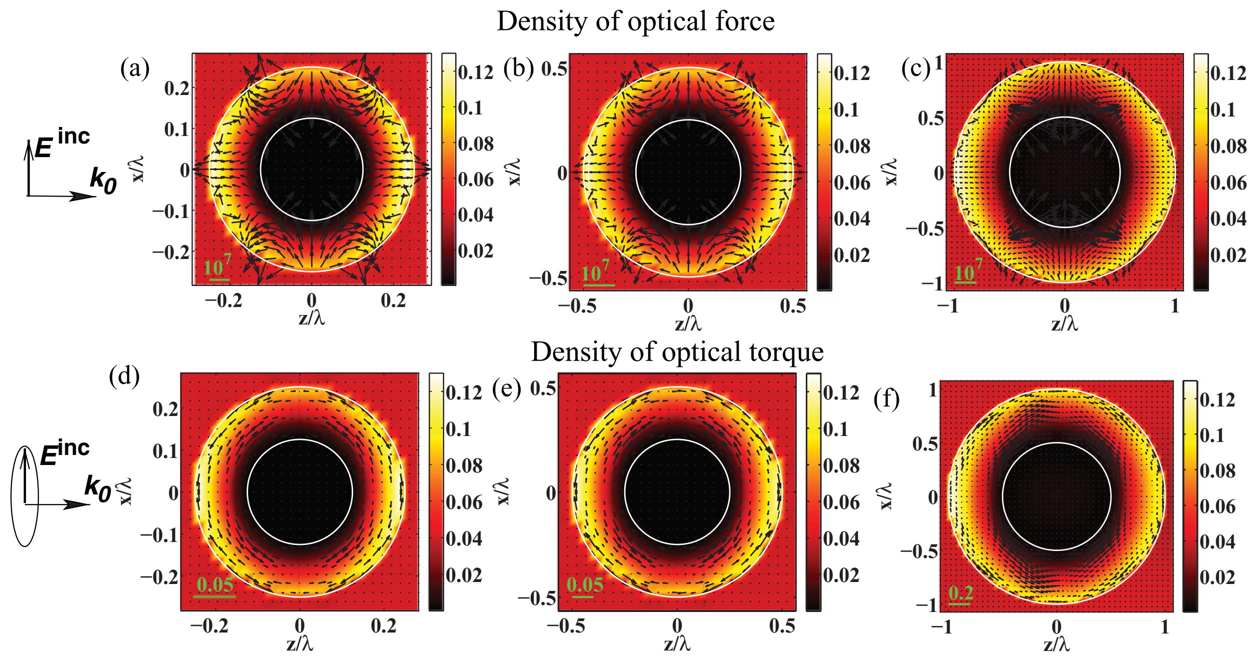

FIG. 3. (Color online) Cross sections in the $y = 0$ plane. Vector fields representing the density of force (first row: a, b, c) for a cloak illuminated by a linearly polarized plane wave and the density of optical torque (second row: d, e, f) on a cloak illuminated by a left circularly polarized plane wave. A sketch of the polarization state of the incident wave is given on the left side of the figure. In all figures, the color scale represents the density of electromagnetic energy. The scale bar for the density of force or torque is given in the bottom left corner of each figure. (a) and (d) $a = \lambda/4$ and $N_l = 23$. (b) and (e) $a = \lambda/2$ and $N_l = 23$. (c) and (f) $a = \lambda$ and $N_l = 41$.

cloak; the particle can be attracted toward or pushed away from the internal edge of the cloak. However, we notice that, inside the cloak, the force on the absorbing particle is stronger than the one on the lossless particle, irrespective of the location inside the cloak. This is due to a stronger in radiation pressure on the absorbing particle [20,21].

To better see the influence of the cloak on the force experienced by the particle we plot in Fig. 4(b) the force normalized to the force the particle would experience in the absence of the cloak, i.e., in free space. For the absorbing particle the result is as expected: The cloak hides the particle, and the optical force experienced by the particle decreases when the particle is within the cloak. This, however, is not the case for the lossless particle as the optical force experienced by the particle *increases* when it is located within the cloak. This somewhat counterintuitive effect is due to the difference between radiation pressure and gradient force. When the particle is inside the cloak, the electromagnetic field is weak (at the center of the cloak the density of energy is less than 0.4% of that of the incident field), hence the radiation pressure is weak. However, the spatial gradient of the fields is not negligible inside the cloak, particularly close to the interior edge. The particle therefore experiences a gradient force (which is proportional to the real part of the polarizability), and the optical force experienced by the particle is stronger within the cloak (near the interior edge), than in free space (no gradient force). Of course, at the center of the cloak, the total optical force experienced by the lossless particle is indeed weaker than in free space as the gradient force vanishes there. When absorption is present we also have a contribution to the optical force that is proportional to the imaginary part of the polarizability. The combination of gradient force and absorption results in, first, a shift between the two curves' [solid line for the lossless particle and dashed line for the absorbing particle in Fig. 4(a)] and, second, a weaker normalized force due to a stronger force on the absorbing particle in the free-space case [dashed line in Fig. 4(b)].

B. Optical force and torque on a dispersive cloak

So far we have neglected material absorption in the cloak so as to highlight the influence of the discrete geometry; however, it is impossible to build a metamaterial with the required optical properties over a wide frequency range without dispersion and absorption. Therefore, in this section we study the influence of dispersion and losses in the cloak by introducing a dispersion profile in the definition of the permittivity and permeability tensors associated with the discrete subunits forming the cloak. Assuming a Lorentz dispersion profile [12], which is commonly used to model the material dispersion of metamaterial structures, we define [22,23]

$$f(\omega) = f_\infty - \frac{F}{\omega^2 + i\Gamma\omega - \omega_g^2}, \tag{12}$$

where ω_g and Γ are the transition frequency and damping rate, respectively. Equation (12) satisfies causality and is Kramers-Kronig consistent [12,24]. The permittivity and permeability tensors defined in Eq. (11) are now multiplied by the function $f(\omega)$ to account for material dispersion. Notice that if $\Gamma = 0$ and $f(\omega) \neq 1$, the cloak is no longer perfect although it is still lossless. However, if Γ is different from zero, then $f(\omega)$ has a nonzero imaginary part and the optical cloak is lossy as a loss term is introduced in both the permittivity and permeability.

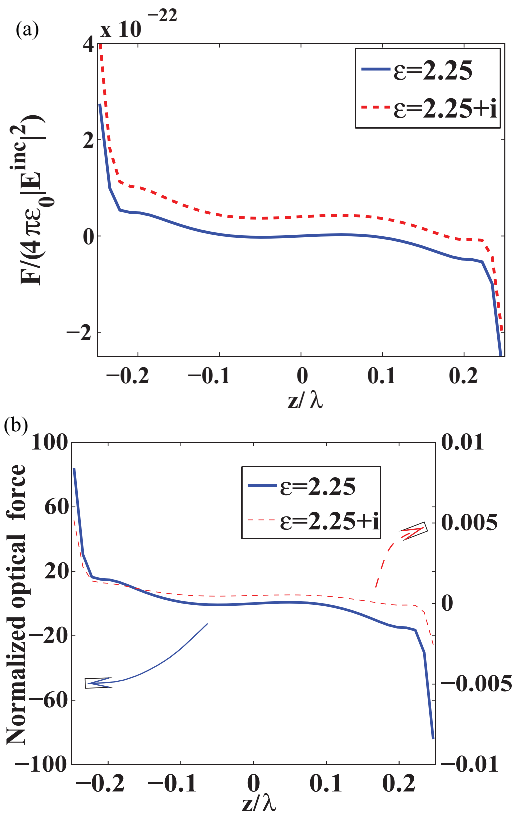

FIG. 4. (Color online) Force on a minute particle of radius $\lambda/72$ inside the cloak with $a=\lambda/2$, $b=a/2$, and $N_l=45$. Lossless particle: $\varepsilon=2.25$; solid line and left vertical axis in (b). Absorbing particle: $\varepsilon=2.25+i$; dashed line and right vertical axis in (b). (a) z component of the optical force is plotted versus z for $x=y=0$. (b) Same as (a), but the optical force is normalized to the optical force experienced by the same particle located in free space.

1. *Dispersive discrete cloak: force*

We choose the illumination wavelength $\lambda_0=2\pi c/\omega_0$ and the parameters of the dispersion profile such that $f(\omega_0)=1$ when $\Gamma=0$. This means that at wavelength λ_0 the relative permittivity and permeability of the cloak matches Eq. (11). With $\Gamma=0$ at $\lambda=\lambda_0$ the cloak is lossless, but if the wavelength of illumination λ is changed from λ_0, the cloak is lossy. Figure 5 shows the optical force on a cloak with outer radius $a=\lambda_0/4$ versus the wavelength of illumination when the cloak is dispersive. The solid line curve without circles (dashed line curve without circles) gives the spectrum of the force for $\Gamma=0$ for an illumination frequency either below the transition frequency ω_g ($\omega_g=3\omega_0$, $F=90\omega_0^2$) or above the transition frequency ω_g ($\omega_g=\omega_0/10$, $F=\omega_0^2/3$) [the inset in the bottom left (right) corner shows $f(\omega)$]. When $\lambda=\lambda_0$ the cloak is an ideal discrete cloak, hence the weak optical force. However, due to the dispersive nature of the cloak, if the wavelength of illumination is moved away from λ_0, the optical force on the cloak increases by several orders of magnitude. If we introduce a small amount of absorption, i.e., $\Gamma=10^{-3}$, the real part of $f(\omega)$ is not noticeably changed over the frequency range of interest and $\mathrm{Im}[f(\omega)]\approx 10^{-3}$. Figure 5 (plots with circles) shows that the introduction of losses entails an increase of the optical force around $\lambda=\lambda_0$, revealing a weakening of the cloaking mechanism by absorption.

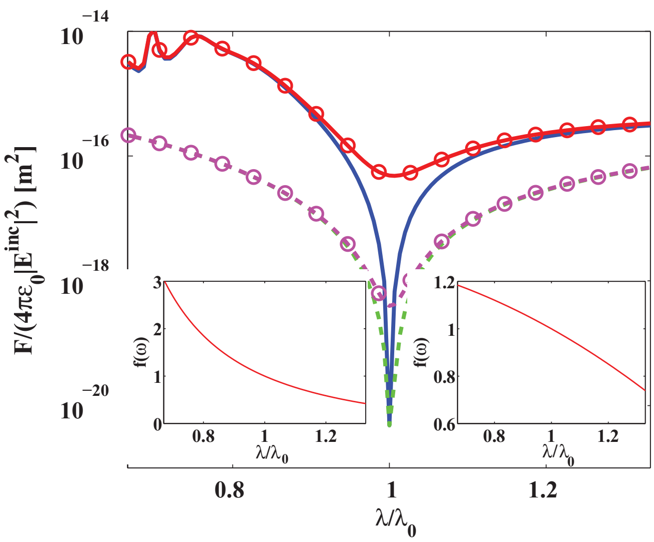

FIG. 5. (Color online) Optical force on a dispersive cloak versus wavelength for $\omega_g=3\omega_0$ and $\Gamma=0$ (solid line), $\omega_g=3\omega_0$ and $\Gamma=0.001$ (solid line with circles), $\omega_g=\omega_0/10$ and $\Gamma=0$ (dashed line), $\omega_g=\omega_0/10$ and $\Gamma=0.001$ (dashed line with circles). The left (right) inset is $f(\omega)$ when $\omega_g=3\omega_0$ ($\omega_g=\omega_0/10$) and $\Gamma=0$.

2. *Dispersive discrete cloak: torque*

The optical torque on the cloak, for the strongly dispersive case $\omega_g=3\omega_0$, is presented in Fig. 6. The cloak is illuminated with a circularly polarized plane wave. One can see that the introduction of a small amount of material absorption ($\Gamma=0.001$) increases the torque significantly. Note, however, the different effect material absorption has on the optical torque and force. For the optical force the magnitude of the force is altered significantly by material absorption only in a narrow spectral region around λ_0. On the other hand, for the optical torque, the lossless ($\Gamma=0$) and lossy ($\Gamma=0.001$) cases are noticeably different over a wider range of frequencies. This is due to the fact that the optical torque consists of two contributions: one from absorption and one from the

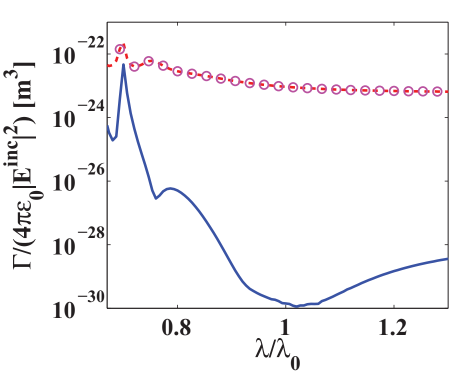

FIG. 6. (Color online) Optical torque versus the wavelength on a dispersive cloak when $\omega_g=3\omega_0$, $\Gamma=0$ (solid line), and $\Gamma=0.001$ (dashed line). Circles: absorbing cross section of the cloak normalized to $8\pi^2/\lambda$.

anisotropy of the permeability and permittivity tensors. In Fig. 6 we plot, using circles (no line), the absorbing cross section of the cloak normalized to $8\pi^2/\lambda$. Clearly the cross-section spectrum follows very closely the spectral evolution of the optical torque, showing that, as far as the optical torque is concerned, the cloak behaves like an absorbing homogeneous sphere [25], i.e., *the optical torque is proportional to the absorbing cross section.*

3. Effect of the dispersion on a hidden particle within the cloak

With invisibility cloaks the focus is traditionally on the concealment of an object from electromagnetic probes. In particular, a cloak is expected to induce very little (none in the ideal case) scattering of electromagnetic waves. However, as we have seen, the scattering of waves by the cloak is only half the story. The electromagnetic probe can affect the cloak mechanically through optical forces and torques. Therefore, it would be interesting to assess the ability of a realistic cloak (with discrete geometry and material dispersion and absorption) to shield an object from optical forces. To this end we introduce a minute test particle at the center of the cloak and study how well the particle is protected from optical forces. The computation of the force experienced by the test particle is done self-consistently. Figure 7(a) shows the spectrum of the optical force experienced by a lossless dielectric particle ($\varepsilon = 2.25$ and $\mu = 1$) at the center of the cloak. In Fig. 7(b) the optical force is normalized to the optical force the particle would experience in the absence of the cloak. We consider two dispersion profile parameters: $\omega_g = 3\omega_0$ (solid line) and $\omega_g = \omega_0/10$ (dashed line) with $\Gamma = 0$. It can be seen that the optical force increases dramatically when the wavelength of illumination moves away from λ_0 [recall that the dispersion profile of the cloak is chosen such that $f(\omega_0) = 1$ in the lossless case] due to the deterioration of the cloaking mechanism: If the field inside the cloak is no longer negligible, the optical force on the test particle increases. This is particularly obvious in Fig. 7(b), where the normalized optical force is larger than one, meaning that the sphere experiences a *stronger* optical force inside the cloak than the force it would experience in free space. Also, the test particle would not remain at the center of the cloak and would move to another equilibrium position or hit the edge of the cloak. Notice that if a weak absorption is added to the permittivity and permeability of the cloak ($\Gamma = 0.001$, curves with circles), the optical force does not change significantly. If the concealed particle is itself absorbing, ($\varepsilon = 2.25 + i$ and $\mu = 1$), we see in Figs. 8(a) and 8(b) that the normalized optical force remains close to the value obtained at λ_0 and always less than one. This illustrates the fact that with a leaky and dispersive cloak, the optical force experienced by an absorbing particle is lower within the cloak than in free space by two orders of magnitude in all the range of wavelength studied, hence the particle is protected by the cloak. This illustrates the more or less intuitive fact that even with a leaky and dispersive cloak (i.e., a nonideal

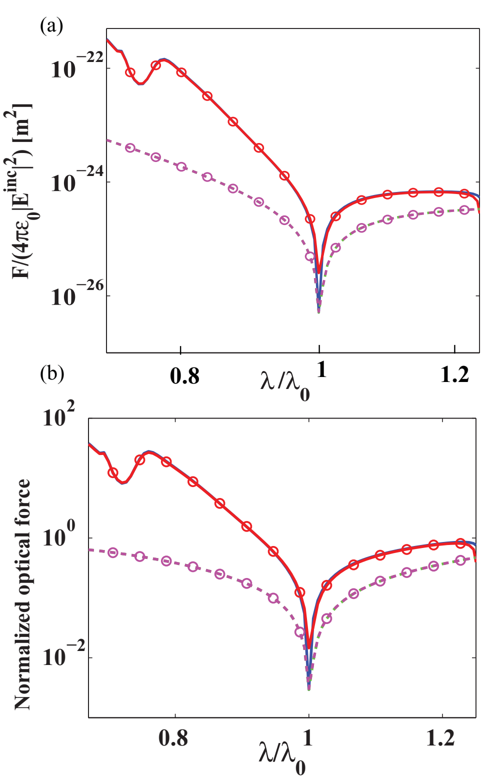

FIG. 7. (Color online) We consider both a lossless and a lossy cloak for two dispersion profiles: $\omega_g = 3\omega_0$, $\Gamma = 0$ (solid line), and $\Gamma = 0.001$ (solid line with circles); $\omega_g = \omega_0/10$, $\Gamma = 0$ (dashed line), and $\Gamma = 0.001$ (dashed line with circles). (a) Spectrum of optical force experienced by a test lossless particle ($\varepsilon = 2.25$ and $\mu = 1$) located at the center of a dispersive cloak. (b) The force is normalized to the optical force the particle would experience in free space (in the absence of the cloak).

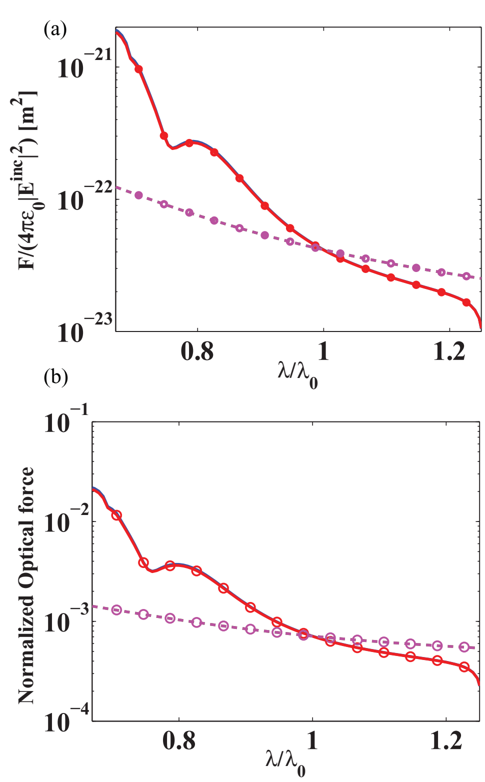

FIG. 8. (Color online) Same caption as in Fig. 7, but the test particle is an absorbing particle: $\varepsilon = 2.25 + i$ and $\mu = 1$.

cloak), an absorbing particle is sheltered by a cloak in contrast to a lossless particle even if the net force experienced by an absorbing particle were larger than that of a lossless particle.

IV. CONCLUSION

We have studied the influence of the discrete geometry of a spherical invisibility cloak on the time-averaged force and torque experienced by the cloak in the time-harmonic regime. We found that a significant cloaking effect can be achieved even with a relatively coarse discrete geometry. We also considered the effect of material dispersion and absorption and showed that a small amount of losses can lead to a significant increase in the optical force and torque experienced by the cloak. The capacity of a cloak to shield an object has been investigated by considering a test particle inside the cloak, and comparing the optical force on the particle to the force it would experience in free space (in the absence of the cloak). While even an imperfect (dispersive and lossy) cloak can reduce the optical force on the particle, the "mechanical shielding" of the particle will greatly depend on the particle's location inside the cloak and whether the particle is lossy. In particular, because optical forces depend not only on the magnitude of the fields but also on their gradients, near the the internal boundary of the cloak, the optical force can exceed what it would be in the absence of the cloak.

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