Optical force on a discrete invisibility cloak in time-dependent fields

Patrick C. Chaumet,[1] Adel Rahmani,[2] Frédéric Zolla,[1] André Nicolet,[1] and Kamal Belkebir[1]
[1]*Institut Fresnel, CNRS, Aix-Marseille Université, Campus de St-Jérôme 13013 Marseille, France*
[2]*Department of Mathematical Sciences, University of Technology, Sydney, Broadway NSW 2007, Australia*
(Received 8 February 2011; published 7 September 2011)

We study, in time domain, the exchange of momentum between an electromagnetic pulse and a three-dimensional, discrete, spherical invisibility cloak. We find that a discrete cloak, initially at rest, would experience an electromagnetic force due to the pulse but would acquire zero net momentum and net displacement. On the other hand, we find that while the cloak may manage to conceal an object and shroud it from the electromagnetic forces associated with the pulse, the cloak itself can experience optomechanical stress on a scale much larger than the object would in the absence of the cloak. We also consider the effects of material dispersion and losses on the electromagnetic forces experienced by the cloak and show that they lead to a transfer of momentum from the pulse to the cloak.

DOI: 10.1103/PhysRevA.84.033808 PACS number(s): 42.25.Fx, 41.20.Jb, 42.50.Md, 42.50.Wk

An invisibility cloak is a device capable of concealing itself, and any object inside it, from electromagnetic (EM) probes [1–5]. A mathematical analysis of cloaking, and of the boundary conditions on the inner boundary of the cloak, can be found in Refs. [6,7]. Typically, the cloaking mechanism is considered in the frequency domain for a monochromatic illumination. Hence, the focus is usually on the scattering properties of the cloak in the time-harmonic regime. Here we study, in the time domain, the interaction of a three-dimensional cloak with a broadband EM pulse (see Refs. [8–12] for the 2D case and Ref. [13] for a discussion of time delay in the 3D case), and we focus on the optomechanical coupling between the pulse and the cloak. Recent studies of EM forces on a cloak, conducted in the time-harmonic regime, have shown that the net time-averaged EM force due to a time-harmonic plane wave is zero [14,15]. However, the case of a time-dependent (pulse) illumination requires a more sophisticated approach. Furthermore, since in practice the metamaterials used to achieve cloaking are composite structures made of discrete elements, it is important to incorporate the discreteness of the cloak in the treatment of EM forces. This is particularly crucial because a discrete cloak would experience some coupling between its interior and the outside world, which would affect its EM properties. We will first consider the case of a lossless and dispersionless cloak and show that, interestingly, the EM stress inflicted by the pulse on a cloak concealing a small object can be larger than the stress the object would experience in the absence of the cloak. Then, because metamaterial structures are in general dispersive and lossy, we will examine the influence of material dispersion and dissipation on the EM force.

Our model of a discrete cloak is based on the discrete dipole approximation (DDA) [16–20]. The DDA has the advantage of allowing us to mimic the discrete structure of an actual cloaking device without having to deal with the specifics of the internal structure of the metamaterial (see Gordon for interaction between a pulse and an atom [21] and Ref. [22] for computation of optical force in time domain for 2D case with finite-difference time-domain method). This way, the cloak is represented as a discrete collection of polarizable elements, with both electric and magnetic polarizabilities [23,24]. The EM forces can be computed for complex geometries using the approach of Refs. [25–31]. It is convenient to split the EM force into two contributions according to how they are derived from the EM fields. The first contribution, which we will label $\mathcal{F}^h(t)$, relates to the conventional term that appears in the time-harmonic treatment of EM forces. This term depends on the spatial derivatives of the EM fields. The second contribution, $\mathcal{F}^p(t)$, comes from a term that vanishes in a time-harmonic picture and which involves the time derivative of the Poynting vector [28]. We recall that the second contribution always gives a zero net momentum transfer between the EM pulse and the scatterer, irrespective of the profile of the pulse and the shape of the object [28]. The total force is the sum of these two contributions, $\mathcal{F}^t(t) = \mathcal{F}^h(t) + \mathcal{F}^p(t)$. For our incident pulse we will consider a plane wave, with Gaussian time envelope, propagating along the z axis and linearly polarized along the x axis (unit vector $\mathbf{e}_x$). The electric field reads

$$\mathbf{E}_{\text{inc}}(z,t) = E_0 \exp\left[-\left(\frac{t-\frac{z}{c}-\tau}{\tau/4}\right)^2\right] \sin\left[\omega_0\left(t-\frac{z}{c}\right)\right]\mathbf{e}_x, \tag{1}$$

where $f_0 = \omega_0/2\pi = c/\lambda_0$ is the central frequency of the pulse and $\tau = 8/f_0$. In our computations we will choose $f_0 = 2$ GHz and $4\pi\varepsilon_0|E_0|^2 = 1$, which gives an irradiance of 0.012 mW/μm^2. The spectral and time profiles of the pulse are plotted in Figs. 1(a) and 1(b), respectively. Note that on the plots, the frequency is normalized to f_0 and the time is normalized to τ. We will start by considering the ideal case of a dispersionless and lossless spherical cloak, centered at the origin. The cloak has inner radius b, outer radius a, and relative permittivity and permeability tensors given by [32],

$$\boldsymbol{\varepsilon}(\mathbf{r}) = \boldsymbol{\mu}(\mathbf{r}) = \frac{a}{a-b}\left(\mathbf{I} - \frac{2br-b^2}{r^4}\mathbf{r}\otimes\mathbf{r}\right). \tag{2}$$

We plot, in Fig. 2(a), the net EM force on the cloak with $a = 2b = \lambda_0/4$. We note that contrary to the case of a time-harmonic incident wave [15], the EM force on the cloak does not vanish. This means that the cloak moves under the influence of the pulse; however, as we shall see, the net displacement of the cloak is still zero. The main contribution to the total EM force comes from the term $\mathcal{F}^p(t)$, which is seldom taken into account in EM force calculations. Using Newton's second

 ©2011 American Physical Society

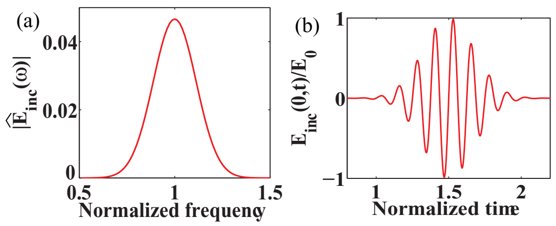

FIG. 1. (Color online) Spectral (a) and time (b) profiles of the incident field. $\widehat{\mathbf{E}}_{\text{inc}}$ denotes the Fourier transform of $\mathbf{E}_{\text{inc}}$.

law we can find the net momentum associated with the EM forces, which is shown in Fig. 2(b). For the total impulse, the instantaneous force is integrated in time [33], and as expected the contribution $\mathcal{P}^p(t) = \int_{-\infty}^{t} \mathcal{F}^p(\gamma)\mathrm{d}\gamma$ vanishes when the pulse ends; however, so does $\mathcal{P}^h(t) = \int_{-\infty}^{t} \mathcal{F}^h(\gamma)\mathrm{d}\gamma$, which means that no net momentum is transferred to the cloak by the pulse. The motion of the cloak can be inferred from Fig. 2(c), where we plot $\int_{-\infty}^{t}[\mathcal{P}^p(\gamma) + \mathcal{P}^h(\gamma)]\mathrm{d}\gamma$ which, in the absence of mechanical damping, is proportional to the position of the center of mass of the cloak (under the assumption that the displacement of the cloak is small enough for us to neglect any variation in the incident field). We see that after interacting with the pulse the cloak returns to its original position, but we also find that in the present case the amplitude of the oscillatory motion of the cloak is perfectly negligible. In summary, for a dispersionless and lossless cloak, although the EM force is not identically zero, the net displacement and momentum transfer to the cloak are both zero.

To better understand how the various contributions to the force behave, we plot in Fig. 2(d) the spectra (Fourier transforms) of the two contributions to the force and their sum. We note that the total force has only one maximum at $2f_0$. This is consistent with the fact that there is no net momentum transferred to the cloak. Note, however, that while $\widehat{\mathcal{F}}^p(\omega)$ and

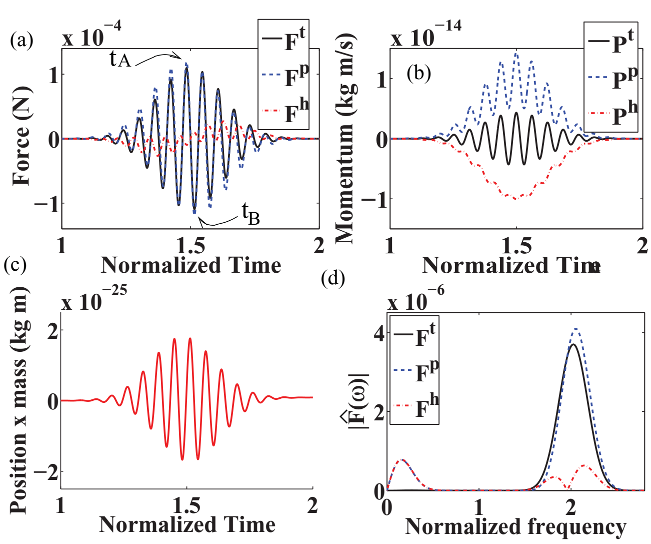

FIG. 2. (Color online) (a) Optical force experienced by the cloak vs time and its different contributions. (b) Momentum of the cloak vs time and its different contribution. (c) Position multiplied by the mass of the cloak of the center of mass of the cloak vs time.(d) Spectrum of the EM force and of its different contribution.

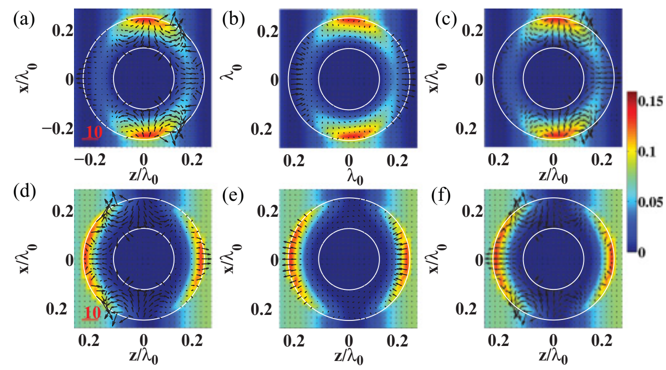

FIG. 3. (Color online) Normalized density of EM force (NDOF); (a)–(c) at time t_A [see Fig. 2(a)] when the total EM force is strongest. (d)–(f) at time t_B [see Fig. 2(a)] when the total EM force is weakest. First column $\mathcal{F}^{\text{h}}$, middle column $\mathcal{F}^{\text{p}}$, last column $\mathcal{F}^{\text{t}}$. The scale for the NDOF is plotted in red in the bottom left-hand corner of the first figure in each row.

$\widehat{\mathcal{F}}^h(\omega)$, which are the Fourier transforms of $\mathcal{F}^p(t)$ and $\mathcal{F}^h(t)$, both exhibit a peak at low frequency, these peaks are equal in magnitude and opposite in sign [this is best seen in Fig. 2(b)], thereby canceling each other and leading to a single resonance for the total force centered at $2f_0$. By contrast, for a simpler scatterer, say, a homogeneous dielectric sphere, the spectrum of the total force (not presented here) would also exhibit a peak at zero frequency as a result of the net transfer of momentum from the pulse to the sphere [29].

We can visualize how the force is distributed within the cloak by plotting (arrows in Fig. 3) the density of EM force (DOF) inside the cloak. On the same figure we also plot the energy density as a color scale. The DOF is normalized to the maximum of the DOF inside a test dielectric particle ($\varepsilon = 2.25$) much smaller than λ_0 illuminated by the same incident pulse (without the cloak). The length of the red segment in the bottom left corner of Figs. 3(a) and 3(d) corresponds to a normalized DOF of 10. Figure 3 highlights the energy buildup near the outer boundary of the cloak. Looking at each contribution to the force we see that the DOF associated with $\mathcal{F}^p(t)$, which involves time derivatives of the EM fields, has a smooth distribution inside the cloak and reaches maximum density near the outer boundary of the cloak. By contrast, the spatial distribution of the DOF associated with $\mathcal{F}^h(t)$, which involves spatial derivatives of the EM fields, is less smooth with maxima of density near both the inner and outer boundaries of the cloak. We also observe that the DOF in the cloak can be an order of magnitude larger than the DOF in the test particle without the cloak. This suggests that if one wants to use a cloak to shield a given structure against the mechanical effect of an EM pulse, most parts of the cloak (and not just those regions where the density of energy is large) may experience significant mechanical stress. In fact, as illustrated in our example, the mechanical stress on the cloak can be stronger than the stress that the object to be protected would experience in the absence of the cloak. Obviously, this is more likely to be an issue for mechanical waves (acoustic waves, pressure and shear waves, etc.) for which similar cloaking approaches exist [34–36]. In such a case, the cloaking structure should be designed taking that

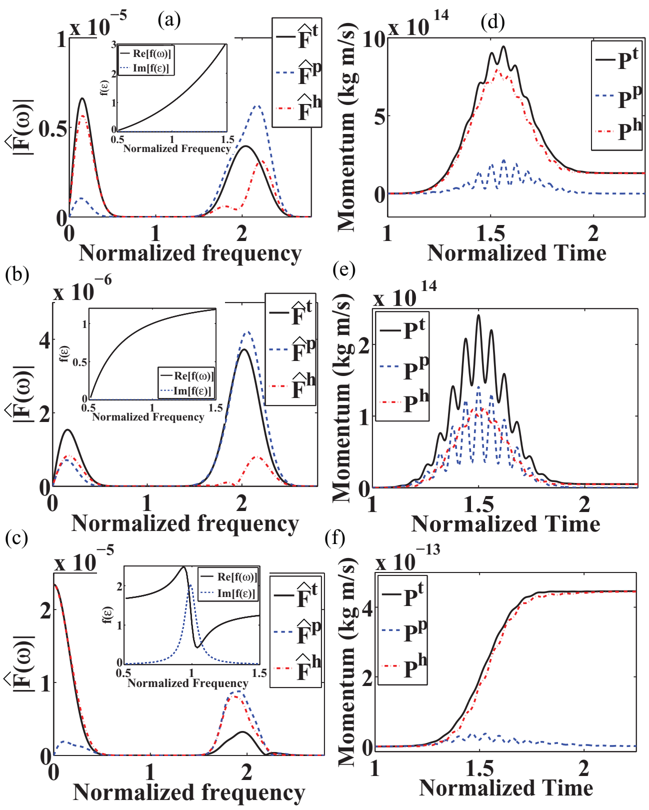

FIG. 4. (Color online) (a), (b), and (c) Spectra of the EM force and of its different contribution. The inserts give the real and imaginary parts of $f(\omega)$ in solid and dashed lines, respectively. (d), (e) and (f) momentum imparted to the cloak versus time.

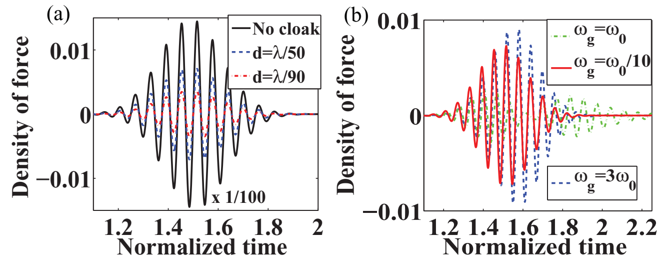

FIG. 5. (Color online) Density of force on a test object located at the origin (center of the cloak). (a) Solid line: no cloak. dashed line: cloak with $d = \lambda/50$. Dot-dashed line with cloak and $d = \lambda/90$. (b) With dispersion parameters corresponding to Figs. 4(a) (dashed line), 4(b) (solid line), and 4(c) (dot-dashed line).

stress into account. This is especially important in the case of earthquakes and tsunami waves where the high mechanical energy involved can strongly strain the cloak. Notice that our computation has been done under the assumption of a rigid cloak, meaning that the mechanical distortion of the cloak by the pulse is assumed to be small enough to be negligible when computing the EM response of the cloak.

Computations (not shown) done for cloaks with larger sizes ($a = \lambda$ and $a = 3\lambda/2$) show a similar behavior of the DOF; however, the thicker the cloak and the lower the stress. This can be understood intuitively by noticing that a thin cloak has to "squeeze" the electromagnetic field into a smaller volume than a thicker cloak, thus resulting in a larger energy density and DOF inside the shell of the cloak. Accordingly, if a cloak is designed to protect a structure against the mechanical effect of a wave one should make the ratio a/b large enough to prevent stress-related damage in the cloak.

So far we have neglected material dispersion and losses. However, any actual cloaking device will most likely suffer from both. Let us now consider a causal dielectric function $f(\omega)$ given by a Lorentz model [37] $f(\omega) = \varepsilon_\infty - F/(\omega^2 + i\Gamma\omega - \omega_g^2)$, where ω_g and Γ are the transition frequency and damping respectively, and F is a constant chosen so that $f(\omega) = 1$. We multiply the relative permittivity/permeability [Eq. (2)] by the function $f(\omega)$ to obtain the new material parameters for the cloak.

In Figs. 4(a) and 4(d) [Figs. 4(b) and 4(e)] we consider the case where the central frequency of the incident pulse is smaller (larger) than the frequency of the critical point, i.e., $\omega_g = 3\omega_0$ with no absorption, $\Gamma = 0$ ($\omega_g = \omega_0/10, \Gamma = 0$). In both cases, after the pulse has died out, the cloak has acquired a nonzero net momentum component in the direction of propagation of the pulse, due to the nonideal cloaking properties of the cloak at frequencies different from the central frequency; in other words, the cloak is pushed along by the pulse. Notice that compared to Fig. 2, the peak of the EM force spectrum near $2f_0$ is blue-shifted for $\mathcal{F}^h(t)$ and $\mathcal{F}^p(t)$ in Figs. 4(d) and 4(e). However, for the total force, the peak remains at $2f_0$ because of the way the two contributions compete with one another. Close to the zero frequency the total force increases slightly due to the aforementioned momentum transfer from the pulse. Notice that the maximum of the normalized DOF is in both cases around 15 compared to about 10 in the nondispersive case. Therefore the stress inside the cloak is increased by the presence of material dispersion. In the case where ω_0 corresponds to the critical point, we should add a damping term ($\Gamma = \omega_g/10$ and $\omega_g = \omega_0$). In that case, as shown by Figs. 4(c) and 4(f), the momentum imparted to the cloak increases dramatically, due to material absorption, and the maximum enhancement of the normalized DOF is about 30. Clearly the introduction of strong losses at the central frequency f_0 of the pulse ruins the cloaking effect, as one would expect, and increases the difficulty of designing a cloak capable of coping with mechanical waves due to the increase of mechanical stress inside the cloak. Some insight into the protective ability of the dispersive cloak can be gained by considering explicitly how a shrouded test object inside the cloak is affected by the pulse. Figure 5 shows the DOF on a minute (Rayleigh) particle located at the center of the cloak. Figure 5(a) shows that when the cloak is made of discrete elements with size $d = \lambda/50$, which roughly corresponds to a five-layer-thick cloak, the DOF inside the particle is not zero (dashed line). This is due to the fact that a discrete cloak cannot achieve perfect cloaking. By comparison, the DOF without the cloak is plotted as a solid line in Fig. 5(a). Keeping in mind that there is a scaling factor of 200 between the two curves, we can see that the cloak, despite not being ideal, is still effective in protecting the object. If d decreases (i.e., the cloak is made of a larger number of smaller elements), the DOF decreases (dashed line with $d = \lambda/90$). This is expected, as a smaller element size means that the material properties of the discrete cloak are getting closer to the idealized case of a continuous cloaking medium. The influence of dispersion is illustrated in Fig. 5(b). We observe that dispersion does not significantly increase the DOF experienced by the test object. Hence even a nonideal, dispersive cloak can be used to shield an object from the mechanical effects of a pulse, with of course the caveat that the cloak must be able to endure the mechanical stress created by the pulse.

In conclusion, we have studied the EM forces in time domain on a discrete, three-dimensional, spherical cloak. We have considered the case of material dispersion and losses. We showed that when interacting with a pulse, the cloak only acquires a nonzero net momentum if it is dispersive and/or lossy. We also showed that the cloak can experience large EM stress even in the dispersionless, lossless case, and that dispersion and more particularly losses would further increase that stress dramatically. Nevertheless, even with those disadvantageous conditions, we have shown that the object within the cloak is still protected by the dispersive cloak.

[1] D. Schurig, J. J. Mock, B. J. Justice, S. A. Cummer, J. B. Pendry, A. F. Starr, and D. R. Smith, Science **314**, 977 (2006).
[2] U. Leonhardt, Science **312**, 1777 (2006).
[3] A. Nicolet, F. Zolla, and S. Guenneau, IEEE Trans. Magn. **44**, 1150 (2008).
[4] F. Zolla, S. Guenneau, A. Nicolet, and J. B. Pendry, Opt. Lett. **32**, 1069 (2007).
[5] S. Zouhdi, A. Sihvola, and A. P. Vonogradov, *Metamaterials and Plasmonics: Fundamentals, Modeling, Applications,* Book Series: NATO Science for Peace and Security Series B–Physics and Biophysics, IEEE Press (Springer, Dordrecht, 2009).
[6] R. Weder, J. Phys. A **41**, 065207 (2008).
[7] R. Weder, J. Phys. A **41**, 415401 (2008).
[8] C. Argyropoulos, Y. Zaho, and Y. Hao, IEEE Trans. Antennas Propag. **57**, 1432 (2009).
[9] C. Argyropoulos, E. Kallos, and Y. Hao, Phys. Rev. E **81**, 016611 (2010).
[10] P. Alitalo, H. Kettunen, and S. Tretyakov, J. Appl. Phys. **107**, 034905 (2010).
[11] C. Li, X. Liu, and F. Li, Phys. Rev. B **81**, 115133 (2010).
[12] S. Yu-Huan, S. Jin-Wei, L. Da He, and Y. Guo-Jian, Chin. Phys. Lett. **27**, 094102 (2010).
[13] B. Zhang, B. I. Wu, and H. Chen, Opt. Express **17**, 6721 (2009) [http://www.opticsexpress.org/abstract.cfm?URI=oe-17-8-6721].
[14] H. Chen, B. Zhang, B. A. Kemp, and B. I. Wu, Opt. Lett. **35**, 667 (2010).
[15] H. Chen, B. Zhang, Y. Luo, B. A. Kemp, J. Zhang, L. Ran, and B. I. Wu, Phys. Rev. A **80**, 011808 (2009).
[16] B. T. Draine, Astrophys. J. **333**, 848 (1988).
[17] M. A. Yurkin and A. G. Hoekstra, J. Quant. Spectrosc. Radiat. Transfer **106**, 558 (2007).
[18] P. C. Chaumet, A. Sentenac, and A. Rahmani, Phys. Rev. E **70**, 036606 (2004).
[19] P. C. Chaumet, A. Rahmani, and G. W. Bryant, Phys. Rev. B **67**, 165404 (2003).
[20] P. C. Chaumet and C. Billaudeau, J. Appl. Phys. **101**, 023106 (2007).
[21] J. P. Gordon, Phys. Rev. A **8**, 14 (1973).
[22] M. Mansuripur and A. R. Zakharian, Phys. Rev. E **79**, 026608 (2009).
[23] P. C. Chaumet and A. Rahmani, Opt. Lett. **34**, 917 (2009).
[24] A. Rahmani, P. C. Chaumet, and G. W. Bryant, Opt. Express **18**, 8499 (2010) [http://www.opticsexpress.org/abstract.cfm?URI=oe-18-8-8499].
[25] P. C. Chaumet, A. Rahmani, and M. Nieto-Vesperinas, Phys. Rev. B **66**, 195405 (2002).
[26] P. C. Chaumet, A. Rahmani, A. Sentenac, and G. W. Bryant, Phys. Rev. E **72**, 046708 (2005).
[27] P. C. Chaumet and A. Rahmani, Opt. Express **17**, 2224 (2009).
[28] P. C. Chaumet, K. Belkebir, and A. Rahmani, Phys. Rev. A **81**, 021803(R) (2010).
[29] P. C. Chaumet, K. Belkebir, and A. Rahmani, Opt. Express **19**, 2466 (2011) [http://www.opticsexpress.org/abstract.cfm?URI=oe-19-3-2466].
[30] A. Rahmani and P. C. Chaumet, Opt. Express **14**, 6353 (2006).
[31] P. C. Chaumet, A. Rahmani, and M. Nieto-Vesperinas, Phys. Rev. B **71**, 045425 (2005).
[32] D. Schurig, J. B. Pendry, and D. R. Smith, Opt. Express **14**, 9794 (2006).
[33] A. Mizrahi and L. Schächter, Phys. Rev. E **74**, 036504 (2006).
[34] M. Farhat, S. Enoch, S. Guenneau, and A. B. Movchan, Phys. Rev. Lett. **101**, 134501 (2008).
[35] M. Farhat, S. Guenneau, and S. Enoch, Phys. Rev. Lett. **103**, 024301 (2009).
[36] S. Zhang, C. Xia, and N. Fang, Phys. Rev. Lett. **106**, 024301 (2011).
[37] J. D. Jackson, *Classical Electrodynamics*, 2nd ed. (Wiley, New York, 1975).