Newton-Kantorovich and Modified Gradient-Inversion Algorithms Applied to Ipswich Data

K. Belkebir[1], J. M. Elissalt[2], J. M. Geffrin[1], and Ch. Pichot[3]

[1]Formerly with the Laboratoire des Signaux et Systèmes
CNRS/Supelec
Supelec, Division Ondes
Plateau de Moulon
91192 Gif-sur-Yvette Cedex
France
Now with Electromagnetics Division
Faculty of Electrical Engineering
Eindhoven University of Technology
PO Box 513
5600 MB Eindhoven
The Netherlands

[2]Formerly with [1], above.
Now with SIMULOG
Les Taissounières HB2
Route des Dolines
06560 Valbonne
France

[3]Laboratoire d'Electronique, Antennes et Télécommunications
CNRS/Université de Nice-Sophia Antipolis
Bât. 4, 250 rue Albert Einstein
06560 Valbonne
France

1. Abstract

The present paper considers the application of the Newton-Kantorovich and modified-gradient methods to the Ipswich data. The object is assumed to be an inhomogeneous lossy dielectric cylinder of arbitrary cross section. Both inverse-scattering methods are based on electric-field integral representations. The Newton-Kantorovich technique builds up the solution by solving successively the forward problem and a linear inverse problem. This method needs regularization, and we use either the identity operator or a gradient operator for regularization. The modified-gradient method is iterative, as is the Newton algorithm, but does not involve a linearization at each step of the nonlinear inverse problem. Results of inversions with both methods, on two known impenetrable targets, are discussed.

2. Introduction

In this paper, the object was supposed to be an infinite cylinder of unknown cross section. The complex permittivity profile of the object was determined, using a Newton-Kantorovich (NK) method [1], and a Tikhonov regularization, with standard identity operator or gradient operator, was applied. As applications will concern impenetrable objects, a priori information was used with the modified-gradient (MG) method [2, 3], and one tried to determine the shape and the location of the object by means of a non-negative characteristic function. The Polak-Ribière conjugate-gradient direction was used in the MG method. No additional regularization procedure was used in the MG method, although recent work indicates that the addition of the total variation as a regularizer is very beneficial [4]. For both methods, the initial guesses were determined with a back-propagation scheme [5, 6] using the adjoint operator, which provided an estimate of the induced current. Reconstructions on two impenetrable objects (a circular cylinder and a strip), with MG and NK methods, were examined, from experimental data.

3. Problem statement

The cylindrical object, characterized by a relative complex permittivity $\varepsilon_r(\mathbf{r})$, is contained in a bounded region, D, and illuminated successively by different incident TM plane waves e_l^i, $l = 1,\ldots,L$. The receivers are located in the domain, S, in the far-field region. For each excitation l, the forward-scattering problem may be formulated as the following domain integral equation

$$e_l = e_l^i + G^D \chi e_l. \qquad (1)$$

The integral representation for the scattered field is

$$e_l^{sca} = G^S \chi e_l, \qquad (2)$$

with complex contrast function $\chi(\mathbf{r}) = \varepsilon_r(\mathbf{r}) - 1$. G^D and G^S are

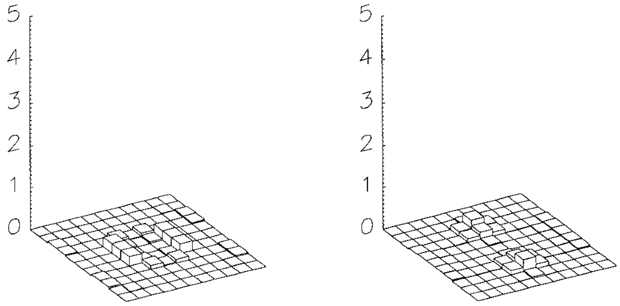

Figure 1a. The circular metallic cylinder: reconstruction of the real (left) and imaginary (right) part of the complex contrast, χ, using the NK method. The initial guess, computed using back propagation, is shown.

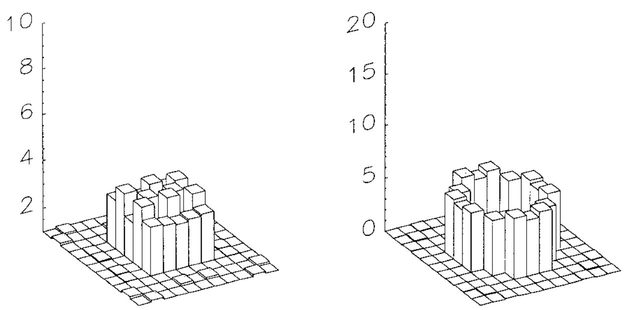

Figure 1b. The circular metallic cylinder: reconstruction of the real (left) and imaginary (right) part of the complex contrast, χ, using the NK method. The reconstruction with identify regularization (iteration 100) is shown.

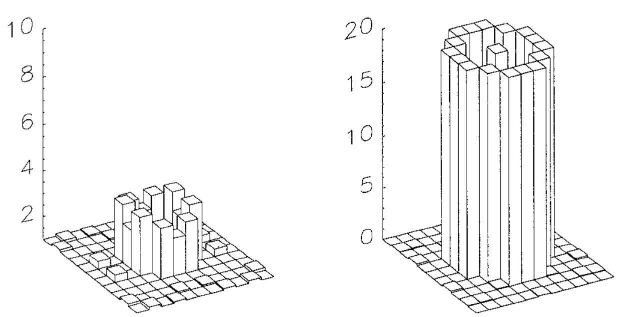

Figure 1c. The circular metallic cylinder: reconstruction of the real (left) and imaginary (right) part of the complex contrast, χ, using the NK method. The reconstruction with gradient regularization (iteration 7) is shown.

two integral operators mapping $L^2(D)$ (square integrable functions in D) into itself, and $L^2(D)$ into $L^2(S)$, respectively, involving the two-dimensional free-space Green's function, $G(\mathbf{r}) = \frac{i}{4} \mathrm{H}_0^{(1)}(k_0|\mathbf{r}|)$. The direct problem is solved using the moment method (MoM), with pulse basis functions and point matching, which transforms the integral equations into matrix equations. The rectangular image (or test domain) containing the region D is discretized into N elementary square cells.

4. Newton-Kantorovich method

The Newton-Kantorovich method builds up an iterative solution of the inverse problem by solving successively the direct problem and a local linear inverse problem [1]. At each iteration, an estimate of the complex contrast function, χ, is given by

$$\chi_n = \chi_{n-1} + \delta\chi \, , \tag{3}$$

where $\delta\chi$ is an update correction. This is obtained by solving, in the least-squares sense, the linearized forward problem

$$A\delta\chi = f - e^{sca} \, . \tag{4}$$

The matrix A is a linearized version of the nonlinear operator relating the scattered field to the contrast function, χ, and f represents the measured data vector [1]. The scattered-field vector, e^{sca}, is calculated through the forward-problem solver with a previous estimate of χ. Unfortunately, the problem of finding the solution of Equation (6) is ill-posed, and needs regularization. For this, we use a Tikhonov regularization, and minimize the functional

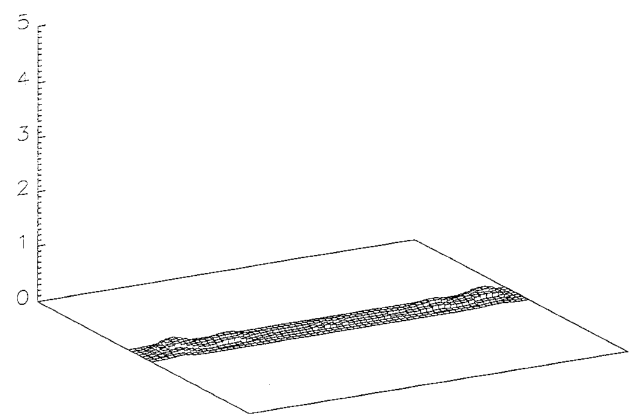

Figure 2a. The metallic strip: reconstruction of the imaginary part of the complex contrast, χ, using the NK method. The initial guess, computed using back propagation, is shown.

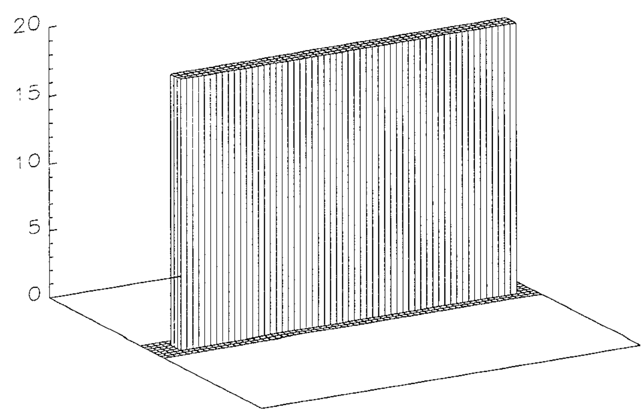

Figure 2b. The metallic strip: reconstruction of the imaginary part of the complex contrast, χ, using the NK method. The reconstruction (iteration 19) is shown.

$$F(\chi) = \left\| A\delta\chi - \left(f - e^{sca}\right)\right\|^2 + \alpha\left\|R\delta\chi\right\|^2. \quad (5)$$

R is the regularization matrix, and α is a regularization parameter chosen according to the general cross-validation method [7]. Two regularization operators have been used: an identity operator and a gradient operator. The gradient operator is based first on the assumption that the object is composed of homogeneous zones of arbitrary geometry, separated by border-like discontinuities, and second on the definition of neighborhood for each elementary cell of the image.

5. Modified-gradient method

With the modified-gradient method, we use a priori information about the nature of the object to be reconstructed, i.e. a high conductivity object ($\chi = \chi_{\max}$). We are interested in finding the location and the shape of the object, using the object characteristic function, ζ (a function taken to be equal to 1 inside the object, and zero outside) [6, 8]. Instead of reconstructing ζ, we propose to reconstruct an auxiliary function, ξ, such that $\zeta = \xi^2$. The definition of the function ξ is relaxed to take any real value. Using the operator notation, the inverse problem is that of finding ζ for given f_l measurements of the scattered field, or solving the equations

$$f_l = G^S \chi_{\max}\xi^2 e_l,\ l = 1,\ldots,L, \quad (6)$$

with domain integral equation

$$e_l = e_l^i + G^D \chi_{\max}\xi^2 e_l,\ l = 1,\ldots,L. \quad (7)$$

The modified-gradient method is iterative, as is the Newton-Kantorovich algorithm, but the approach is very different. The iterative procedure minimizes two residual errors at each iteration, with the cost functional [5, 6]

$$F = \frac{\sum_{l=1}^{L}\left\|e_l - G^D\chi_{\max}\xi^2 e_l\right\|_D^2}{\sum_{l=1}^{L}\left\|e_l^i\right\|_D^2} + \frac{\sum_{l=1}^{L}\left\|f_l - G^S\chi_{\max}\xi^2 e_l\right\|_S^2}{\sum_{l=1}^{L}\left\|f_l\right\|_S^2}. \quad (8)$$

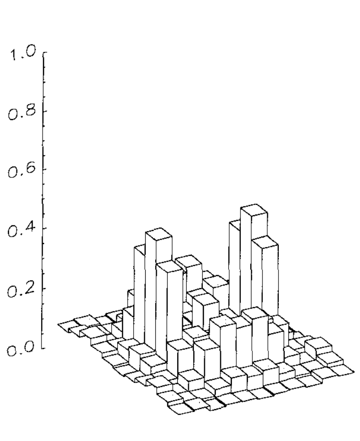

Figure 3a. The circular metallic cylinder: reconstruction of the characteristic function, ζ, using the MG method. The initial guess, computed using back propagation, is shown.

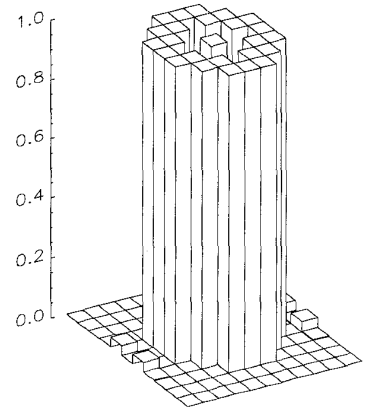

Figure 3b. The circular metallic cylinder: reconstruction of the characteristic function, ζ, using the MG method. The reconstruction (iteration 32) is shown.

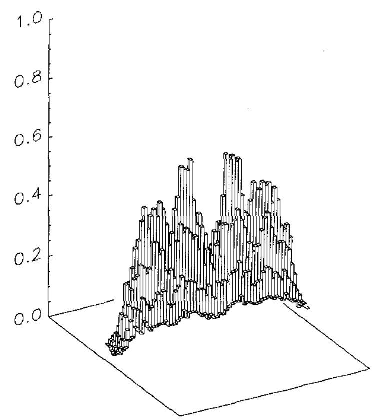

Figure 4a. The metallic strip: reconstruction of the characteristic function ζ using the MG method. The initial guess, computed using back propagation, is shown.

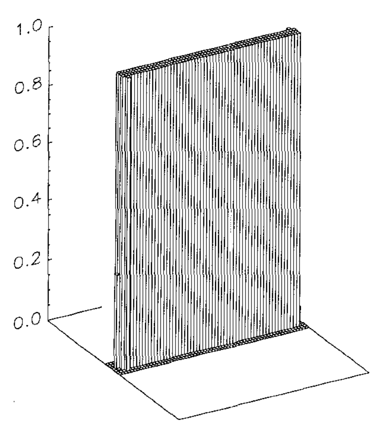

Figure 4. The metallic strip: reconstruction of the characteristic function ζ using the MG method. The reconstruction (iteration 32) is shown.

$\|\bullet\|_D^2$ and $\|\bullet\|_S^2$ are the norms associated with the inner product on $L^2(D)$ and $L^2(S)$, respectively. Sequences are constructed with the recurrence relations

$$e_{l,n} = e_{l,n-1} + \alpha_{l,n} v_{l,n}, \quad (9)$$

$$\xi_n = \xi_{n-1} + \beta_n d_n, \quad (10)$$

where the two functions $v_{l,n}$ and d_n are the update directions for functions $\{e_{l,n}\}$ and $\{\xi_n\}$, respectively. The complex numbers $\alpha_{j,n}$ and the real parameter β_n are weights that are chosen at each step so as to minimize the cost functional, F_n. Once the $v_{l,n}$ and d_n update directions are found, F_n is but a nonlinear expression in L complex variables, $\alpha_{j,n}$, and one real variable, β_n. The minimization of F_n is accomplished using a Polak-Ribière conjugate-gradient algorithm. The $v_{l,n}$ and d_n update directions are chosen as in [6].

6. Reconstructions and numerical results

In this section, we present the results on two cylindrical impenetrable objects: a circular cylinder, of 1.59 cm diameter, and a strip, 12 cm × 4 cm. The data were collected for eight incident angles, of {0, 5, 10, 15, 20, 45, 60, 90} degrees, over the observation sector $0 \le \theta < 360°$, with a sample spacing of $\Delta\theta = 10°$ (36 measurement stations). Taking into account the symmetry, the scattered field was composed of a set of 28 illuminations. For both iterative methods, we did not start from a zero estimate for the object or contrast function. The initial guesses were computed using a back-propagation method [5, 6]. With the NK method, the process was stopped when $\mathrm{Im}(\varepsilon_r) \ge 20$, which corresponds to a conductivity of 11.1 S/m, and an attenuation of -56 dB/cm at 10 GHz. With the MG method, $\chi_{\max} = 2i$, which corresponds to a conductivity of 1.1 S/m, and an attenuation of -14 dB/cm. For the circular object, we used a test domain of 5.5 × 5.5 cm^2 ($\approx 1.8\lambda_0 \times 1.8\lambda_0$), divided into 11 × 11 subsquares. For the strip object, the test domain was divided into 63 × 7 subsquares of 2 × 2 mm^2. For the circular object, and with the NK reconstruction method, we compare the results using either the identity operator or the gradient operator. For the strip, the reconstruction was effected with the NK method, using only the gradient operator.

7. Conclusion

Good results were obtained on the two impenetrable objects with both methods. With the NK method, the convergence was faster with the gradient operator than with the identity operator. The NK method with gradient regularization reconstructed larger contrast ($\chi_{\max} = 20$) than the MG method, but the MG method gave a good shape description with $\chi_{\max} = 2i$. The number of iterations used for the NK method was less than for the MG method. For the circular cylinder, 17 iterations were used with the NK method, and 32 iterations were used with the MG method. For the strip, 19 iterations and 50 iterations were used, respectively. Nevertheless, one can note that the MG method is faster: for the circular cylinder, 30 sec with the MG method instead of 2 min with NK, and for the strip, 4 min 30 sec instead of 20 sec, on a DEC Station 5000/240.

8. Acknowledgment

The authors wish to express their gratitude to Prof. R. E. Kleinman and Prof. P. M. van den Berg for helpful discussions concerning the modified-gradient method. The measured data were provided courtesy of Rome Laboratory, RL/ER, 31 Grenier St., Hanscom AFB, MA 01731-3010.

9. References

1. N. Joachimowicz, Ch. Pichot, and J. P. Hugonin, "Inverse scattering: an iterative numerical method for electromagnetic imaging," *IEEE Transactions on Antennas and Propagation*, **AP-39**, 1991, pp. 1742-1751.

2. R. E. Kleinman and P. M. van den Berg, "A modified gradient method for two-dimensional problems in tomography," *Journal of Computational and Applied Mathematics*, **42**, 1992, pp. 17-35.

3. R. E. Kleinman and P. M. van den Berg, "An extended range-modified technique for profile inversion," *Radio Science*, **28**, 1993, pp. 877-884.

4. P. M. van den Berg, R. E. Kleinman, "A total variation enhanced modified gradient algorithm for profile reconstruction," *Inverse Problems*, **11**, 1995, pp. L5-L10.

5. R. E. Kleinman and P. M. van den Berg, "Two-dimensional location and shape reconstruction," *Radio Science*, **29**, 1994, pp. 1157-1169.

6. K. Belkebir, R. E. Kleinman, and Ch. Pichot, "Microwave imaging–Location and shape reconstruction from multifrequency scattering data," submitted to *IEEE Transactions on Microwave Theory and Techniques*.

7. A. Franchois, Ch. Pichot, "Generalized cross validation applied to a Newton-type algorithm for microwave tomography," in M. A. Fiddy (ed.), *Proceedings Inverse Problems in Scattering and Imaging*, San Diego, 1992, pp. 232-240.

8. R. Kleinman, D. Lesselier, and B. Duchêne, "A nonlinearized iterative approach of the eddy current characterization of voids in a conductive half-space," *Proceedings PIERS*, Noordwijk, 1994, p. 408.