Review on spatial nonlinearity in plasmonic waveguides: single interface and slot configurations

Gilles Renversez

Athena team, Institut Fresnel (CNRS), Aix-Marseille Univ., Marseille, France
gilles.renversez@fresnel.fr

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Outline

1. What is a plasmon–soliton?
2. Motivations and context
3. Single interface configuration
4. Simple nonlinear slot waveguides
5. Slot with a metamaterial nonlinear core
6. Conclusion
Plasmon–soliton wave building blocks

Surface plasmon polariton

Solution of a linear wave equation

Spatial optical soliton

Solution of a nonlinear wave equation

Propagation constant

\[ \beta_p = k_0 \sqrt{\frac{\epsilon_1 \epsilon_2}{\epsilon_1 + \epsilon_2}} \]

Propagation constant

\[ \beta_s = k_0 n_0 \sqrt{1 + \frac{n_2 I}{n_0}} \]
What is a plasmon–soliton wave?

A nonlinear optical wave combining a spatial soliton and a plasmon field with a single propagation constant
What is a plasmon–soliton wave?

A nonlinear optical wave combining a spatial soliton and a plasmon field with a single propagation constant

\[ n = n_0 + n_2 I \]
Motivation — Plasmon-soliton coupling in the semi-infinite NL region case

Geometry of the 3-layer nonlinear model

Seminal articles:

- V. M. Agranovich et al.
  Nonlinear surface polaritons. 

- J. Ariyasu et al.
  Nonlinear surface polaritons guided by metal films. 
Motivation — Plasmon-soliton coupling in the semi-infinite NL region case

More recent articles:

- Using the 'interaction picture' approach:

- Starting from nonlinear Schrödinger’s equation:
Motivation — Plasmon-soliton coupling in the semi-infinite NL region case

More recent articles:

- Starting from Maxwell’s equations:
  
  A. R. Davoyan, I. V. Shadrivov, and Y. S. Kivshar. Self-focusing and spatial plasmon-polariton solitons.
  


Single interface configuration
Choice of a proper structure

Can we design a **feasible simple** structure supporting **low power plasmon-soliton** that can be **excited directly and recorded**?
Single interface configuration
Choice of a proper structure

Can we design a feasible simple structure supporting low power plasmon-soliton that can be excited directly and recorded?


\[ \epsilon_{NL} = \epsilon_1 + \alpha |E|^2 \]

Results
- no low power plasmon-soliton
- plasmon part cannot be recorded using NFO experiments
Single interface configuration
Choice of a proper structure

Can we design a feasible simple structure supporting low power plasmon-soliton that can be excited directly and recorded?

3-layer model (J. Ariyasu et al., Appl. Phys. 58(7), 2460 (1985)):

\[ \epsilon_{NL} = \epsilon_1 + \alpha |E|^2 \]

Our results

- No low power plasmon-soliton when air or water is chosen as external medium
**Single interface configuration**

**Choice of a proper structure**

Can we design a feasible simple structure supporting low power plasmon-soliton that can be excited directly and recorded?

A 4-layer model must be developed

\[
\epsilon_{\text{NL}} = \epsilon_1 + \alpha |E|^2
\]

Low-power plasmon-soliton in realistic nonlinear planar structures.

W. Walasik, G. Renversez, Y. Kartashov
Stationary plasmon–soliton waves in metal-dielectric nonlinear planar structures: Modeling and properties.
Formulation of our 1D model: TM vector case assumptions
Stationary waves and nonlinear eigenvalue problem

Kerr nonlinear dielectric
\[ \epsilon_{NL} = \epsilon_1 + \alpha |E|^2 \]

Linear dielectric \( \epsilon_2 \)
Metal \( \epsilon_3 \)
External medium \( \epsilon_4 \)

Geometry of the 4-layer nonlinear model used to study 3-layer structures.

- Metal layer \( \rightarrow \) **TM waves:**
  \[ E = [E_x, 0, iE_z] \text{ and } H = [0, H_y, 0] \]

- **Kerr nonlinearity**

- Transverse field prevails \( \rightarrow \) \( |E_z| \ll |E_x| \) (verified *a posteriori*)

- We look for **stationary solutions:**
  \[ E(x, z, t) = E_{NL}(x) \exp[i(\beta_{NL} k_0 z - \omega t)] \]
  \[ k_0 = \frac{2\pi}{\lambda}, \text{ and } \beta_{NL} \text{ is the effective index of this nonlinear wave} \]
Nonlinear wave equation in the frame of the nonlinear eigenvalue problem

Maxwell’s equations $\rightarrow$ nonlinear wave equation for $E_{NL,x}$

$$\frac{d^2 E_{NL,x}}{dx^2} - k_0^2 q^2(x) E_{NL,x} + k_0^2 \alpha(x) E_{NL,x}^3 = 0,$$

with $q^2(x) = \beta_{NL}^2 - \epsilon(x)$,
and $\alpha(x) = \mathcal{H}(-x) \epsilon_0 c \epsilon(x) n_2$

$\mathcal{H}(x)$ — Heaviside step function

Analytical solutions in the whole structure

- In nonlinear region: the well-known solitonic-type solution:

$$E_{NL,x}(x) = E_{0,x}(\beta_{NL}) \text{sech}[k_0(\beta_{NL}^2 - \epsilon_1)^{\frac{1}{2}}(x - x_0)]$$

where $x_0$ denotes the center of the soliton

- In the linear regions: decreasing and/or increasing exponentials
Nonlinear dispersion relation (NDR) for the single interface configuration

Boundary conditions $\rightarrow$ closed form for the NDR of the 4-layer model

\[
\Phi_+ \left( \tilde{q}_4 + \tilde{q}_3 \right) \exp(2k_0 \tilde{q}_3 \epsilon_3 d) + \Phi_- \left( \tilde{q}_4 - \tilde{q}_3 \right) = 0, \tag{2}
\]

\[
\Phi_{\pm} = \left( 1 \pm \frac{q_{1\text{NL}}}{\tilde{q}_3} \right) + \left( \frac{q_{1\text{NL}}}{\tilde{q}_2} \pm \frac{\tilde{q}_2}{\tilde{q}_3} \right) \tanh(k_0 \tilde{q}_2 \epsilon_2 L), \tag{3}
\]

with $\tilde{q}_j = q(x)/\epsilon(x)$ in the $j$-th layer, and $q_{1\text{NL}} = \tilde{q}_1 \tanh(k_0 \tilde{q}_1 \epsilon_1 x_0)$

All the plasmon-soliton characteristics can be evaluated

1. Eq. (2) $\rightarrow$ allowed $\beta_{\text{NL}}$ of 1D nonlinear problem
2. $\beta_{\text{NL}}$ and $E_{\text{NL},x}(x)$ $\rightarrow$ other field components ($E_{\text{NL},z}(x)$ and $H_{\text{NL},y}(x)$)
3. $\beta_{\text{NL}}$ and $E_{\text{NL},x}(x)$ $\rightarrow$ power $P$
4. limiting cases: 3-layer and 2-layer model results
First example of low power plasmon-soliton waves

Realistic opto-geometric structure parameters

- Chalcogenide glasses ($\epsilon_1 = 2.4707^2$, $n_2 = 10^{-17} m^2/W$) $\rightarrow$ high nonlinear coefficient
  - Coated planar chalcogenide waveguides already fabricated (*V. Nazabal et al., Int. J. Appl. Ceram Technol.*, 8, 2011)
  - Spatial solitons already observed in planar chalcogenide waveguides (*M. Chauvet et al., Opt. Lett.*, 34, 2009)

- Silica ($\epsilon_2 = 1.443^2$, $L = 15\text{nm}$) $\rightarrow$ well known, good compatibility

- Gold ($\epsilon_3 = -96$, $d = 40\text{nm}$) $\rightarrow$ low loss, good compatibility

- Air as external medium ($\epsilon_4 = 1$) $\rightarrow$ Near field optics to record the plasmon part of the field
First example of low power plasmon-soliton waves

Realistic soliton parameters $\rightarrow$ feasible excitation of the plasmon-soliton peak power $P \simeq 1.07 \text{GW/cm}^2$

($P \simeq 2\text{GW/cm}^2$ reported by *M. Chauvet et al., Opt. Lett., 34, 2009*)

![Graphs showing intensity vs. position](image)

Recordable plasmon field ($E \simeq 4.5 \text{MV/m}$)
Soliton center position influence

- \( x_0 = 20\lambda \)
- \( \beta = 2.4707317 \)
- total power = 11.28 kW
- FWHM = 34 \( \mu m \)

- soliton peak intensity = 0.63 \( GW/cm^2 \)
- metal/air interface intensity = 0.49 \( MW/cm^2 \)
- metal/air interface electric field = 3.04 \( MV/m \)

\( \lambda = 1.5 \mu m \quad n_{NL} = 2.4707 \quad n = 1.624 \quad \epsilon_m = -96 \quad L = 22 \)

\( d = 40 \text{ nm} \)
**Soliton center position influence**

- \( x_0 = 10\lambda \)
- \( \beta = 2.4707486 \)
- total power = 10.01 kW
- FWHM = 27 \( \mu \)m

- soliton peak intensity = 0.97 GW/cm\(^2\)
- metal/air interface intensity = 2.01 MW/cm\(^2\)
- metal/air interface electric field = 6.11 MV/m

\[ \lambda = 1.5 \, \mu m \quad n_{NL} = 2.4707 \quad n = 1.624 \quad \epsilon_m = -96 \quad L = 22 \]
Soliton center position influence

- $x_0 = 5\lambda$
- $\beta = 2.4707859$
- total power = 8.88 kW
- FWHM = 20 $\mu$m

- soliton peak intensity = 1.71 GW/cm$^2$
- metal/air interface intensity = 5.36 MW/cm$^2$
- metal/air interface electric field = 9.99 MV/m

$\lambda = 1.5 \mu m$
$nm$
d = 40 nm
Model parameters for the experimental design: \( L \) and \( d \), with \( P_{\text{peak}} \leq 3\text{GW/cm}^2 \)
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![3D graph showing the relationship between metal and buffer thicknesses for different gold and SiO2 chalco1 configurations.](image-url)
Model parameters for the experimental design: $L$ and $d$, with $P_{peak} \leq 3\text{GW/cm}^2$
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Other composition for the buffer: silica $\rightarrow$ mulit ($n = 1.624$)
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Other composition for the buffer: silica $\rightarrow$ mullit $(n = 1.624)$
**Metal thickness & soliton center position scan (Max 3GW/cm²)**

### Peak intensity GW/cm²

- **x₀ [µm]**
  - values from -30 to 0
- **Metal [nm]**
  - values from 46 to 63

### FWHM [µm]

- **x₀ [µm]**
  - values from -30 to 0
- **Metal [nm]**
  - values from 46 to 63

### Plasmon peak MW/cm²

- **x₀ [µm]**
  - values from -30 to 0
- **Metal [nm]**
  - values from 46 to 63

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- **λ = 1.5 µm**
- **chalcogenide glass \( n_{NL} = 2.4707 \)**
- **buffer: \( n = 1.624 \)**
- **buffer thickness = 21 nm**
- **metal: gold \( \varepsilon = -96 \)**
First fabrications using chalcogenide glass

with the chalcogenide layer above a silica film on a BK7 glass substrate

with the chalcogenide layer above a silica film on a silicon substrate

Structures made at the University of Rennes I by EVC-ISCR (courtesy of V. Nazabal)
Nonlinear slot waveguide — Introduction

Linear case:

- V. R. Almeida et al.
  Guiding and confining light in void nanostructure,

- J. A. Dionne et al.
  Plasmon slot waveguides: Towards chip-scale propagation with subwavelength-scale localization,
Nonlinear slot waveguide — Introduction

Nonlinear case:


No experimental results on plasmon–soliton in nonlinear slot

Too high nonlinear index change \( \Delta n = n_2 I \)
Nonlinear slot waveguide — Introduction

- **Waveguide configuration**
- **Subwavelength focusing**
- **Control the solutions with the power**
- **Peculiar nonlinear effects**

![Waveguide configuration diagram](image.png)
Nonlinear slot waveguide — Models: eigenvalue problem with nonlinearity

Approach: the spatial dependency of transverse field components is kept

"Modal" nonlinear solutions of Maxwell’s equations for TM stationary waves using field continuity conditions in 1D structure

Both $n_{\text{eff}}$ and field profiles that depend on total power $P_{\text{tot}}$ are computed
Nonlinear slot waveguide — Models: eigenvalue problem with nonlinearity

Common hypotheses to our two models

- **Stationary solutions of Maxwell’s equations:**
  \[
  \begin{align*}
  \{ E(x, z, t) \} & = \{ E_{NL}(x) \} \exp[i(\beta_{NL}k_0z - \omega t)] \\
  \{ H(x, z, t) \} & = \{ H_{NL}(x) \}
  \end{align*}
  \]
  where \( k_0 = 2\pi/\lambda \), and \( \beta_{NL} \) is the effective index \( n_{eff} \) of this nonlinear wave.

- **TM waves:**
  \( E = [E_x, 0, iE_z] \) and \( H = [0, H_y, 0] \)

- **Kerr nonlinearity**

- Maxwell’s equations + boundary conditions → **Nonlinear dispersion relation**
Nonlinear slot waveguide — Models: eigenvalue problem with nonlinearity

Jacobi Elliptic function based Model (JEM)

Extension to slot configuration of
W. Chen and A. A. Maradudin

- Low nonlinearity depending only on the transverse electric field
- **Analytical formulas for field shapes and nonlinear dispersion relation**

Finite Element Method (FEM)

Adaptation to slot configuration of:
F. Drouart, G. Renversez, *et al.*

- **Exact treatment of Kerr-type nonlinearity**
- Field shapes and dispersion curves obtained numerically
Nonlinear slot waveguide — Models: eigenvalue problem with nonlinearity

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Nonlinear slot waveguide — Dispersion relations

A. Davoyan, I. Shadrivov, and Y. Kivshar, Nonlinear plasmonic slot waveguides, *Opt. Express*, 16(26), 2008
Nonlinear slot waveguide — Dispersion relations

(a) Dispersion relations for different slot configurations. The graph plots the propagation constant ($\beta$) against the waveguide’s index of refraction ($\Delta n$). Different colors represent different slot types: AN, S, and AS.

(b) - (h) Field distributions for various slot configurations. The graphs show the magnetic field ($H_y$) distribution along the slot for different x positions.
Nonlinear slot waveguide — Dispersion relations

\[ \Delta n \]

\[ H_y \text{ [MA/m]} \times [\mu m] \]

\[ H_y \text{ [GA/m]} \times [\mu m] \]

\[ H_y \text{ [MA/m]} \]

\[ H_y \text{ [GA/m]} \]
Energy interpretation for the emergence of the asymmetric mode from the symmetric mode in a symmetric nonlinear simple slot

Power as a function of normalized propagation constant $\beta$ for the first three modes (S: symmetric, AN: anti-symmetric, AS: asymmetric), single interface is the structure with a semi-infinite nonlinear dielectric medium and a semi-infinite metal region. First: global view, Second: zoom on the bifurcation region where the asymmetric mode emerges.
Energy interpretation for the emergence of the asymmetric mode from the symmetric mode in a symmetric nonlinear simple slot

Power as a function of normalized propagation constant $\beta$ for the first three modes (S: symmetric, AN: anti-symmetric, AS: asymmetric), single interface is the structure with a semi-infinite nonlinear dielectric medium and a semi-infinite metal region. First: global view, Second: zoom on the bifurcation region where the asymmetric mode emerges.
Size and permittivity contrast effects in nonlinear slot

Metal
$\epsilon_1 = \epsilon_{l,1}$

Nonlinear dielectric
$\epsilon_2 = \epsilon_{l,2} + \alpha |E|^2$

Metal
$\epsilon_1 = \epsilon_{l,1}$

Bifurcation — spontaneous symmetry breaking
Asymmetric modes in symmetric structures
Parameter rules to lower power needed for nonlinear effects
Size and permittivity contrast effects in nonlinear slot

\[ \begin{align*}
\text{Metal} & : \quad \varepsilon_1 = \varepsilon_{l,1} \\
\text{Nonlinear dielectric} & : \quad \varepsilon_2 = \varepsilon_{l,2} + \alpha|\mathbf{E}|^2 \\
\text{Metal} & : \quad \varepsilon_1 = \varepsilon_{l,1}
\end{align*} \]

\[ \begin{array}{c}
\begin{array}{cc}
\text{Metal} & \text{Nonlinear dielectric} \\
\varepsilon_1 = \varepsilon_{l,1} & \varepsilon_2 = \varepsilon_{l,2} + \alpha|\mathbf{E}|^2 \\
\text{Metal} & \varepsilon_1 = \varepsilon_{l,1}
\end{array}
\end{array} \]

- Bifurcation — spontaneous symmetry breaking
- Asymmetric modes in symmetric structures
- Parameter rules to lower power needed for nonlinear effects
Stability analysis: introduction

D. J. Mitchell and A. W. Snyder
Stability of fundamental nonlinear guided waves

Vakhitov–Kolokolov criterion
linear stability analysis → geometrical approach

- possible stability of a fundamental mode
- weak guiding approximation
- first use to nonlinear slot waveguides

\[ \beta \]

possibly stable

\[ n \]

\[ n-1 \]

unstable

\[ P_0 \]

\[ P \]
Stability analysis: introduction

Temporal propagation simulation over 15\lambda for the asymmetric mode |E| (comsol based)

Numerical results confirm the conclusions drawn from the geometrical method
Stability analysis: introduction

Temporal propagation simulation over $15\lambda$
MEEP full vector nonlinear FDTD simulation for the asymmetric mode $H_y$
Stability analysis: full vector numerical results from nonlinear FDTD simulations

Dispersion curves with stability results for the symmetric and asymmetric modes from FDTD simulations, $d = 500$ nm amorphous Si core surrounded by gold.
Intermediate conclusions

- Two semi-analytical models for nonlinear slot waveguide configuration with a finite size nonlinear core
- Prediction of the existence of higher order modes in nonlinear slot waveguides
- Study of size and permittivity contrast effects on bifurcation threshold → ways to reduce it
- Stability study of plasmon–solitons using two numerical methods → stable asymmetric mode


Intermediate conclusions

- Two semi-analytical models for **nonlinear slot waveguide** configuration with a finite size nonlinear core

- Prediction of the existence of **higher order modes** in nonlinear slot waveguides

- Study of size and permittivity contrast effects on **bifurcation threshold** → ways to reduce it

- **Stability study** of plasmon–solitons using two numerical methods → stable asymmetric mode

- But losses and bifurcation threshold are still high for realistic and useful parameters.
Slot with a metamaterial nonlinear core — Introduction

The idea to use metamaterial and/or epsilon-near-zero (ENZ) materials to enhance nonlinear effects was already proposed several times:

- **A. Husakou and J. Hermann**
  Steplike transmission of light through a Metal-Dielectric Mutilayer Structure due to an Intensity-Dependent Sign of the Effective Dielectric Constant

- **A. Ciattoni et al.**
  Extreme nonlinear electrodynamics in metamaterials with very small linear permittivity,

- **A. D. Neira et al.**
  Eliminating material constraints for nonlinearity with plasmonic metamaterials

Nevertheless, **nonlinear ENZ waveguide problems and the key role of anisotropy seem to have been partially overlooked.**
Slot with a metamaterial nonlinear core

\[ \varepsilon_{\text{core}} \rightarrow \bar{\varepsilon}_{\text{core}} = \begin{pmatrix} \varepsilon_x = \varepsilon_{\perp} & 0 & 0 \\ 0 & \varepsilon_y = \varepsilon_{//} & 0 \\ 0 & 0 & \varepsilon_z = \varepsilon_{//} \end{pmatrix} \] (4)

- Effective Medium Theory (EMT) \( \rightarrow \bar{\varepsilon}_{\text{core}} \) tensor for uniaxial anistropic medium as:

\[ \begin{aligned} \varepsilon_y &= \varepsilon_z = r\varepsilon_2 + (1 - r)\varepsilon_1 = \varepsilon_{//} \\
\varepsilon_x &= \frac{\varepsilon_1\varepsilon_2}{r\varepsilon_1 + (1 - r)\varepsilon_2} = \varepsilon_{\perp} \end{aligned} \]

\[ r = \frac{d_2}{d_1 + d_2} \] is the ratio of the 2nd material in the layered structure.
Slot with a metamaterial nonlinear core

Metamaterial based nonlinear core

Full slot with its metamaterial nonlinear core

$\varepsilon_{core} \rightarrow \bar{\varepsilon}_{core} = \begin{pmatrix} \varepsilon_x = \varepsilon_\perp & 0 & 0 \\ 0 & \varepsilon_y = \varepsilon_\parallel & 0 \\ 0 & 0 & \varepsilon_z = \varepsilon_\parallel \end{pmatrix}$ \hspace{1cm} (4)

- As first order approximation, the nonlinear part of the permittivity is isotropic. We have recently extended our methods to deal with anisotropic nonlinearity. In this case, only the nonlinear FEM method can be used.
Equations for the TM waves with linear anisotropy and Kerr type isotropic nonlinearity

\[ E_x(x) = \frac{\Re(n_{eff})H_y(x)}{\varepsilon_0 \varepsilon_x(x)x} \]  
\[ E_z(x) = \frac{1}{\varepsilon_0 \varepsilon_z(x)\omega} \frac{dH_y(x)}{dx} \]  
\[ k_0 \Re(n_{eff})E_x(x) - \frac{dE_z(x)}{dx} = \omega \mu_0 H_y(x) \]  
\[ \varepsilon_j = \varepsilon_{jj} + \alpha |E_x^2 + E_z^2|, \ \forall j \in \{x, y, z\} \ \text{(full nonlinearity)} \]  
\[ \text{or} \]  
\[ \varepsilon_j = \varepsilon_{jj} + \alpha |E_x^2|, \ \forall j \in \{x, y, z\} \ \text{(full adapted to EJEM nonlinearity)} \]

For the FEM:
We use the two continuous components across the interfaces, \( H_y \) and \( E_z \), to write the weak formulation.

\[ \Rightarrow \text{A nonlinear eigenvalue problem made of 2 coupled equations with nodal elements only.} \]
Slot with a metamaterial nonlinear core — Effective nonlinearity

- In the frame of our semi-analytical 1D model (Maxwell’s equations & stationary TM waves), we obtain for the **effective nonlinearity** $a_{nl}$ using Eq.(4):

$$a_{nl}^{EJEM} = -\tilde{\alpha} n_{eff}^2 \left( n_{eff}^2 (\varepsilon_{xx} - \varepsilon_{zz}) - \varepsilon_{xx}^2 \right) / \left( \varepsilon_{xx}^4 c^2 \varepsilon_0^2 \right)$$

(11)  

with $\tilde{\alpha} = \varepsilon_0 c \Re(\varepsilon_{1})(1 - r)n_{2,1}$.

- $a_{nl}^{EJEM} \rightarrow a_{nl,ISOTROPIC}$ when $\varepsilon_{xx} \rightarrow \varepsilon_{zz}$

- Metamaterial properties $\Rightarrow \varepsilon_{xx} = \varepsilon_{\perp} = f(\varepsilon_1, d_1, \varepsilon_2, d_2)$ and $\varepsilon_{zz} = \varepsilon_{//} = g(\varepsilon_1, d_1, \varepsilon_2, d_2)$

- $n_{eff}$ depends on the total power $P_{tot}$ and on the opto-geometric parameters of the slot

**Consequences in the elliptical case:** $\varepsilon_{\perp} \gg \varepsilon_{//} > 0$

- Lowering the bifurcation threshold for symmetry breaking:
  1 GW/cm$^2$ $\rightarrow$ 10 MW/cm$^2$

Slot with a metamaterial nonlinear core — Numerical results with our 4 models for the isotropic case: semi-analytical Jacobi Elliptic function based Model (JEM), nonlinear FEMs (full or adapted), and the Interface Model (IM)

Isotropic case: nonlinear dispersion relation for the symmetric and asymmetric modes as a function of total power $P_{tot}$. $\lambda = 1.55 \mu m$, $d_{core} = 400$ nm, and $n_2 = 2 \times 10^{-17} m^2/W$, and for $\varepsilon_\perp = 0.042$, $\varepsilon_{//} = 9.07$.

- Here, in the FEM adapted to JEM only $E_x$ is in the nonlinear term in order to correspond with JEM
Slot with a metamaterial nonlinear core — Numerical results
with our 3 models for the anisotropic case: semi-analytical Extended Jacobi Elliptic function based Model (EJEM), nonlinear FEMs (full or adapted)

**Anisotropic case:** zoom of nonlinear dispersion relation for the symmetric and asymmetric modes as a function of total power $P_{tot}$, around the bifurcation for $\varepsilon_{\perp} = 0.042$, $\varepsilon_{//} = 9.07$,.
Slot with a metamaterial nonlinear core — Numerical results

Influence of the linear anisotropy on the bifurcation threshold

**Anisotropic case:** Power threshold $P_{tot,th}$ as a function of linear transverse permittivity $\varepsilon_{xx}$ in the elliptical case for two longitudinal permittivity $\varepsilon_{zz}$ values. Isotropic case is shown by the black curve. Inset: $P_{tot,th}$ as a function of $\varepsilon_{zz}$ for two $\varepsilon_{xx}$ values.
**Conclusions**

- **Stability of the asymmetric mode** of the isotropic slot waveguide

- 2 or 3 orders of magnitude reduction of the bifurcation threshold using realistic metamaterial based nonlinear core
  → Important nonlinear effects in plasmonic waveguides at low power

**2D models:**

- We also investigated the 2D model: a rib waveguide with a nonlinear layer using vector FEM with edge elements...for the next time.
- We computed the nonlinear figures of merit taking into account the losses for comparison with state of the art full dielectric nonlinear waveguides.