NUMERICAL MODELIZATION OF TRANSIENT CURRENT IN RELAY

W. Legros - A. Nicolet - M. Paganini
University of Liège - Institut Montefiore - Sart Tilman B28
B-4000 Liège - Belgium

Abstract

This paper describes the application of a magnetic field computation software to the evaluation of transient current in the coil of a relay. The software AZIMUT is used to compute induced current in the coil and electromagnetic force acting on the mobile part of the relay. This force is introduced in a simple mechanical equation to determine the change of position of the mobile part.

The field computation software has been modified to take into account the electromotive force induced by the movement. Some of the difficulties met in coupling the mechanical and electrical problems are discussed. The current measured in a relay fed with a voltage step is then compared to a computed curve.

Introduction

The computation of transient current in a relay involves two interacting domains : an electrical one and a mechanical one. The electrical domain influences the mechanical one through the forces it creates on the mechanic parts. The mechanical domain influences the electrical one through the changes of geometry.

The modelling's basis is the magnetic field computation software AZIMUT. It allows a two-dimensional field computation taking into account the linear magnetic materials and the induced currents in the conductors. A routine computes the forces acting on the modelized objects. A modification has been brought about in the software to take into account the electromotive force induced by the movement. Other routines compute the mechanical aspect and modify the geometry directly in the data of the electromagnetic software. The solutions of the electrical and mechanical equations are uncoupled and we will discuss the conditions of validity and the advantages of this approach.

Magnetic field computation

The two-dimensional transient magnetic field is computed using the software AZIMUT based on the boundary element method [1]. The magnetic materials are taken into account, assuming the vector potential $\vec{A}$ and its normal derivative $\frac{\partial \vec{A}}{\partial n}$ are unknown on the boundary, and using the relation :

$$c\,\vec{A} = \oint_{\Gamma} \vec{A}\,\frac{\partial G}{\partial n}\,d\Gamma - \oint_{\Gamma} \frac{\partial \vec{A}}{\partial n}\,G\,d\Gamma \tag{1}$$

where :

c = 0 outside the domain,
1 inside the domain,
1/2 on a smooth boundary,
G is the Green function,
$\vec{n}$ is the exterior normal of the boundary Γ;

with the continuity conditions :

$$\vec{A}_{in} = \vec{A}_{out}$$
$$\left[\frac{1}{\mu_{in}}\frac{\partial A}{\partial n}\right]_{in} = \left[\frac{1}{\mu_{out}}\frac{\partial A}{\partial n}\right]_{out} \tag{2}$$

The discretization of the magnetic materials' boundary leads to a system of linear equations.

Transient induced currents are computed by expressing the local Ohm's law $\left(\rho\,\vec{J} = \vec{E}\right)$ and the Faraday induction law $\left(\vec{E} = -\frac{d\vec{A}}{dt} - \overrightarrow{\text{grad}}\,V\right)$:

$$\rho\,\vec{J} = -\frac{d\vec{A}}{dt} - \overrightarrow{\text{grad}}\,V \tag{3}$$

where :

$\vec{J}$: is the current density,

ρ : is the resistivity,

$\frac{d\vec{A}}{dt}$: is the induced emf,

grad V : is the external voltage source.

The influence of a current density is :

$$\vec{A} = \int_{v} \mu_0\,\vec{J}\,G\,dV \tag{4}$$

The discretization of the inside of the conductors leads to a linear system of differential equations.

The complete system is obtained by expressing cross-influences between induced currents and magnetic boundaries. Some additional equations are necessary to express forced currents and/or voltages.

Forces and movement

The forces acting on an object are computed using the following formulae [2] :

- Lorentz's force :

$$\vec{f}_l = \int_{v} \vec{J} \times \vec{B}\,dV \tag{5}$$

- Magnetic force :

$$\vec{f}_m = \oint_{\Gamma} \frac{\mu_{in} - \mu_{out}}{2}\left(\vec{H}_{in} \cdot \vec{H}_{out}\right)\vec{n}\,d\Gamma \tag{6}$$

Formula (6) is well adapted to the boundary element method because the field's value on both sides of the boundary can be computed directly from the values of $\vec{A}$ and $\frac{\partial \vec{A}}{\partial n}$ on the boundary.

Equation of the movement is :

$$m\frac{d^2x}{dt^2} + f\frac{dx}{dt} + kx = F_{gravitationnal} + F_{electromagnetic} \tag{7}$$

where :

m is the mass,

0018-9464/89/0900-3593$01.00©1989 IEEE

x is the position,
f is the friction coefficient,
k is the elastic coefficient of a spring.

Motional induced electromotive force

After discretization, expression (1) becomes :

$$M_1 \underline{X} + M_2 \underline{Y} = \underline{b}_1 \qquad (1')$$

and (3) becomes :

$$\frac{d}{dt}\left(M_3 \underline{X} + M_4 \underline{Y}\right) + R\,\underline{Y} + \underline{U} = \frac{d}{dt}\underline{b}_2 \qquad (3')$$

where :

$\underline{X}$: is the boundary element method unknowns vector,

$\underline{Y}$: is the induced currents vector,

M : are the influence matrices,

$\underline{b}$: is the influence of forced currents vector,

R : is the resistance matrix,

$\underline{U}$: is the terminal voltage vector.

If the structure is mobile, (3') becomes :

$$\underbrace{\left(M_3 \frac{d}{dt}\underline{X} + M_4 \frac{d}{dt}\underline{Y}\right)}_{(a)} + \underbrace{\left(\left(\frac{dM_3}{dt}\right)\underline{X} + \left(\frac{dM_4}{dt}\right)\underline{Y}\right)}_{(b)} + R\,\underline{Y} + \underline{U} = \frac{d\,\underline{b}_2}{dt} \qquad (3'')$$

The expression of the flux in the relay coil is $\phi = L\,I$ where I is the current and L the inductance.

According to Lenz's law :

$$emf = \frac{d\phi}{dt} = L\frac{dI}{dt} + \frac{dL}{dt}I \qquad (8)$$

term (a) is the expression of $L\frac{dI}{dt}$, i.e. the flux variation due to the time variation of excitation;

term (b) is the expression of $I\frac{dL}{dt}$, i.e. the flux variation due to the movement (motional-induced electromotive force).

Note

1) Standard electrotechnical approximations are assumed (Displacement current neglected).

2) The distinction between motional-induced electromotive force and electromotive force due to time changing flux depends on the observer [3].

Time discretization of system (1'), (3") (Euler implicit scheme) gives the linear system that relates values at time t_i with values at time $t_{i+1} = t_i + \Delta t$.

$$\begin{bmatrix} M_{1(i+1)} & M_{2(i+1)} \\ 2\,M_{3(i+1)} - M_{3(i)} & R_{(i+1)}\Delta t + 2\,M_{4(i+1)} - M_{4(i)} \end{bmatrix} \begin{bmatrix} \underline{X}_{(i+1)} \\ \underline{Y}_{(i+1)} \end{bmatrix} + \begin{bmatrix} 0 \\ \underline{U}_{(i+1)}\Delta t \end{bmatrix} = \begin{bmatrix} \underline{b}_{1(i+1)} \\ \underline{b}_{2(i+1)} - \underline{b}_{2(i)} \end{bmatrix} + \begin{bmatrix} 0 \\ M_{3(i+1)}\underline{X}_{(i)} + M_{4(i+1)}\underline{Y}_{(i)} \end{bmatrix} \qquad (9)$$

In the case of a linear system with constant resistances and inductances, it can be shown that the Euler implicit scheme is always stable [4].

Coupling magnetical and mechanical systems

Table 1 describes the main loop of the simulation. We start with current and position values I_n and x_n at time t_n and we compute I_{n+1} and x_{n+1} at time $t_{n+1} = t_n + \Delta t$.

Problem	*Initial conditions*	*Hypothesis*	*Method of Resolution*	*Solution*
Electrical problem (equations (1') (3"))	$x(t_n)$ $I(t_n)$	Flux variation between t_n and t_{n+1} $= \frac{\phi(t_n) - \phi(t_{n-1})}{t_n - t_{n-1}}$	Euler implicit time discretization scheme	$I(t_{n+1})$
Forces	$I(t_{n+1})$ $x(t_n)$	x and I do not vary too much on a time step	Formulae (5) and (6)	$F(t_{n+1})$
Mechanical problem (equation (7))	$x(t_n)$ $F(t_{n+1})$	F is constant on a time step	Analytical	$x(t_{n+1})$

Table 1

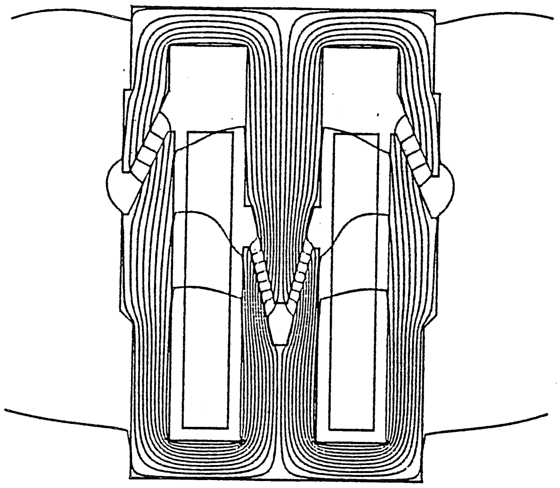

Figure 1

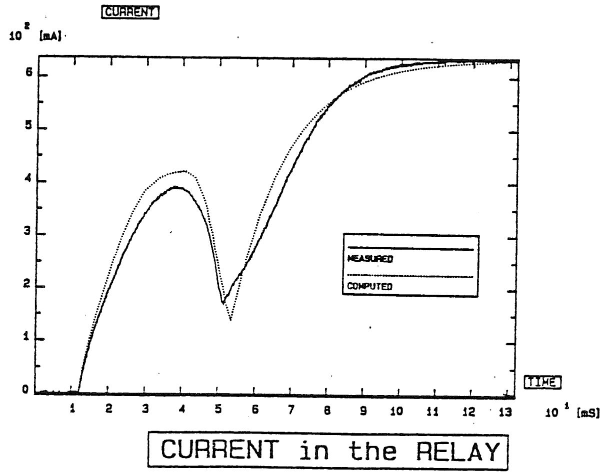

Figure 2

All hypothesis can be sum up by saying that current I and position x can only have small variations on a time step. A variable time step method has been used to fulfil those requirements. In the case of a complete coupling between electrical and mechanical problems we can say that :

- Hypothesis of small variation of x and I on a time step remain;
- An implicit scheme is difficult to use because the force depends quadratically on the electromagnetic values and therefore the system is non-linear.
- The resolution depends on the kind of movement. However, in our approach, it requires little changes in the program to force a movement by expressing a position changing independently of the electromagnetic force.

Results

Figure 1 shows the field lines in a two-dimensional representation of an open relay.

Figure 2 shows the computed and the measured currents in the relay. The end of the movement is given by the discontinuity in the slope of the curves. At the end of the movement, the current is decreasing because of the motional-induced electromotive force. It shows the importance of this phenomenon in this kind of study.

The shape of the curves are similar.

Differences between the curves may be explained by :

- unaccuracy of the physical values in the model,
- the real structure is three-dimensional,
- non linearities in the magnetic materials.

Conclusion

We have modified a field computation software to allow simple movements. The example shows that the dynamic phenomena are well taken into account. In the case of the relay, the movement is forced along one direction and only one object moves but the program is more general : several objects can move with translation (free or forced along an axe) and/or rotation.

Improvements of the modelling would be three-dimensional computation and non-linearities for magnetic materials but it will lead to very important computation time.

References

[1] A. Genon, W. Legros, J.P. Adriaens, A. Nicolet. "Computation of Extra Joule Losses in Power Transformers". International conference on electrical machines, Pisa, Italy, September 88.

[2] A. Frühling. "Cours d'électricité". Dunod, 1966.

[3] P. Lorrain, D. Corson. "Electromagnetic Fields and Waves". Freeman, 1970.

[4] J. Stoer, R. Bulirsch. "Introduction to Numerical Analysis". Springer Verlag, 1976.