Comparison of various methods for the modeling of thin magnetic plates[a)]

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Computation of the magnetic field in thin plates is particularly difficult because nodes on opposite sides of the plate are very close together and because the elements inside the plate are very flat. Three different methods, together with corresponding variations and combinations were used to compute the magnetic field in the plate: the boundary element method, the finite element method, and use of thin plate transmission conditions. With classical methods, the results are good for all cases, provided the computation of coefficients is sufficiently accurate. For very thin plates, a fine meshing is required to avoid failure of the computation. A new method that avoids those problems was designed. Since the vector potential varies very steeply across the plate, the idea is to replace the real plate by an equivalent double layer of current. The result is a special transmission condition, relating the values of the vector potential and of the tangential magnetic field on both sides of the plate. This condition does not depend on the kind of numerical method used and has been introduced in finite element and boundary element methods. Comparison with previous results shows that this highly economical method is both accurate and valid. This is the case even for very thin plates, where other methods fail, particularly if the meshing is too coarse.

I. INTRODUCTION

Computation of the magnetic field in thin steel plates is particularly difficult because the nodes on opposite sides of the plate are very close together.[1] In numerical computations, the coefficients associated with the nodes are very similar and lead to an ill-conditioned system. The most common methods used in magnetic-field numerical computation are the finite element method and the boundary element method. To provide the desired level of accuracy for the coefficients we used adaptative integration methods and required a relative precision of 10^{-4}.

As those methods are quite costly in computation time, a new method which avoids those problems was designed. Since the vector potential varies very steeply across the plate, the idea is to replace the real plate by an equivalent double layer of current.

Accordingly, the steep variation of the vector potential is replaced by a discontinuity depending on the characteristics of the plate and on the tangential field. The result is a special transmission condition, relating the values of the vector potential and of the tangential magnetic field on both sides of the plate. This condition does not depend on the kind of numerical method used and has been introduced in finite element and boundary element methods.

In this paper, we restrict ourselves to the two-dimensional case where the potential vector has only one component.

II. NUMERICAL METHODS

The direct boundary element method (BEM) is based on the Green's function and the Green's identity.[1,2] In two-dimensional magnetostatics the vector potential has only one component and we use

$$cA = \oint_\Gamma A\, \partial G/\partial n\, d\Gamma - \oint_\Gamma G\, \partial A/\partial n\, d\Gamma, \qquad (1)$$

where G is the free-space Green's function of the two-dimensional laplacian; $c=0.5$ on a smooth boundary; A is the vector potential; and $\partial/\partial n$ is the normal derivative. Integrals are taken on the boundary Γ of the domains and the method involves no internal nodes.

The finite element method (FEM) is based on the Galerkin method[3] and uses, for a domain Ω of boundary Γ,

$$\int_\Omega (\nu \operatorname{grad} A \operatorname{grad} w - Jw)\, d\Omega - \oint_\Gamma w\, \nu\, \partial A/\partial n\, d\Gamma = 0, \qquad (2)$$

with A the unknown vector potential, ν the magnetic reluctivity, J the current density, and w the weighting function. This method involves internal points.

The boundary term allows us to take A and $\partial A/\partial n$ as unknown on the boundary. This can be used to couple the boundary element method and the finite element method. At a boundary point between two domains, A and $\partial A/\partial n$ are supposed unknown in each domain. BEM or FEM give an equation for each domain.

Two more equations are necessary to close the system. In the interface between two magnetic media those conditions are the continuity of the potential vector A and of the tangential magnetic field $\nu\, \partial A/\partial n = H_t$.

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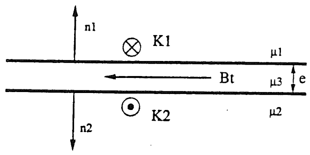

FIG. 1. Thin magnetic plate with equivalent surface currents.

III. TRANSMISSION CONDITIONS FOR THIN MAGNETIC PLATES

A thin magnetic plate of thickness e and permeability μ_3 separating two media of permeability respectively μ_1 and μ_2 is considered (see Fig. 1). In the indirect boundary element method the interface between two media is replaced by a surface current. If this technique is applied to the thin plate it is replaced by equivalent surface currents, K_1 on one side and K_2 on the other, which are very close together.

The approximation is to consider those two layers as a superposition of a single layer of current K_s and a double layer of current K_d at the middle of the plate. Using the indirect boundary element formulation we find

$$K_s = K_1 - K_2 = 2(\mu_1 - \mu_2)[(B_t/\mu_0)/(\mu_1+\mu_2)], \quad (3)$$

$$K_d = (K_1+K_2)/2$$
$$= (\mu_1+\mu_2-2\mu_3)[(B_t/\mu_0)/(\mu_1+\mu_2)], \quad (4)$$

where B_t is the magnetic field tangential to the middle axis of the plate and given by

$$B_t = (\partial A/\partial n_1 + \partial A/\partial n_2)/2. \quad (5)$$

Subscripts 1 and 2 are for media 1 and 2, respectively, and μ_0 is the free-space permeability.

Returning to the direct formulation, the expressions for K_d and K_s can be used to express the discontinuity of A and $\partial A/\partial n$ (tangential induction) due, respectively, to the equivalent double layer and single layer of current:

$$[(\mu_1-\mu_2)/(\mu_1+\mu_2)](\partial A/\partial n_1 + \partial A/\partial n_2)$$
$$= \partial A/\partial n_1 - \partial A/\partial n_2, \quad (6)$$

$$[(\mu_1+\mu_2-2\mu_3)/(\mu_1+\mu_2)](\partial A/\partial n_1 + \partial A/\partial n_2)e/2$$
$$= A_1 - A_2. \quad (7)$$

Equation (6) can be rewritten as

$$\nu_1\, \partial A/\partial n_1 = \nu_2\, \partial A/\partial n_2. \quad (8)$$

These conditions can be used as transmission conditions at the boundary between two domains to take into account a thin magnetic plate. They can be used without regard to the numerical method and need no addition of mesh points. For instance, the transmission conditions can be used in the finite element method as follows.

The thin steel plate to be modeled must be defined as a part of the boundary between two subdomains (named domain 1 and domain 2). At each node of this boundary, four degrees of freedom are assumed: the vector potential values A_1 in domain 1 and A_2 in domain 2 and the tangential induction values $\partial A/\partial n_1$ in domain 1 and $\partial A/\partial n_2$ in domain 2.

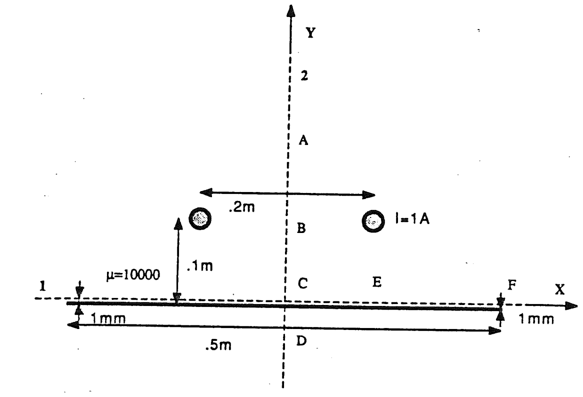

FIG. 2. Test problem: a coil above a thin magnetic plate.

Equation (2) leads in each domain to a finite element equation involving the vector potential and the tangential induction in this domain. So we have two finite element equations: one for domain 1 and one for domain 2. Two equations are missing and therefore the transmission conditions (7) and (8) complete the system.

IV. RESULTS

As a test computation the simple case of a "2D" coil 0.1 m above a thin magnetic plate of length 0.5 m, thickness e=1 mm (aspect ratio 1/500), and relative permeability 10 000 (Fig. 2) is taken. The "coil" consists of wires carrying equal and opposite currents of 1 A, separated by 0.2 m. The field lines in this structure are shown in Fig. 3. This computation has been made with second-order finite element and special transmission conditions. The discontinuity of the potential vector and the screening effect of the plate are obvious.

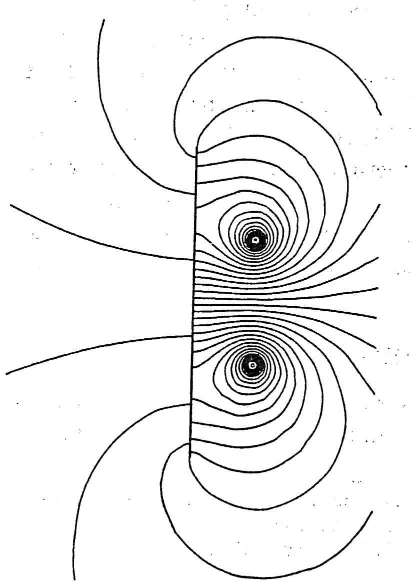

FIG. 3. Field lines around the thin magnetic plate.

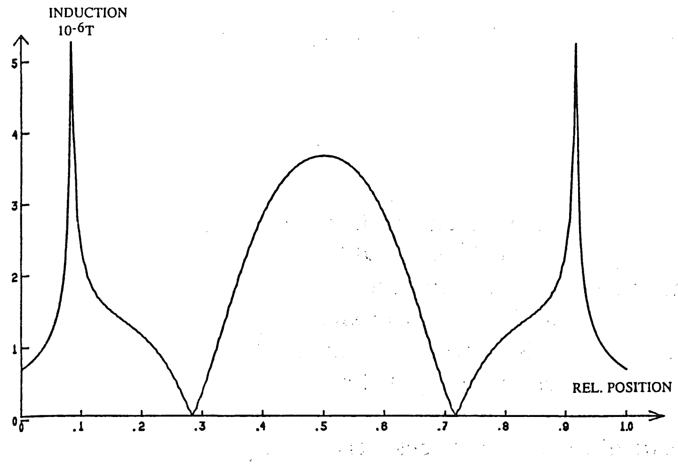

FIG. 4. Induction variation 1 mm above the thin magnetic plate.

Figure 4 shows the variation of the magnetic induction just above the plate (line 1 of Fig. 2). Here the computation has been made with first-order classical boundary elements. At the ends of the plate there are high peaks due to a kind of point effect.

Figure 5 shows the variations of the magnetic induction through the plate (line 2 of Fig. 2). A discontinuity occurs across the plate, and the screening effect appears clearly. As a comparison of various numerical methods,

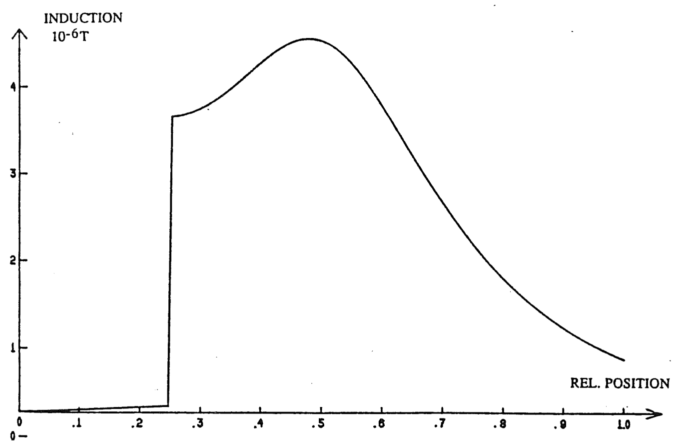

FIG. 5. Induction through the thin magnetic plate.

TABLE I. Comparison of $B(\mu T)$ for various methods.

Point	$X(m)$	$Y(m)$	BEM	FEM	FEMBEM	TCFEM	TCBEM
A	0	0.2	2.70	2.18	2.18	2.44	2.19
B	0	0.1	4.71	4.50	4.50	4.59	4.52
C	0	0	3.69	3.59	3.59	3.61	3.67
D	0	− 0.003	0.324	0.422	0.422	0.346	0.327
E	0.1	0	0.974	0.599	0.599	0.741	0.628
F	0.25	0	3.39	3.06	3.06	3.50	3.68

Table I gives the values of the induction at various characteristic points (Fig. 2). The methods compared are: the finite element method (FEM); the boundary element method (BEM); the coupling of the boundary element method outside the plate and the finite element inside (FEMBEM); the transmission conditions used with the finite element method (TCFEM); and the transmission conditions used with boundary element method (TCBEM).

The length of the plate is divided in 50 elements and all the elements are of the first order. The quality of the various methods is very similar, with a small advantage to the boundary element methods only due to the fact that finite element methods assume a constant induction on each element.

V. CONCLUSION

The modeling of a thin magnetic plate is very difficult and care is necessary in the numerical integration and in the algebraic system resolution. Finite elements and boundary elements with adaptative numerical integrations give good results but are time-consuming. The method proposed is much more economical. The inside of the plate is not explicitly taken into account. The meshing of very thin plates is often cumbersome but can be completely avoided with this method. Moreover there is no theoretical limitation to the lower limit of the thickness; the thinner the better. Numerical results confirmed the validity of the method.

[1] J. P. Adriaens, P. Bourmanne, F. Delince, A. Genon, A. Nicolet, and W. Legros, IEEE Trans. Magn. **MAG-26**, 2376 (1990).
[2] W. Legros, A. Nicolet, and M. Paganini, IEEE Trans. Magn. **MAG-25**, 3593 (1989).
[3] P. P. Silvester and R. L. Ferrari, *Finite Elements for Electrical Engineers* (Cambridge University Press, Cambridge, 1990).