Numerical computation of the magnetostriction effect in ferromagnetic materials

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This paper describes a model of the magnetostriction by a finite element scheme for magnetostatic cases. Given a structure, the magnetic field is first computed taking into account the ferromagnetic saturation. Magnetic induction over elements is then used to compute the magnetoelastic energy and the minimization of the mechanical functional leads to the determination of displacement, strain, etc., due to the magnetostriction process.

INTRODUCTION

Magnetostriction is the phenomenon where the shape of a ferromagnetic specimen changes during the process of magnetization.[1,2] The deformation dl/l due to magnetostriction is as small as 10^{-6}–10^{-5}. Nevertheless, it is responsible for vibrations, noise, and mechanical losses in magnetic circuits. It also has important applications such as high-frequency oscillators and ultrasound generators. However, magnetostriction is usually studied experimentally because of the difficulties arising from the magnetic field computation and the coupling between magnetic and mechanical problems.

MAGNETOSTRICTION

When a magnetic field is applied to a ferromagnetic material it has its dimensions changed during the process of magnetization. The strain tensor due to magnetostriction ϵ_{ij} can be divided into two distinct parts:

$$\epsilon_{ij}=e^v\delta_{ij}+\hat{e}_{ij}, \quad (1)$$

where δ_{ij} is the Kronecker tensor,

$$e^v=\tfrac{1}{3}\epsilon_{ii} \quad (2)$$

is the volume dilatation, and

$$\hat{e}_{ij}=\epsilon_{ij}-e^v\delta_{ij}. \quad (3)$$

The kind of magnetostriction responsible for e^v is called the volume magnetostriction, while the other components $\hat{e}_{ij}$ of the strain tensor come from the Joule magnetostriction.

The volume magnetostriction appears in very high magnetic fields far above magnetic saturation of the material. On the other hand, the Joule magnetostriction (i.e., incompressible part of magnetostriction) takes its maximum value at saturation. The behavior of volume magnetostriction can be linearized by taking

$$e^v=0 \quad \text{for } H<H_v \quad (4)$$

and

$$e^v=a(H-H_v) \quad \text{for } H>H_v. \quad (5)$$

For Joule magnetostriction the strain is supposed to be a function of the magnetization M

$$\hat{e}_{ij}=\hat{e}_{ij}(M). \quad (6)$$

For an isotropic material, at saturation, we admit that the strain takes its maximum value λ_s. The tensor becomes

$$\hat{e}_{ij}=\begin{pmatrix}\lambda_s & 0 & 0\\ 0 & -\lambda_s/2 & 0\\ 0 & 0 & -\lambda_s/2\end{pmatrix} \quad (7)$$

for a magnetization along the x axis. λ_s represents the relative change of dimension of the material at saturation and is easily measured by experiment. If the material is assumed to be linear between the demagnetizated state and saturation, then there is a strict proportionality between magnetization and deformation:

$$\hat{e}=\lambda=\lambda_s|M|/|M_s|, \quad (8)$$

where M_s is the magnetization at saturation.

MECHANICAL PROBLEM

A body V is taken with internal forces f_i and with prescribed stress T_i on a part of its surface S_t while the other part S_u is subject to prescribed displacement U_i. The solution of the linear elasticity problem is given by the principle of minimum of energy.[3] The exact solution minimizes the classical functional (i.e., annulation of the first variation).

$$I(U_i)=\int_V W^e\,dV-\int_V f_iU_i\,dV-\int_{S_t} T_iU_i\,dS. \quad (9)$$

The first term represents the elastic energy stored in the body, and the others are, respectively, the work of the internal and external forces. The density of elastic energy W^e is a quadratic form of the strain tensor (using the Einstein summation convention)

$$W^e=\tfrac{1}{2}C_{ijkl}\epsilon_{ij}\epsilon_{kl}, \quad (10)$$

where C_{ijkl} is called Hooke's tensor (of order 4) but which can be reduced to only two independent components C_{11} and C_{12} for an isotropic material.

The components C_{11} and C_{12} are related to the Young's modulus E and the Poisson's ratio σ by[4]

$$C_{11}=\frac{E(1-\sigma)}{(1+\sigma)(1-2\sigma)} \quad (11)$$

and

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$$C_{12}=\frac{E\sigma}{(1+\sigma)(1-2\sigma)}. \quad (12)$$

In this case, the density of energy becomes

$$W^e=\tfrac{1}{2}(C_{11}-C_{12})\epsilon_{ij}\epsilon_{ij}+\tfrac{1}{2}C_{12}\epsilon_{jj}\epsilon_{ii}. \quad (13)$$

COUPLED PROBLEM

A new term can be added to the functional representing the magnetoelastic energy associated to the magnetostriction effect

$$I(U_i)=\int_V W^e\,dV-\int_V f_iU_i\,dV-\int_{S_t} T_iU_i\,dS - \int_V W^m\,dV. \quad (14)$$

Note that this functional supposes that the external forces are not depending on the displacement. In the following, only the magnetostrictive effect is considered and other kinds of forces are neglected.

In this case, the functional becomes

$$I(U_i)=\int_V W^e\,dV-\int_V W^m\,dV. \quad (15)$$

The density of magnetoelastic energy is

$$W^m=s_{ij}\epsilon_{ij}, \quad (16)$$

where s_{ij} are the stresses due to magnetostriction. Utilization of Hooke's law and mathematical developments lead to

$$\begin{aligned} W^m=&\tfrac{3}{2}(C_{11}-C_{12})\\ &\times \hat{e}\,[\epsilon_{11}(\alpha_1^2-\tfrac{1}{3})+\epsilon_{22}(\alpha_2^2-\tfrac{1}{3})+\epsilon_{33}(\alpha_3^2-\tfrac{1}{3})]\\ &+3(C_{11}-C_{12})\,\hat{e}\,(\epsilon_{12}\alpha_1\alpha_2+\epsilon_{23}\alpha_3\alpha_2+\epsilon_{13}\alpha_1\alpha_3)\\ &+(C_{11}+2C_{12})e^v(\epsilon_{11}+\epsilon_{22}+\epsilon_{33}), \end{aligned} \quad (17)$$

where C_{11} and C_{12} are the elastic coefficients taken from Hooke's tensor and the α_i are the unit vectors of the direction of the magnetization M.

HYPOTHESIS AND FINITE ELEMENT

Only the Joule magnetostriction is taken into account in the finite element implementation because it is the more usual kind of magnetostriction. In addition, although the previous relations were three dimensional, we will restrict ourselves to a two-dimensional model.

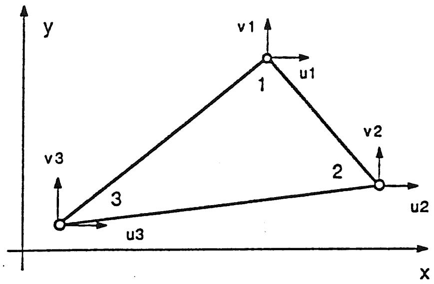

FIG. 1. First-order triangular element for elasticity.

In the mechanical problem there are two different points of view: plane strain or plane stress. The first case corresponds to an infinite body in which the deformations are the same in every perpendicular plane. Note that this case is not incompressible which is incompatible with a Joule magnetostriction. On the other hand, the plane stress which corresponds to a thin body (for which all stresses are in the plane of the body) is incompressible.

The magnetostatic state of the structure is first computed and then the values of the magnetic induction are used in the magnetoelastic computation. First-order triangular elements (Fig. 1) are used for the finite element implementation of the magnetoelastic model. Two components of displacement are defined at each node. The element has six degrees of freedom given by the vector of displacement:

$$\{q\}^T=(u_1,v_1,u_2,v_2,u_3,v_3). \quad (18)$$

Linear variations of displacement are assumed on the element. According to standard computations for this kind of element in elasticity,[5] the following contribution I^e of the element to the discretized functional is derived:

$$I^e=\tfrac{1}{2}\{q\}^T[K]\{q\}-\{g\}^T\{q\}, \quad (19)$$

where $[K]$ is the classical element stiffness matrix[5] and $\{g\}$ is a generalized force vector due to the magnetostriction. This vector is given by

$$\{g\}=\frac{S}{2|S|}\begin{Bmatrix} -\tfrac{3}{2}\eta_1(C_{11}-C_{12})\,\hat{e}\,(\alpha_1^2-a/3)-\eta_1(C_{11}+2C_{12})ae^v+\tfrac{3}{2}\xi_1(C_{11}-C_{12})\,\hat{e}\,\alpha_1\alpha_2\\ \tfrac{3}{2}\xi_1(C_{11}-C_{12})\,\hat{e}\,(\alpha_1^2-a/3)+\xi_1(C_{11}+2C_{12})ae^v-\tfrac{3}{2}\eta_1(C_{11}-C_{12})\,\hat{e}\,\alpha_1\alpha_2\\ -\tfrac{3}{2}\eta_2(C_{11}-C_{12})\,\hat{e}\,(\alpha_1^2-a/3)-\eta_2(C_{11}+2C_{12})ae^v+\tfrac{3}{2}\xi_2(C_{11}-C_{12})\,\hat{e}\,\alpha_1\alpha_2\\ \tfrac{3}{2}\xi_2(C_{11}-C_{12})\,\hat{e}\,(\alpha_1^2-a/3)+\xi_2(C_{11}+2C_{12})ae^v-\tfrac{3}{2}\eta_2(C_{11}-C_{12})\,\hat{e}\,\alpha_1\alpha_2\\ -\tfrac{3}{2}\eta_3(C_{11}-C_{12})\,\hat{e}\,(\alpha_1^2-a/3)-\eta_3(C_{11}+2C_{12})ae^v+\tfrac{3}{2}\xi_3(C_{11}-C_{12})\,\hat{e}\,\alpha_1\alpha_2\\ \tfrac{3}{2}\xi_3(C_{11}-C_{12})\,\hat{e}\,(\alpha_1^2-a/3)+\xi_3(C_{11}+2C_{12})ae^v-\tfrac{3}{2}\eta_3(C_{11}-C_{12})\,\hat{e}\,\alpha_1\alpha_2 \end{Bmatrix}, \quad (20)$$

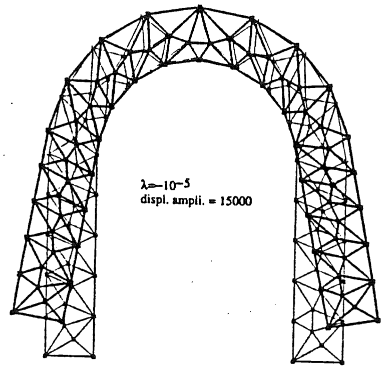

FIG. 4. Deformation of the electromagnet under a negative magnetostriction.

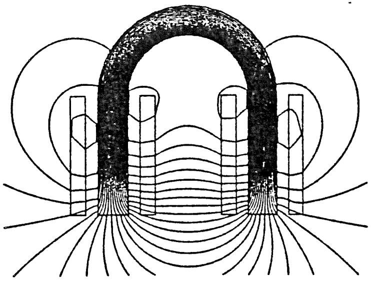

FIG. 2. Field lines in an electromagnet.

where

$$\xi_1=x_3-x_2,\quad \xi_2=x_1-x_3,\quad \xi_3=x_2-x_1$$

$$\eta_1=y_3-y_2,\quad \eta_2=y_1-y_3,\quad \eta_3=y_2-y_1 \qquad (21)$$

$$S=\xi_2\eta_3-\xi_3\eta_2,\quad a=1-(C_{12}/C_{11}).$$

The element contribution to be assembled to the global system is therefore

$$[K]\{q\}=\{g\}. \qquad (22)$$

Geometry variations due to the magnetostriction can be neglected from the magnetic field computation point of view because deformations are very weak (relative deformation $\approx 10^{-5}$).

RESULTS

Figure 2 shows the field lines in an electromagnet. The values of the induction are used to compute the magnetostriction. The maximum induction in the electromagnet is about 1.4 T. Its elastic constants are $C_{11} = 2.75 \times 10^{11}$ J/m^2 and $C_{12} = 1.25 \times 10^{11}$ J/m^2. The width of the magnetic horseshoe is 1 m. The parameters of magnetostriction are $M_s = 1.6 \times 10^6$ A/m and $\lambda_s = 10^{-5}$.

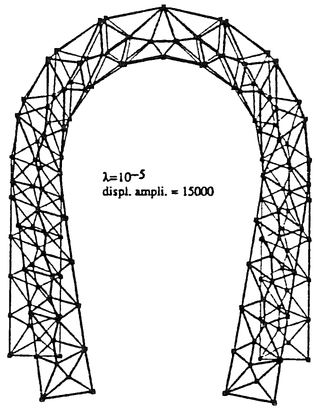

FIG. 3. Deformation of the electromagnet under a positive magnetostriction.

Figure 3 shows the deformation of the structure under positive magnetostriction and Fig. 4 illustrates that under a negative one. Displacements are amplified by a factor of 15 000. The positive magnetostriction gives a dilatation along the field lines and a contraction in the perpendicular direction. The negative magnetostriction leads to the opposite behavior.

CONCLUSION

The integration of the magnetoelastic computation as a part of the postprocessing of a magnetic field computation software furnishes a very simple tool to take the magnetostriction into account in the design of electrotechnical devices. It allows the computation of the magnetostriction with the same geometry as in the electromagnetic model.

Numerical computations provide coherent results but an experimental confirmation is not yet available. The plane strain hypothesis leads to better results. This case corresponds to thin objects which is not so restrictive in electrotechnics because most of the magnetic yokes are constituted of thin sheets in order to limit eddy currents. Further improvements will include dynamic problems. The introduction of the inertial terms in the mechanical problem will allow the determination of the eigenmodes of vibrations of the structures.

[1] S. Chikazumi, *Physics of Magnetism* (Wiley, New York, 1964).
[2] C. Kittel, *Introduction to Solid States Physics* (Wiley, New York, 1953).
[3] L. Scipio Albert, *Principles of Continua* (Wiley, New York, 1967).
[4] M. Gaudaire, *Propriétés de la matière* (Dunod, Paris, 1969).
[5] K. H. Huebner, *The Finite Element Method for Engineers* (Wiley, New York, 1975).