Influence of Hysteresis on the Behaviour of Coupled Finite Element - Electric Circuit Models

F. Delincé*, A. Nicolet, F. Henrotte, A. Genon, W. Legros.

University of Liège - Dept of Electrical Engineering
Institut Montefiore - Sart Tilman Bât. B28 - B-4000 Liège (Belgium)
* This author is a Research Assistant with the Belgian National Fund for Scientific Research.

Abstract - This paper describes the computation of waveforms of electrical current and voltage in inductances, taking into account the hysteresis phenomenon and electric circuit equations. The non-linear transient magnetic field is computed with the finite element method. The classical non-linear case is compared to the hysteretic one : it is shown that hysteresis has a crucial importance on the waveform by introducing asymmetry and damping, and by modifying the natural frequency of the oscillations.

I. INTRODUCTION

Finite element models of inductances are essentially based on a simple, imposed current excitation. The magnetic materials are either linear or non-linear. In this paper, a method is presented that considers the precise waveform of the excitation generated by electric circuits with lumped elements. The hysteretic behaviour of the magnetic materials is also taken into account. It is shown that neglecting the dissipative effect of hysteresis can lead to a bad evaluation of the current and voltage waveforms.

II. NUMERICAL MODELS

2D Finite element modelization :

The equation for two-dimensional magnetostatics is :

$$\text{curl}\ (\nu\ \text{curl}\ \underline{A}) = \underline{J} \tag{1}$$

where ν is the magnetic reluctivity and $\underline{A}$ and $\underline{J}$ are respectively the vector potential and the current density. These vectors have only the z component different from zero. Using Ohm's law, the case of eddy currents can be dealt with by introducing (2) as the expression for the current density in (1), [1].

$$J = \sigma E = -\left(\frac{\partial A}{\partial t} + U\right) \tag{2}$$

Manuscript received November 1, 1993. This text presents research results of the Belgian programme on Interuniversity Poles of attraction initiated by the Belgian State, Prime Minister's Office, Science Policy Programming. The scientific responsibility is assumed by its authors.

The conductor is characterised by its electrical conductivity σ, and U can be interpreted as the terminal voltage of the conductor (per unit of length).

The finite element formulation is based on the Galerkin method and we have (3) for the domain Ω of boundary Γ, where w is a weighting function. The boundary term is usually used to apply a Neumann boundary condition. Here it is used at the interface between two finite element subdomains. The tangential field $\nu\partial A/\partial n = H_t$ is taken as an unknown on the boundary and the boundary term of (3) is introduced. It leads to the same solution for the vector potential A as the classical formulation but it also provides the value of the tangential field on the boundary [2].

$$\begin{aligned}&\iint_\Omega \left[\nu\, \text{grad}\, A . \text{grad}\, w + \sigma\left(\frac{\partial A}{\partial t} + U\right) w\right] d\Omega \\ &-\oint_\Gamma w \nu \frac{\partial A}{\partial n} d\Gamma = 0\end{aligned} \tag{3}$$

Handling circuit equations :

As the finite element formulation involves the tangential field on the boundary, the total current I can be expressed as the line integral of the magnetic field (Ampere's law) [3] :

$$I = \oint_\Gamma \nu \frac{\partial A}{\partial n} d\Gamma \tag{4}$$

From a more general point of view we can systematically relate to every conductor an equation like (4) and two global degrees of freedom : the total current I and the terminal voltage U. To do so, a circuit equation is necessary for each conductor.

Magnetic materials :

Magnetic materials are usually considered as non-linear but reversible. The constitutive law of such magnetic materials is (5). In order to take hysteresis into account, this classical relation has to be modified to (6) which ensures that the reluctivity is always positive and well defined [4] :

$$\underline{H} = \nu(B)\ \underline{B} \tag{5}$$

$$\underline{H} = \nu(B)\ \underline{B} + \underline{H}_c \tag{6}$$

The term $\underline{H}_c$ is the representation of the irreversible component introduced by hysteresis. It depends on the magnetic history of the material and can be seen as the parameter determining which hysteresis branch the magnetic state is on. By introducing (6) in (1), it is possible to show that the derivatives of $\underline{H}_c$ behave exactly like an additional current density. The finite element equations are thus exactly similar to the classical non-linear case, except for the additional current density representing the hysteresis behaviour. The whole problem is then reported on their evaluation.

Hysteresis model

Equation (6) must be associated to an hysteresis model. The well known Preisach model [5] is considered here with adjunction of a purely reversible component. Practically, the flux density is computed by taking a linear combination of the Everett function (7) evaluated at the vertices of the staircase line (Fig. 1) representing the magnetic history {h} in the Preisach plane (Γ(a,b) is the Preisach density function). In the same way, hysteretic losses Q [6] and the magnetic energy W stored reversibly in the material can be computed by the formulae (8) and (9).

$$E(H, H_e) = \iint_{T(H,H_e)} \Gamma(a,b)dS \tag{7}$$

$$Q(H, H_e) = \iint_{T(H,H_e)} (a-b)\Gamma(a,b)dS \tag{8}$$

$$W(H, H_e) = \operatorname{sign}(H - H_e)\iint_{T(H,H_e)} (a+b)\Gamma(a,b)dS. \tag{9}$$

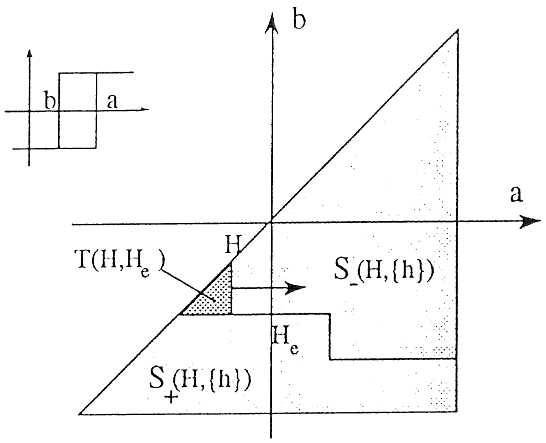

Figure 1 : Preisach Plane.

Those quantities will be helpful when computing the energy balances. It must be noticed that if the Everett function is known, it is possible to transform the expression (8) of the losses Q in order to involve only line integrals in the Preisach plane [7]; a similar transformation is also possible for the function W. These two densities are represented in Figures 2 and 3 over the Preisach plane.

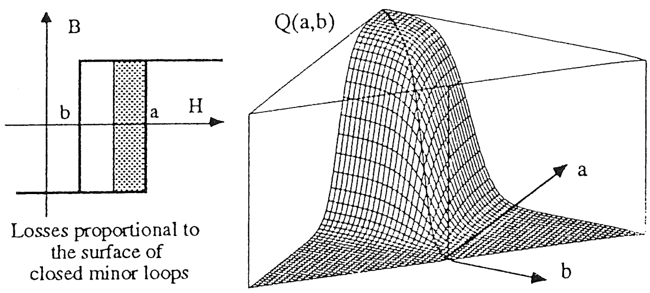

Figure 2 : Hysteretic losses density Q(a,b).

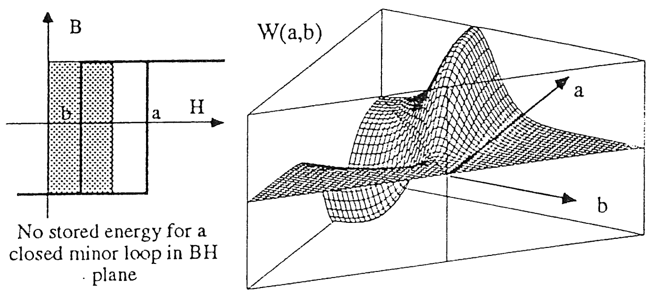

Figure 3 : Magnetic energy density W(a,b) (increasing evolution).

III. APPLICATION

The following problem is considered as an example (Fig. 4) : a non-linear inductance constituted by a primary coil in series with a capacitor of initial voltage U_{Co}, and by a secondary coil in open circuit. The two coils are linked by a ferromagnetic core considered first as non-linear (and reversible), and then as hysteretic (i.e. irreversible).

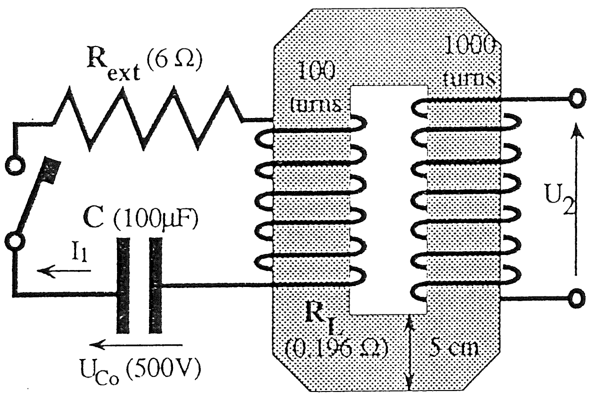

Figure 4 : Non-linear inductance.

Figure 5 shows the magnetic characteristics used for the computations. The non-linear characteristic is the same as the anhysteretic magnetisation curve. The capacitor voltage U_C has to be chosen as a state variable, and the circuit equations (10) and (11) must be considered.

$$U_C + (R_{ext} + R_L)\, I_1 + U_L = 0 \qquad (10)$$

$$I_1 + C\frac{\partial U_C}{\partial t} = 0 \qquad (11)$$

The current I_1 in the primary coil and the voltage U_2 in the secondary coil are analysed. Figure 6 (dotted line) shows the classical current waveform inside an non-linear inductance. The response is a symmetric damped oscillation. As the level of current decreases, the saturation becomes less important and the harmonic rate diminishes. The period of oscillation increases with time, as the apparent inductance increases with desaturation. In the case of the hysteretic core, it is shown that the oscillations are no longer symmetric, and are more damped because hysteresis is associated with energy dissipation [6,7]. After 0.3 s, the oscillations have a smaller period and become symmetric. This is due to the stabilisation of the magnetic state on a smaller and almost reversible cycle with a small differential permeability.

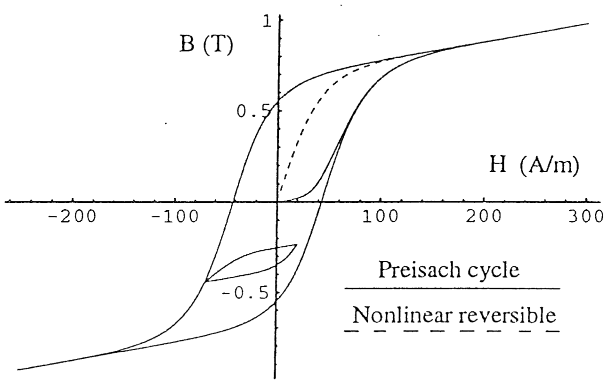

Figure 5 : Magnetic characteristics.

The voltage waveforms on the secondary coil (Fig. 7) show clearly the asymmetry introduced by hysteresis.

It is also interesting to take a look at the energy balance of the system. The case without hysteresis is considered first. There are three kinds of energy in the system : the magnetic energy stored in all the magnetic materials (i.e. the entire finite element region), the electrostatic energy in the capacitor and the Joule losses in the resistances (the total dissipation is considered as a particular type of energy in order to check the energy conservation). If these three quantities are added together at any time, the initial energy of the system (i.e. $1/2.C.U^2{}_{Co}$ = 12.5 Joules) must be found back. This has proved to be a good criterion of the solution quality and it is very sensitive to the time integration scheme. Figure 8 shows the conservation of energy for the example. Two points should be noticed : the first one is the exchange of energy between the inductance and the capacitor and the second one is that Joule dissipation takes place when the magnetic energy is important (i.e. the current too). The total energy of the system is conserved.

The conservation of energy in the hysteretic inductance (fig. 9) is more interesting, because hysteretic losses are also present. The formulae (8) and (9) have to be used to compute these losses and the magnetic energy in the core. The damping effect is obviously more important although there is less total Joule dissipation than in the reversible case. Interlacement of the two types of dissipation is also remarkable : when the hysteresis loss power (i.e. the slope of the curve) is important, the Joule power is negligible. It is because the hysteresis losses are more important at low level of flux density but the Joule dissipation is not important at this time since the current is low; the opposite is also verified. In this case, the conservation of energy is also verified.

IV. Conclusions

Preisach hysteresis model and electric circuit equations have been integrated into a finite element model. Numerical experiments demonstrate the importance of hysteresis on current and voltage waveforms in inductances. Computations based only on the non-linear behaviour of the magnetic core are unable to predict the actual time constants and dampings associated with such devices.

V. References

[1] P. Bourmanne, F. Delincé, A. Genon, W. Legros, A. Nicolet, *"Skin Effect and Proximity Effect in Multiconductor Systems with Applied Currents and Voltages"*, Journal of Applied Physics, 69 (8), 1991, pp. 5035-5037.

[2] F. Delincé, P. Dular, A. Genon, W. Legros, J. Mauhin, A. Nicolet, *"Modelling of Inductive Effects in Thin Wire Coils"*, IEEE T-MAG 28 n° 2, 1992, pp. 1190-1192.

[3] A. Nicolet, F. Delincé, N. Bamps, A. Genon, W. Legros, *"A Coupling Between Electric Circuits and 2D Magnetic Field Modelling"*, IEEE T-MAG 29, n°2, 1993, pp. 1697-1699.

[4] F. Henrotte, A. Nicolet , F. Delincé, A. Genon, W. Legros, *"Modelling of Ferromagnetic Materials in 2D Finite Element Problems Using Preisach's Model"*, IEEE T-MAG 28 n° 5, 1992, pp. 2614-2616.

[5] F. Preisach, *"Uber die magnetische nachwirkung"*, Zeitschrift für Physik, Vol. 94, 1935, pp. 277-302.

[6] G. Biorci, A. Ferro, *"Hysteresis losses along open transformations"*, Journal de Physique et le Radium 20, 1959, pp. 237-240.

[7] I.D. Mayergoyz, G. Friedman, *"The Preisach Model and Hysteretic Energy Losses"*, Journal of Applied Physics 61 (8), 1987, pp. 3910-3912.

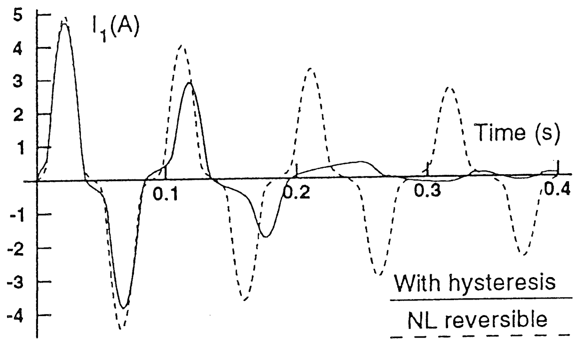

Figure 6 : Primary current.

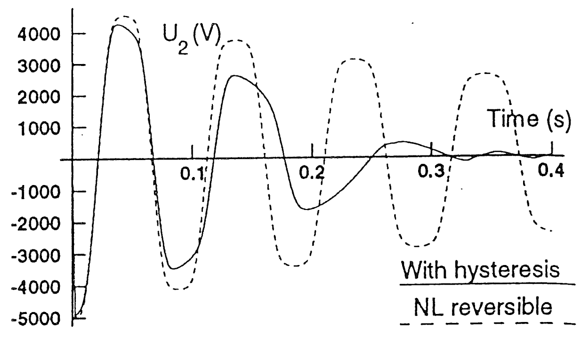

Figure 7: Secondary voltage

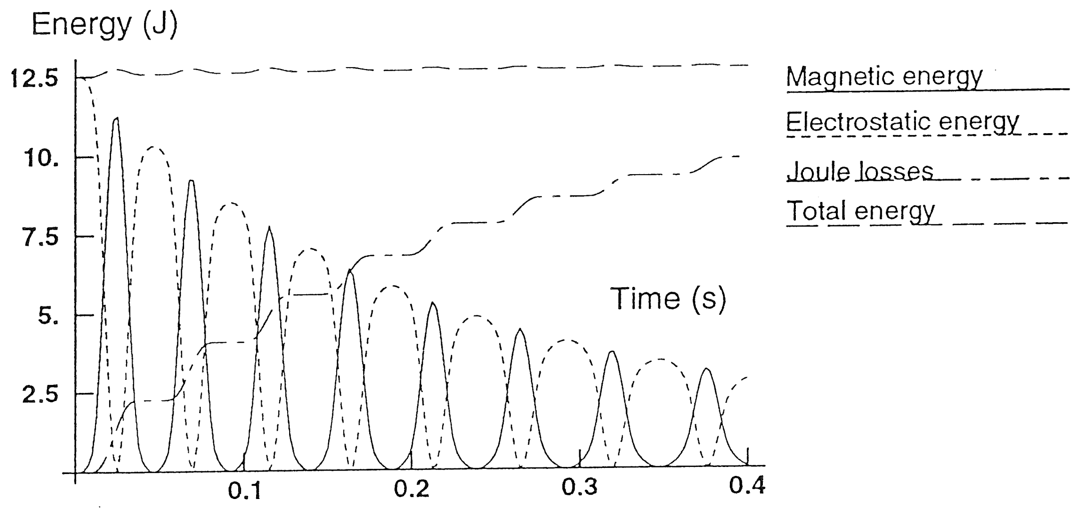

Figure 8 : Energy conservation in the reversible system.

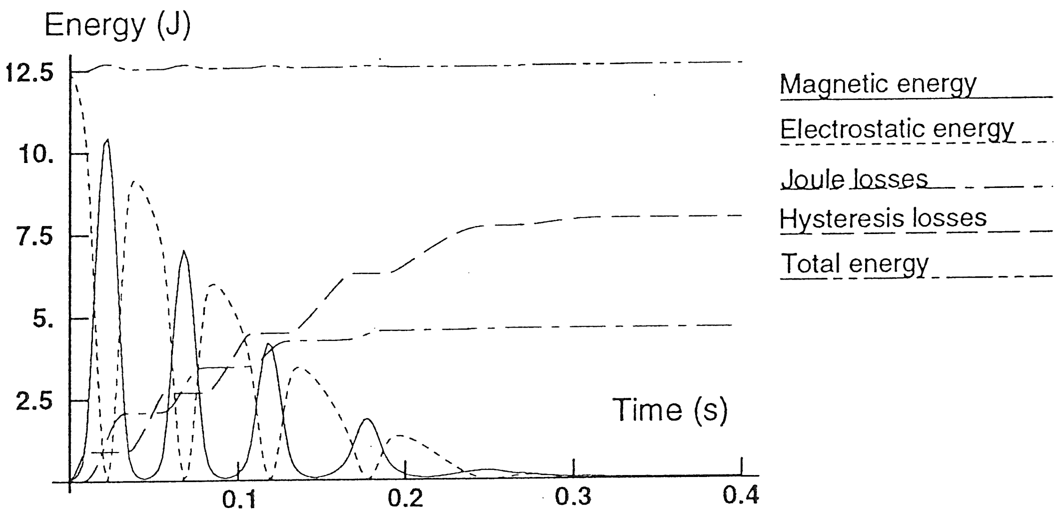

Figure 9 : Energy conservation of the hysteretic system.