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Analysis of ferroresonance with a finite element method taking hysteresis into account †

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Abstract

This paper describes the analysis of ferroresonance in power systems with the help of 2D finite element method. It leads to the realistic evaluation of the nonlinear inductance behaviour. The effect of hysteresis on waveforms and the energy balances are analysed for various modes of ferroresonance.

1. Introduction

Ferroresonance is the special case of a resonance jump in which the nonlinearity is the iron core magnetization. If an incremental change in the amplitude or frequency of the sinusoidal input to the system causes a sudden jump in signal amplitude somewhere in the system, jump resonance is said to have occurred [1]. The problem of ferroresonance in power systems is usually tackled by the lumped circuit equations approach, combined with a suitable representation of the nonlinearity of inductances. However, the real behaviour of the inductance is not taken into account. Field computation is the solution to that problem: local saturations and leakage fluxes are implicitly computed by the finite element method which has to be extended in order to take circuit equations and hysteresis (based on the Preisach model [2]) into account.

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2. Numerical methods

2.1. 2D finite elements

The equation for the two-dimensional magnetostatics is:

$$\mathrm{rot}(\nu\ \mathrm{rot}\ A) = J, \tag{1}$$

where A is the vector potential which has only one component, J is the current density and ν is the magnetic reluctivity. The case of eddy currents can be dealt with by introducing (2) in (1). The conductor is characterized by its conductivity σ, and U can be interpreted as the terminal voltage of the conductor (per unit of length). The finite element formulation, based on the Galerkin method, gives (3) for the domain Ω of boundary Γ, where ω is the weighting function:

$$J = \sigma E = -\left(\frac{\partial A}{\partial t} + U\right), \tag{2}$$

$$\iint_\Omega \left[\nu\ \mathrm{grad}\ A \cdot \mathrm{grad}\ \omega + \sigma\left(\frac{\partial A}{\partial t} + U\right)\omega\right] \mathrm{d}\Omega - \oint_\Gamma \omega\nu\frac{\partial A}{\partial n}\ \mathrm{d}\Gamma = 0. \tag{3}$$

2.2. Circuit equations

The current I can be expressed as:

$$NI = -\sigma_e\left(\frac{\partial A^*}{\partial t} + SU\right), \tag{4}$$

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where N is the number of turns, σ_e an equivalent conductivity (defined by multiplying the actual conductivity by the filling factor of the coil), A^* is the integral of the potential over the surface of the coil, and S is the equivalent area of a thin wire (defined by dividing the total surface of the coil by the number of turns) [3].

For a series RLC circuit, the capacitor voltage U_C has to be chosen as a state variable, and the circuit equations (5) must be considered (U_L is the inductance voltage and $E(t)$ the excitation source):

$$I + C\frac{\partial U_C}{\partial t} = 0, \quad U_C + RI + U_L = E(t). \tag{5}$$

2.3. Magnetic materials

In order to take hysteresis into account, the constitutive law of magnetic materials has to be modified [4]:

$$H = \nu(B)B + H_c. \tag{6}$$

The supplementary term H_c is the representation of the irreversible component introduced by hysteresis and depends on the magnetic history. By introducing (6) into (1), it is possible to show that the derivatives of H_c behave exactly like an additional current density [4]. The finite element equations are thus exactly similar to the classical nonlinear case, except that the additional current density represents the hysteresis behaviour.

Eq. (6) must be associated with an hysteresis model. We consider the classical Preisach model [2] with adjunction of a purely reversible component. The change in flux density from the last reversal field H_e to the actual field H, is given by the Everett function $E(H, H_e)$ (7a), a double integration of the Preisach density function $\Gamma(a, b)$ over the triangle $T(H, H_e)$ on the Preisach plane. The hysteretic losses and the magnetic energy stored in the media are computed in the same way using the functions $Q(H, H_e)$ (7b) [5] and $W(H, H_e)$ (7c), respectively. These quantities will be useful for the energy balance computation.

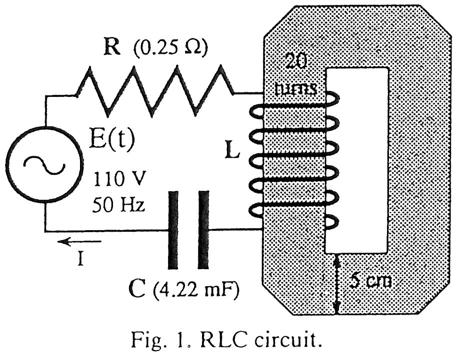

Fig. 1. RLC circuit.

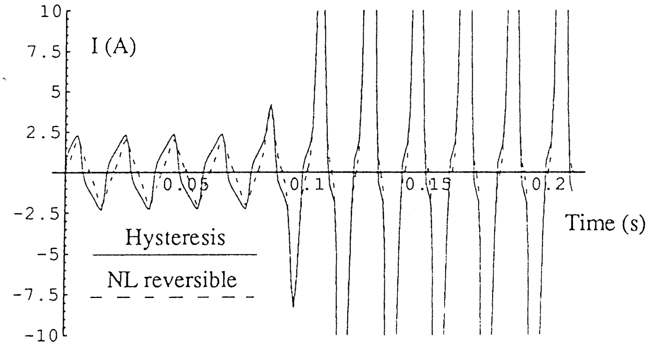

Fig. 2. Ferroresonant current waveform.

$$E(H, H_e) = \iint_{T(H,H_e)} \Gamma(a, b)\, dS, \tag{7a}$$

$$Q(H, H_e) = \iint_{T(H,H_e)} (a-b)\Gamma(a, b)\, dS, \tag{7b}$$

$$W(H, H_e) = \text{sign}(H - H_e) \iint_{T(H, H_e)} (a+b) \times \Gamma(a, b)\, dS. \tag{7c}$$

If the Everett function is known, $Q(H, H_e)$ and $W(H, H_e)$ can be transformed in order to involve only line integrals instead of surface integrals, which makes their computation much faster [6].

3. Application

The following problem is considered as an example (Fig. 1): a nonlinear inductance computed by finite element method is in series with a capacitor and a resistance. Two cases are considered: one with a nonlinear but reversible magnetic core, and a second with

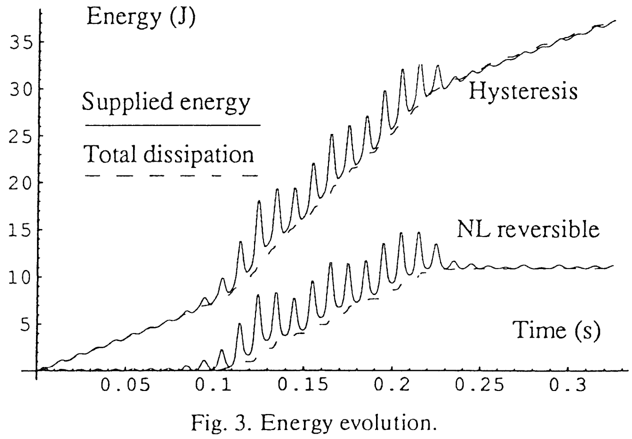

Fig. 3. Energy evolution.

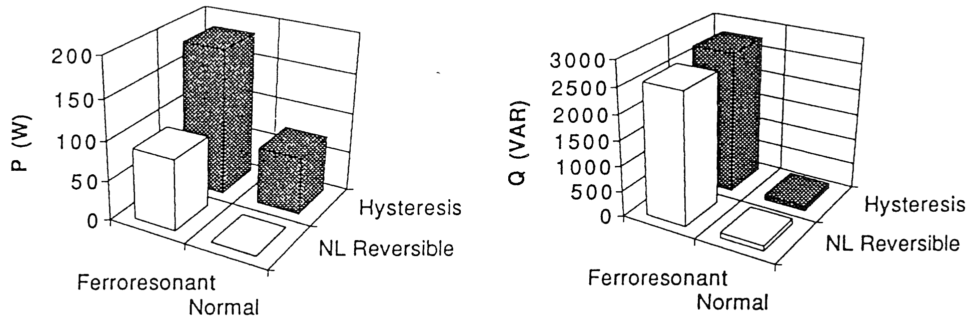

Fig. 4. Image of the powers.

an hysteretic core ($B_r = 0.77$ T, $H_c = 82.6$ A/m). The excitation source is the sinusoidal voltage $E(t)$. The amplitude of the excitation suddenly grows to cause fundamental ferroresonance. After some time, the excitation comes back to its initial value. The system can then either return to its initial state, stay on fundamental ferroresonance or switch on a stable sub-harmonic ferroresonant mode depending on the perturbation endured. Fig. 2 shows the current waveform for the fundamental ferroresonance (the peaks have been truncated in order to see the waveform distortion). It is

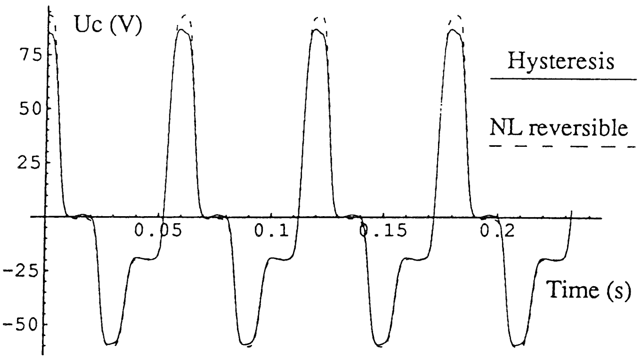

Fig. 5. Capacitor voltage for 1/3 sub-harmonic ferroresonance.

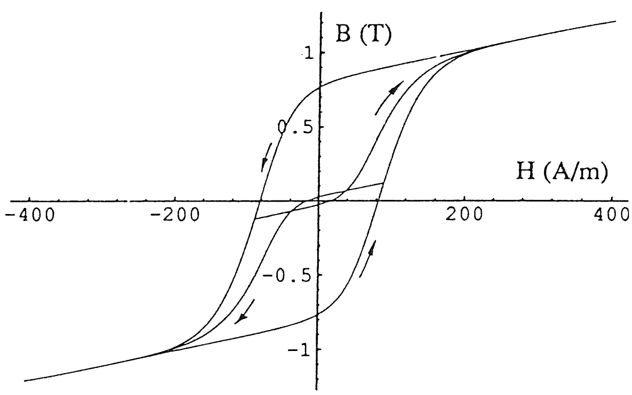

Fig. 6. BH cycle for 1/3 sub-harmonic ferroresonance.

shown that the current is more distorted with hysteresis, but principally in normal state. The hysteresis does not have a large influence on the ferroresonant waveforms. Fig. 3 shows the energy and dissipation evolutions. Note the fluctuations in the energy supplied at ferroresonance.

The active and reactive powers (P and Q) are only defined in the linear case, so that it is not possible to quantify them. However, as the excitation voltage is sinusoidal, we can compute an *indicator* of the magnitude of these powers. P can be seen as the mean power over one period T (8), and Q as the mean power computed with the excitation voltage considered $T/4$ ahead of time (9).

$$P = \frac{1}{T}\int_t^{t+T} U(t) I(t) \, dt, \tag{8}$$

$$Q = \frac{1}{T}\int_t^{t+T} U\left(t + \tfrac{1}{4}T\right) I(t) \, dt. \tag{9}$$

The results are shown in Fig. 4. The ferroresonant state is associated with an energy exchange between the power source and the reactive elements of the circuit. Of course, as the level of current increase with ferroresonance, P also increases, but not in accordance with the increase in Q. Note that P is more important in the hysteretic case, as hysteretic losses are consuming active power.

Finally, a 1/3 sub-harmonic state is analysed. Fig. 5 shows the capacitor voltage waveform, and Fig. 6 the associated BH path. The two minor cycles are characteristic of the 1/3 sub-harmonic ferroresonance.

4. Conclusions

The Preisach hysteresis model and electric circuit equations have been integrated into a finite element model in order to study the ferroresonance phenomenon. Numerical experiments demonstrate that the hysteresis effect is not really important in waveform modification. However, the power supplied is very sensitive to hysteresis. The appearance of ferroresonance depends on energy dissipation [7], so that hysteresis cannot be neglected in the simulations.

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