Modeling of ferromagnetic materials in 2D finite element problems using Preisach's model

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Abstract — In this paper, a complete 2D finite element modelization for ferromagnetic materials is described. It is based upon the definition of an adapted constitutive law which has to be completed by an hysteresis model like Preisach's one. A representation of the irreversible part of the ferromagnetic behaviour by equivalent currents is given; a classical non-linear system has to be solved finally. Some numerical results are joined to underline the physical capacities of the method.

I. Introduction

Ferromagnetic materials are widely used in electrotechnical applications. Their essential features are non-linearity and irreversibility. Both are important but, when the way to deal with the first in numerical calculations is quite clear, the second is often overlooked. This paper presents a method allowing one to involve in finite element problems materials with irreversibility as well as non-linearity properties.

When dealing with a two-dimensional problem, it is convenient to choose the vector potential $\underline{A}$, defined by (1) as an unknown because equation (2) is automatically satisfied and only a scalar field has to be discretized. Thus, the only remaining equation that must be dealt with is (3) where $\underline{H}$ depends on $\underline{A}$ through $\underline{B}$.

$$\underline{B} = \text{curl}\,\underline{A} \quad (1)$$

$$\text{div}\,\underline{B} = 0 \quad (2)$$

$$\text{curl}\,\underline{H} = \underline{J} \quad (3)$$

II. Constitutive laws

Usually, classical constitutive laws, such as $\underline{H} = \nu\,\underline{B}$ or $\underline{H} = \nu(B)\,\underline{B}$, are used that are both reversible and respectively linear and non-linear. In the case of ferromagnetic materials, these relations are not useful because they are not able to take irreversibility into account. This being so, our work is based on the definition of a more complete constitutive law for such materials :

$$\underline{H} = \nu(B)\,\underline{B} + \underline{H}_c. \quad (4)$$

The additional term $\underline{H}_c$ can be understood as the coercive field, it is the representation of the irreversible part of the ferromagnetic behaviour and depends on the past-history of the material. In another way, $\underline{H}_c$ can also be seen as the parameter determining on which hysteresis branch the magnetic state is at, at time t. The reluctivity is then defined as (Fig. 1)

$$\nu(B) = |\,\underline{H}(B) - \underline{H}_c\,| / B. \quad (5)$$

It is always positive and finite in spite of the division by a possibly zero-induction. The non-linearity of ferromagnetic behaviour is described by the dependence on induction of that reluctivity.

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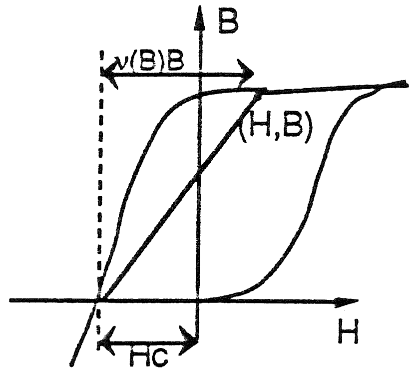

Fig. 1. Definition of the reluctivity and of the coercive field

III. Hysteresis model

Equation (5) must be completed by a hysteresis model which will give the actual values of $\nu(B)$ and $\underline{H}_c$ at each time and each point of discretization. There are many kinds of hysteresis models, they can usually be seen as an expression giving $\underline{B}$ from $\underline{H}$ and a set of parameters which represent the important values of the "past-history" of the material (more precisely the turning-point values of the local variable magnetic field, further called "dates"). Those dates are obviously different from one point to another in the magnetic domain,that compells us to memorize one set of such values at least for each finite element.

Moreover, hysteresis models are mostly scalar ones. So, attributing a direction to the vector $\underline{H}_c$ makes it usually necessary to hypothesize about (or model) the two-dimensional behaviour of the material and about a possible non-alignment of $\underline{H}$ and $\underline{B}$. In this study, we decided to neglect that misalignment, as a first approximation.

The natural formulation of most hysteresis models is to determine B(t) knowing H(t); unfortunately, seeing that our unknown is the vector potential (and consequently B), it appears that we must solve the model in the reversed fashion. That leads to an iterative search for the value of H corresponding to a given value of B. In view of the particular shape of hysteresis branches, the "Regula Falsi method" is well suited for that search (Fig. 2).

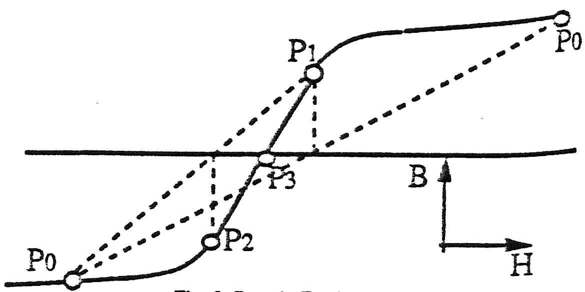

Fig. 2. Regula Falsi method

IV. PREISACH'S MODEL

Among the plethora of hysteresis models, we chose the Preisach's one due to its great accuracy and generality and despite a certain amount of complexity and heaviness. It is a phenomenological model for the hysteretic magnetization M(t). A reversible component is added afterwards to obtain the induction. The model has the following form :

$$M(t) = \int_{a \geq b} \mu(a,b)\, G_{ab}[H(t)]\, da\, db \, . \tag{6}$$

In that expression, $\mu(a,b)$ is a weight function defined on the triangular half-plane domain $\{ a,b \mid a \geq b \}$, and $G_{ab}[f(t)]$ is a two-valued operator characterized by a pair of switching values (a,b) with $a \geq b$. It switches (Fig. 3) :

from (–1) to (+1) if $f(t)=a$ and $f'(t)>0$
from (+1) to (–1) if $f(t)=b$ and $f'(t)<0$.

It can be shown that at any time t the two subdomains defined by $S\pm = \{ a,b \mid G_{ab}[f(t)] = \pm 1 \}$ are simply connected and separated by a polygonal line L(t) whose vertices coordinates (a_i,b_i) are coinciding with the local extrema of H(t) at previous times (Fig. 4); M(t) is by the way completely defined by L(t). If we introduce the function

$$T(a,b) = \int_b^a \left(\int_b^y \mu(a,b)\, dx \right) dy \, ,$$

the integration (6) can be reduced to a simple summation of values of that function on the vertices of L(t) which makes the evaluation of B(t) very fast (See Mayergoyz [1] for more information about that model).

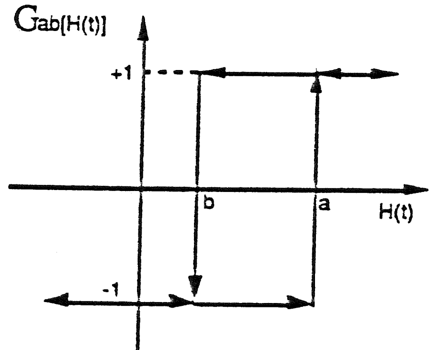

Fig. 3. $G_{ab}[f(t)]$: elementary hysteresis operator

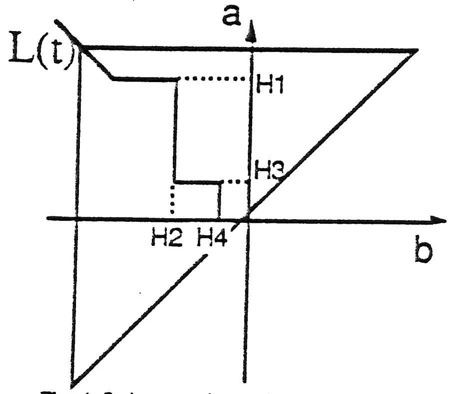

Fig. 4. Staircase polygonal boundary L(t)

The important point to keep in mind is that any hysteresis model, together with the constitutive law (4), makes a whole and coherent modelization of ferromagnetic media. Ideally, several interchangeable hysteresis models should be implemented. Afterwards, the user must choose a balance between the amount of calculation required and the desired accuracy.

V. ADDITIONAL CURRENTS EQUIVALENT TO IRREVERSIBILITY

After substituting (4) in (3), the equations to solve become :

$$\text{curl}\ \nu(B)\underline{B} = \underline{J} - \text{curl}\ \underline{H}_c = \underline{J}' .$$

Note that, in this expression, $-\text{curl}\ \underline{H}_c$ behaves like a fictional additional current density. Knowing the value of $\underline{H}_c$ in each finite element of the magnetic domain, we are able to evaluate that fictional current density and add it to the real one. In that way, we have reduced an irreversible problem into a reversible and still non-linear one, which can now be solved by an iterative method like Newton-Raphson.

The whole problem is then brought down to the evaluation of that fictional current density

$$\underline{J}_{fic} = -\,\text{curl}\ \underline{H}_c \, . \tag{7}$$

For that purpose, let us consider the following particular $\underline{H}_c$ distribution (Fig. 5) :

$$\underline{H}_c^{\,n}(x,y) = \underline{h} \quad \text{if } (x,y) \in D_n$$
$$\underline{H}_c^{\,n}(x,y) = 0 \quad \text{if } (x,y) \notin D_n$$

where $\underline{h}$ is a constant vector and D_n is the n^{th} finite element.

If we apply Stokes' theorem to an arbitrary external curve C_1 and to an arbitrary internal curve C_2, respectively enclosing the surfaces S_1 and S_2 (Fig. 6), the following expressions are obtained

$$\int_{S_1} J_{fic}\, dS_1 = 0 = \int_{S_2} J_{fic}\, dS_2$$

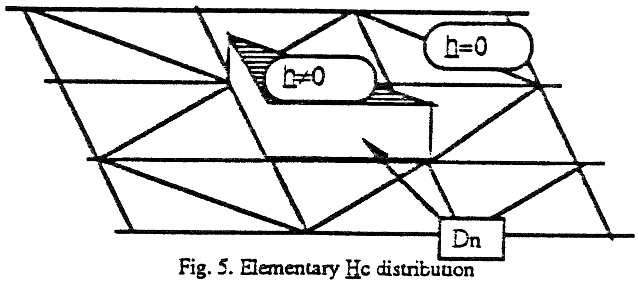

Fig. 5. Elementary $\underline{H}c$ distribution

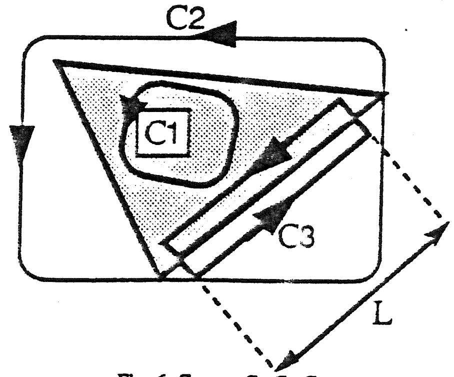

Fig. 6. Curves C_1,C_2,C_3.

showing that $\underline{J}_{fic}$ is in fact confined on the edges of the element. Moreover, the same theorem applied to curve C_3 leads to :

$$\int_{S_3} J_{fic}\, dS_3 = \int_0^L j_{fic}\, ds = h\, L \cos\phi$$

where L is arbitrary and ϕ is the angle between $\underline{h}$ and the edge under consideration. It can thus be deduced that the edge density $j_{fic}(s)$ has a constant value on each side of the element given by

$$j_{fic} = h \cos\phi.$$

As usually done in the finite element method, the edge densities are converted into nodal currents by a weighting operation. The expression of those nodal currents is very simple :

$$I^n(\text{node } i) = \underline{H}_c{}^n . \underline{L}_{jk}{}^n / 2,$$

where (ijk) is a direct permutation of (123) and $\underline{L}_{jk}$ is the vector going from node j to node k in the n^{th} element.

Using first degree triangular elements (which are really the best adapted to hysteresis modulation), $\underline{H}_c(x,y)$ is precisely a summation of states like the one shown Fig. 5. The interesting feature of this approach is that the superposition principle can be applied now (curl is a linear operator). The additional currents will consequently be evaluated element by element without needing information about the magnetic state of neighbouring elements and immediately assembled in the system matrix.

VI. Results

As an example, let us consider the system consisting of a magnetic circuit including an air gap whose thickness is e. On both sides, time-dependent currents oriented respectively upwards and downwards induce a variable magnetic flux in the circuit (Fig. 7). In the case of a simple loading-unloading excitation, successive magnetic states of two representative elements of the system are given in Fig 8a, 8b and 8c respectively for e=0.0mm (no air gap), 0.5mm and 10mm (while the average length of the circuit is 1400mm).

Computation with this method fits correctly the physical behaviour. The remanent induction, B_R, and the demagnetizing field H_D, appear clearly on the diagrams, respectively when the magnetic circuit is closed (Fig. 8a) and when the air gap is large (Fig. 8c). A real computation of the hysteresis phenomenon, as attempted in this paper is really the only way to show those typically irreversible characteristics and furthermore to obtain quantitative values

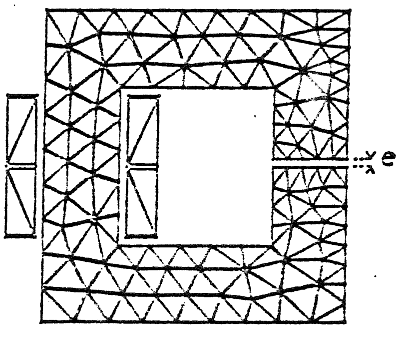

Fig. 7. Magnet ic circuit with airgap e

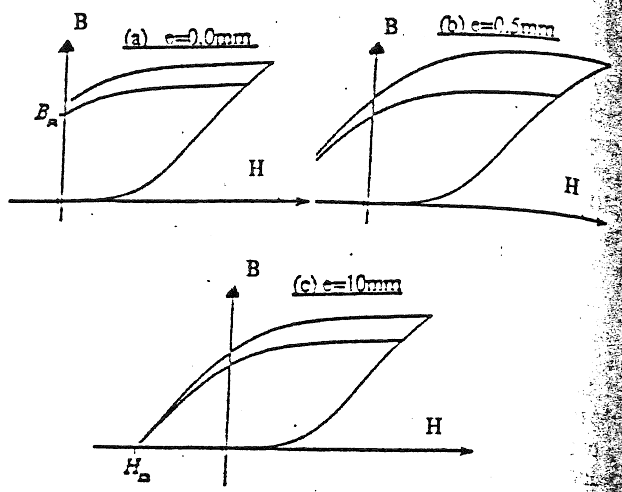

Fig. 8. Plot of the successive magnetic states in two different elements of the system; (a) with no air gap, (b) with an average air gap, (c) with a large air gap. Note that the ascending path is always common because the initial state is "story-less"

of them. As a matter of fact, any reversible law would have led in the three cases to a uniform zero-solution after unloading.

VII. Difficulties

Some properties of the non-linear system obtained by this method have to be pointed out. (1°) The functional of the variational principle associated is not always convex and therefore areas of decreasing reluctivity exist. (2°) Because of the non-linearity of the system, the Jacobian matrix has to be evaluated, which involves the derivative of the reluctivity ($\partial\nu / \partial B^2$). At turning-points, the latter is not continuous (the left and right limit values are different). These properties make the Newton-Raphson solving method no longer unconditionnally convergent [3]. Some work must then be done to obtain a really reliable solving method.

VIII. Conclusion

We have in fact demonstrated a specialized finite element scheme that can be used in any situation where ferromagnetic materials have to be modeled. The irreversible part of the behaviour is transformed into a set of equivalent nodal currents which leads us to a classical non-linear system. The physical background of the hysteresis phenomenon appears in numerical results; quantities like remanent induction and demagnetizing field can be calculated by this method.

References

[1] I.D. Mayergoyz, "Méthodes numériques en électromagnétisme, modèles d'hystérésis," *coll. de la direction des études d' Electricité de France*, Eds Eyrolles, 1991.

[2] A. Nicolet, "Modélisation du champ magnetique dans les systèmes comprenant des milieux non-linéares," *thèse de doctorat Université de Liège Fac. Sc. Appliquées*, Mai 1991.

[3] Claes Jonhson, "Numerical solution of partial differential equations by the finite element method," *Cambridge University Press*.