A Coupling Between Electric Circuits and 2D Magnetic Field Modeling

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Abstract— This paper presents a method which enables coupling between equations of electric circuits consisting of lumped elements RLC and a magnetic field model. The coupling between the finite element and the boundary element methods is used to compute the magnetic field produced by conductors excited by an electric circuit. The conductors involved in this computation may be connected according to any circuit topology and mixed with lumped elements.

I. INTRODUCTION

One important point in eddy current computation is the control of the excitation of conductors. It is sometimes possible to superimpose eddy currents on a given static current density. But this approach is often too inaccurate and a more sophisticated control of the excitations is required [1]. From an electric circuit point of view, the conductors of the magnetic field model appear as self and mutual inductances (with resistance). An approach may be to extract the impedance matrix from the numerical model. Unfortunately, because of skin and proximity effects, this matrix depends on the frequency. Moreover, in nonlinear systems, it also depends on the level of excitation and the very notion of inductance may be ambiguous [2].

In the case of a transient nonlinear eddy current problem, the only accurate method seems to be the direct coupling between the circuit equations and the finite element equations [3]. In a magnetic field computation with simple excitations, only resistive and inductive phenomena are present. Consequently, the only possible free behaviour is damping. Oscillations, in such models, are externally forced by the excitation and their frequencies are given.

The corollary coupling with circuits allows the modelling of RLC oscillating systems whose L and R elements may depend on the magnetic field models. In this case, frequencies and damping time constants depend on phenomena such as skin effect and saturation. Such a system may exhibit a very complex time behaviour.

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II. FINITE ELEMENT - BOUNDARY ELEMENT COUPLING

The equation for the two-dimensional magnetostatics is:

$$\operatorname{div} \nu \operatorname{grad} A = -J \tag{1}$$

where:

- A is the vector potential which has only one component;
- J is the current density;
- ν is the magnetic reluctivity.

In linear magnetic non-conducting media, the direct boundary element method [4] is based on the relation (2):

$$cA = \int_\Gamma A \frac{\partial G}{\partial n} d\Gamma - \int_\Gamma G \frac{\partial A}{\partial n} d\Gamma \tag{2}$$

where:

- G is the free space Green function of the two-dimensional Laplace operator;
- c = 0.5 on a smooth boundary;
- $\partial./\partial n$ is for the normal derivative.

Integrals are taken on the boundary Γ of the subdomains and the method involves only A and $\partial A/\partial n$ (tangential flux density) on the boundaries.

The finite element formulation [4] is based on the Galerkin method and we have for the domain Ω of boundary Γ:

$$\int_\Omega [\, \nu \operatorname{grad} A \ \operatorname{grad} w - J\, w \,]\, d\Omega - \int_\Gamma w\, \nu \partial A/\partial n \; d\Gamma = 0 \tag{3}$$

where ν is the magnetic reluctivity, J the current density and w the weighting function. The Galerkin method is obtained by choosing the same basis functions w_i for the weighting of the residue and the discretization of A given by $A = \Sigma A_i w_i$ where A_i are the nodal values. Here the w_i are chosen as piecewise linear functions on triangles.

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The boundary term is usually used to apply a Neumann boundary condition. Here it is used at the interface between the finite element and the boundary element domains to couple the methods. This can be extended to the boundary between two finite element subdomains. The tangential field $\nu\partial A/\partial n = H_t$ is taken as an unknown on the boundary and the boundary term of (3) is introduced. It leads to the same solution for the vector potential A as the classical formulation but it also provides the value of the tangential field on the boundary [4].

III. EDDY CURRENTS

Using Ohm's law, the case of eddy currents can be dealt with by introducing

$$J = \sigma E = \sigma\left(-\frac{\partial A}{\partial t} - U\right) \tag{4}$$

as the expression for the current density in equation (3). The conductor is characterized by its electrical conductivity σ. The time derivative of A expresses the inductive effects since $U = \text{grad } V$ is due to an externally imposed electromotive force and can be interpreted as the terminal voltage of the conductor (per unit of length).

In this case (3) becomes:

$$\int_\Omega \left[\nu \,\text{grad}\, A \;\; \text{grad}\, w + \sigma\left(\frac{\partial A}{\partial t} + U\right) w \right] d\Omega - \int_\Gamma w\, \nu \frac{\partial A}{\partial n}\, d\Gamma = 0 \tag{5}$$

The presence of the dynamical terms does not change the method of coupling. The numerical method used to discretize this transient problem is a semi discrete Galerkin method. The first step is a spatial discretization of the domains similar to the one used in the static case. This discretization uses the purely spatial weighting function w_i and leaves a continuous time variation of the degrees of freedom. It leads to a system of ordinary differential equations:

$$[S]\frac{\partial \underline{A}(t)}{\partial t} + [M]\,\underline{A}(t) = \underline{b} \tag{6}$$

where $\underline{A}$ is the vector of the nodal values of A and also of the other degrees of freedom such as $\nu\partial A/\partial n$ and U. [S] and [M] are the matrices of the system. The nonlinear nature of the problem is found in the dependence of [M] on $\underline{A}$.

The next step is a time discretization that gives an algebraic system. The backward Euler method leads to:

$$\left(\frac{[S]}{\Delta t} + \left[M(\underline{A}_{n+1})\right]\right) . \underline{A}_{n+1} = \underline{b} + \frac{[S]}{\Delta t}\underline{A}_n \tag{7}$$

where $\underline{A}_n$ and $\underline{A}_{n+1}$ are the values of $\underline{A}$ at the times t_n and t_{n+1} with $t_{n+1} - t_n = \Delta t$

This leads to a time stepping method where each step requires the resolution of the nonlinear algebraic system (7).

IV. COUPLING WITH CIRCUIT EQUATIONS

If U is given for all the conductors of the problem, the differential system can be solved. Unfortunately, in most cases, it is the total current I in a conductor which has to be imposed. As the finite element formulation involves the tangential field on the boundary, the total current I can be expressed as the line integral of the magnetic field (Ampere's law) [5] :

$$I = \int_\Gamma \nu\, \partial A/\partial n \, d\Gamma \tag{8}$$

From a more general point of view we can systematically relate to every conductor an equation like (8) and two global degrees of freedom: the total current I and the terminal voltage U. To do so, a circuit equation is necessary for each conductor. The simplest cases correspond to imposing U or I for an individual conductor but any case of conductor interconnection and electric circuit coupling can be considered. In this case, the number of circuit equations must be equal to the number of conductors involved in the field modelling plus the number of state variables necessary to describe the external circuit (For RLC circuits this number is simply the number of inductances and capacitors). Those equations together with equations coming from (8) are added to system (6). Circuit equations that are differential are discretized according to the backward Euler method. It should be noted that this treatment of conductors may be extended to thin wire conductors. In this case, no skin effect is computed but global inductive effects are taken into account. Although the equations are different, they may be processed the same way from an electric circuit point of view [5]. No special treatment is made in order to symmetrize circuit equations because symmetry is lost anyway with the FEM-BEM coupling. The drawback of having a non-symmetric system is compensated for by the reduction of elements involved in the boundary element method. In fact, the balance between the two effects depends strongly on the particular problem considered.

The algebraic system is linearized by the Newton Raphson method and the linear system is solved by LU decomposition. The solution is improved by iterative refinement.

V. EXAMPLE

The following problem is considered as an example: a nonlinear inductance consisting of a coil with a ferromagnetic saturable core is fed by a capacitor and a voltage source in series (Fig. 1, 2 ,and 3).

The inductance is modelled with finite elements. The coil is represented by two conductors. There are four global degrees of freedom associated to those finite element objects: total currents I_1, I_2 and the terminal voltages U_1, U_2 . To describe the electric circuit, the capacitor voltage U_C has been chosen as a state variable. This introduces five degrees of freedom related to circuit equations and five more equations are needed. Two equations are given by formula (8) expressed for each finite element conductors. The three following equations (two algebraic and one differential) are added to the system:

$$U_C + U_1 - U_2 = V(t)$$
$$I_1 + I_2 = 0$$
$$I_1 + C\, \partial U_C / \partial t = 0 \qquad (9)$$

Fig. 4 shows the time evolution of the current when a voltage step is applied. The response is a damped oscillation. Harmonics due to the saturation of the core appear clearly. As the level of current decreases, saturation becomes less important and the harmonic rate diminishes. As the oscillations are free, their frequency is determined by the resonance frequency of the circuit. It must be noticed that the period increases with time as the apparent inductance increases because of desaturation.

Fig. 5 shows the time evolution of the flux density in the core between the two conductors.

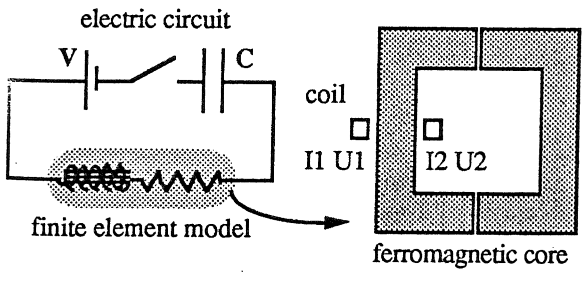

Fig. 1. Nonlinear inductance fed by a capacitive circuit.

VI. CONCLUSION

From an electric circuit point of view, conductors involved in the numerical magnetic field modelling form an RL network whose terminals each correspond to a conductor. The complete determination of the characteristics of this network is not feasible because of the complicated dependence on the frequency and amplitude of currents resulting from skin effect and saturation. It is therefore necessary to solve the problem simultaneously for the magnetic field model and the electric circuits connected to the conductors. The method presented here is general and allows a transient simulation of eddy current nonlinear problems with conductors excited by electric circuits. Those electric circuits may contain current and voltage sources (with various wave forms), resistances, inductances and capacitances and there is no restriction on the topology. Conductors in the magnetic field modelling may be included anywhere in the circuit.

This flexible method allows realistic modelling of electromagnetic devices. Some devices may exhibit new behaviour such as natural oscillation with a period varying with time.

VII. REFERENCES

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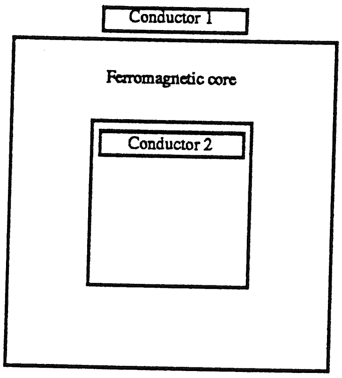

Fig. 2. Geometry of the nonlinear inductance. The width of the magnetic circuit is 1 cm and the conductivity of the conductors is 1.13 S/m. Feeding circuit: Voltage step = 5000 V, capacitance = 100 nF.

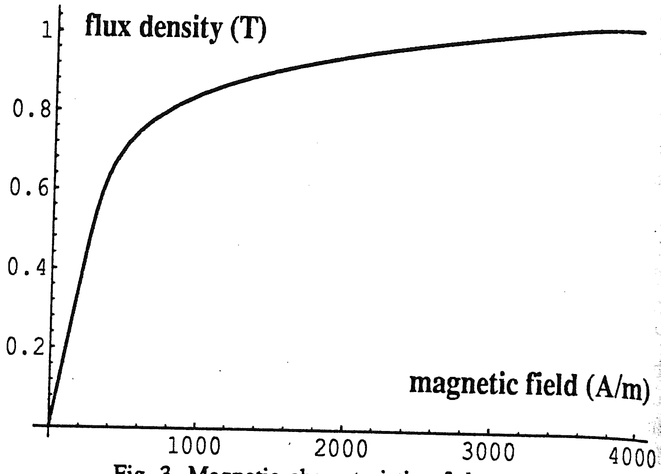

Fig. 3. Magnetic characteristic of the core.

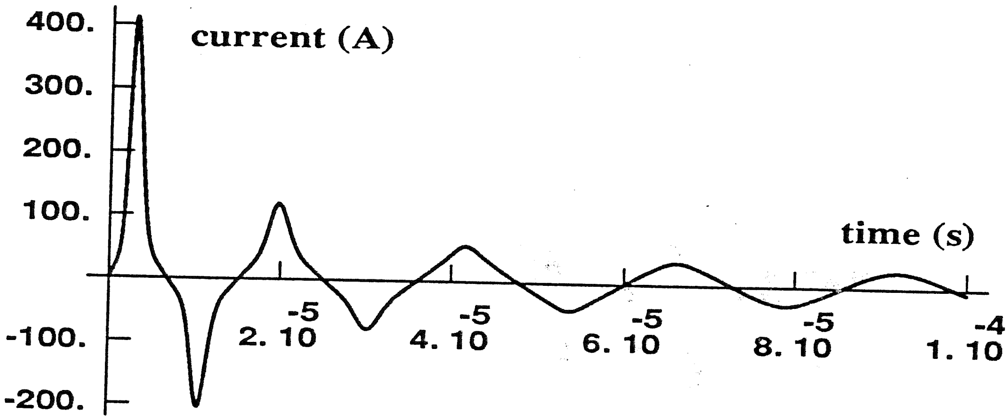

Fig. 4. Time evolution of the current in the conductors.

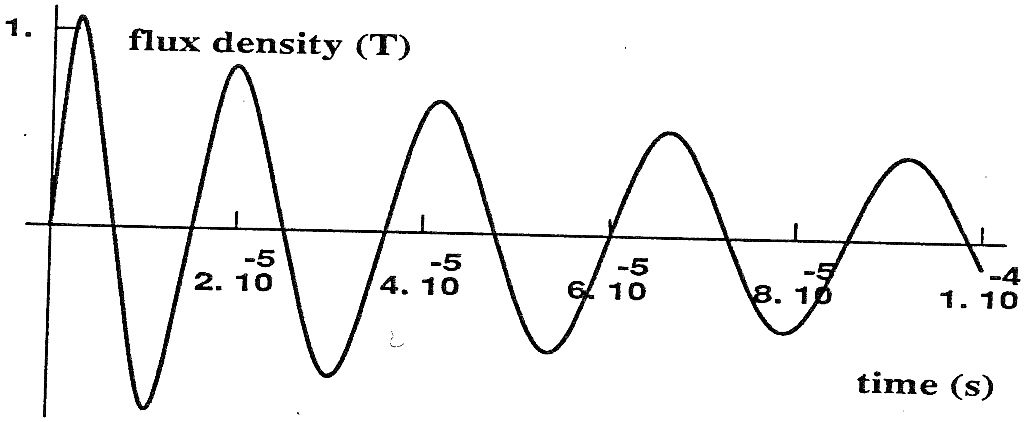

Fig. 5. Time evolution of the flux density in the core.