Modelling of Electromechanical Relays taking into account Movement and Electric Circuits

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Abstract - This paper presents a numerical modelling of an electromechanical relay connected with an electric excitation circuit. This transient modelling not only takes into account the classical electromagnetic equations of the device but also the movement and circuit equations. The use of the finite element–boundary element coupling method facilitates the computation of the movement while the actual coupling with circuit equations is necessary for an accurate and reliable representation of transient phenomena.

INTRODUCTION

In the majority of practical applications, electromagnetic phenomena are used to cause movement or forces (motors, relays, ...) and are inserted in external electric circuits (transformers, ...). Magnetic field programs that are only able to treat electromagnetic quantities are therefore not very effective in numerical modelling of such apparatus. Moreover the coupling is often strong in well designed systems which are precisely conceived to transform one kind of energy into another (e.g. a motor has to transform electrical energy into mechanical energy). In a relay for example, the excitation circuit generates current in the coil and the resulting magnetic field causes force and torque on moving parts. In the reversed way, motion of moving parts acts on geometry, induces flux variations in the system and modifies the inductance in the circuit.

I MAGNETIC FIELD MODELLING

To compute the electromagnetic part of the problem (and to evaluate forces), a program called LUCIE is used which is based on the coupling of the finite element method and the boundary element method (F.E.M.–B.E.M. coupling).

a. Subdomain method

To carry out that coupling, the system is divided into N subdomains $\{D_i, i=1, \dots ,N\}$ whose respective boundaries are the contours $\{\Gamma_i, i=1, \dots ,N\}$. The unknown fields are the vector potential $A(\underline{x},t)$ on the subdomains D_i and the tangent magnetic field $H_t(\underline{x},t)$ on the boundaries Γ_i. Under these conditions, the 2D magnetic field equation (1) (which is a set of partial differential equations) is well posed in each subdomain D_i.

$$\mathrm{div}\,(\nu_i\ \mathrm{grad}\ A) = -J_i \qquad (1)$$

Either the finite element method (F.E.M.) or the boundary element method (B.E.M.) can be used to solve (1) in each subdomain D_i. Both methods are based on spatial discretization relations such as :

$$A(\underline{x},t) = \sum_{k=1}^{N} A_k(t)\ w_k(\underline{x})$$
$$H_t(\underline{x},t) = \sum_{k=1}^{M} H_{tk}(t)\ w_k(\underline{x}) \qquad (2)$$

where $w_k(\underline{x})$ are nodal weighting functions, A_k are the N time varying nodal values of the vector potential and H_{tk} are the M time varying nodal values of the magnetic tangential field on the boundaries Γ_i.

The finite element method with trial functions, $\{w_n(\underline{x}), i=1,\dots,N\}$, identical to the weighting functions (Galerkin method) gives N equations such as (3).

$$\int_{D_i} (\nu_i\ \mathrm{grad}\ A.\mathrm{grad}\ w_n - J_i\, w_n)\ dD_i - \int_{\Gamma_i} w_n\, H_t\ d\Gamma_i = 0 \qquad (3)$$

In a subdomain made of a linear and nonconducting material, it is also possible to use the boundary element method which is based on equation (4) where G is the free space Green function for 2D Laplace operator, c is a constant depending on the smoothness of Γ_i (c=1/2 on a regular boundary) and n is the outer normal vector for D_i.

$$c\,A = \int_{\Gamma_i} \left(A\frac{\partial G}{\partial n} - \frac{1}{\nu_i} G\, H_t \right) d\Gamma_i \qquad (4)$$

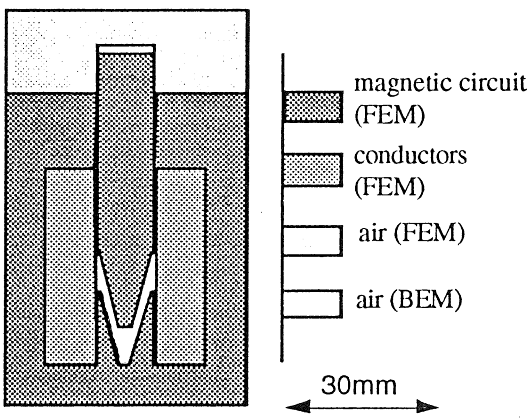

Fig. 1. Representation of the sub-domains defined in the modelling of the relay. The solving method is specified for each subdomain.

b. F.E.M.–B.E.M. coupling

The philosophy of the F.E.M.–B.E.M. coupling is to use the B.E.M. for linear magnetic and non conducting media wherein rigid parts can move (generally air), remeshing is

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therefore avoided. The B.E.M. is also used to provide a rigorous treatment for open boundaries. The F.E.M. is chosen everywhere else and especially in media that can be the seat of phenomena such as saturation or eddy currents.

II COUPLING WITH MOVEMENT [1]

a. Mechanical model

Only rigid motions in 2D models are considered. Three degrees of freedom are associated with each moving part, two degrees of translation and one degree of rotation. The mechanical model is the classical Newton equation taking into account electromagnetic forces and torque. Connection of the different objects with springs, viscous dampers or constant forces is considered.

The movement first order differential equations are :

$$m\frac{dv}{dt}+cv+kx=F(t) \quad , \quad v=\frac{dx}{dt} \tag{5}$$

where the following quantities are defined :

- position : x(t);
- speed : v(t);
- mass or momentum of inertia : m;
- elastic constant of the spring : k;
- viscous damper constant : c;
- applied force or torque including magnetic forces and constant forces: F(t).

Time discretization by the backward Euler method gives :

$$\begin{aligned} v(t+\Delta t) &= \frac{mv(t) + F(t)\Delta t - k\,x(t)\Delta t}{k\Delta t^2 + c\Delta t + m} \\ x(t+\Delta t) &= \frac{mx(t) + mv(t)\,\Delta t + F(t)\Delta t^2 + cx(t)\Delta t}{k\Delta t^2 + c\Delta t + m} . \end{aligned} \tag{6}$$

b. Computation of magnetic forces

The F.E.M.–B.E.M. coupling method gives, at each time step, not only the knowledge of the vector potential field A but also the knowledge of the tangential magnetic field H_t on the boundaries Γ_i. That allows an easy computation of magnetic forces by the Maxwell stress tensor method [3]. That method gives an equivalent normal force density $\mathbf{F_n}$ and an equivalent tangential force density $\mathbf{F_t}$ on the boundaries of objects whose expressions are :

$$\mathbf{F_n}=(\frac{B_n^2}{2\mu_0}-\frac{\mu_0}{2}H_t^2)\mathbf{n} \quad , \quad \mathbf{F_t}=B_nH_t\mathbf{t} \tag{7}$$

Note that the quantities B_n and H_t which are involved in those expressions are well defined even on surfaces of discontinuity ; the scalar field H_t is directly given by the coupling method while B_n is computed from the vector potential by :

$$\begin{aligned} B_n &= \mathbf{curl}(A).\mathbf{n}=\frac{\partial A}{\partial y}n_x-\frac{\partial A}{\partial x}n_y \\ &= (-\frac{\partial A}{\partial y}\frac{\partial y}{\partial \xi}-\frac{\partial A}{\partial x}\frac{\partial x}{\partial \xi})\,/\,J(\xi)=-\frac{\partial A}{\partial \xi}\,/\,J(\xi) \end{aligned} \tag{8}$$

where n_x and n_y are the components of the outer normal vector $\mathbf{n}$, ξ is the local coordinate of the shape functions used for the boundary element discretization and $J(\xi)$ is the corresponding Jacobian.

c. Coupling with the magnetic system

Due to the fact that the expression of the force is an intricate nonlinear function of the magnetic degrees of freedom, it is cumbersome to solve simultaneously the finite element equations and the mechanical equations. Consequently, the following algorithm is adopted to model the movement :

- Compute electromagnetic state at time t
- Compute the magnetic force and torque for this state
- Solve the mechanical equations to find the displacements of the moving parts.
- Verify coherence:
  - Keep position in physical limits
  - Modify time step if necessary
- Modify the geometry in the database using the computed displacements
- Next step

This kind of algorithm can be qualified as "weak coupling" because the magnetic and mechanical equations are not solved simultaneously; that means that the magnetic force is supposed to be a constant on one time step.

III COUPLING WITH CIRCUITS [2]

From an electrical point of view, the conductors of a magnetic field model can be seen as self and mutual inductances with resistances. An approach for coupling them with an external excitation circuit could be to extract an equivalent impedance matrix from the numerical model and to introduce it in the electrical circuit equations. Unfortunately, because of saturation, movement and skin effect, that matrix would depend on the level of excitation, on the position and on the frequency. That makes it necessary to realize a real coupling of the finite element equations with the circuit equations.

a. Eddy currents

In transient problems where eddy currents exist in the conductors, the current density term $J(\underline{x},t)$ in equation (2) is not known a priori and is then replaced by its expression in accordance with Ohm's law :

$$J_i=\sigma_i E=-\sigma_i(\frac{\partial A}{\partial t}+U_i). \tag{9}$$

The conductor, D_i, is characterized by its conductivity σ_i. The time partial derivative $\partial A/\partial t$ stands for the inductive effect and U_i is the terminal voltage p.u. of length imposed on the conductor.

A crucial problem is to determine the effect of movement on eddy currents. According to Faraday's induction law, the induced electromotive force in a circuit is equal to the total magnetic flux variation through the circuit whatever the origin of the variation : time variation of the field or movement of the circuit. Equation (9) is therefore only applicable for non moving conductors and the correct expression for a conductor moving at speed v_i is [4]:

$$J_i = \sigma_i (E + v_i \times B) = -\sigma_i (\frac{\partial A}{\partial t} + U_i - v_i \times B) \qquad (10)$$

where B is the induction. Since $-v_i \times B = (v_i.\text{grad})\, A$ (as it may be verified component by component), the generalization of (9) for moving conductors is :

$$J_i = -\sigma_i (\frac{dA}{dt} + U_i) \qquad (11)$$

where d/dt is the material derivative, i.e. the variation for a given particle. We have pointed out before that with the F.E.M.–B.E.M. coupling method the mesh of moving parts is involved as a whole in the movement. Each node of the system corresponds to the same particle during all the motion. The material derivative dA/dt is simply evaluated by difference of nodal values; there is no need for an explicit term for the motional induced electromotive force.

b. External degrees of freedom for conductors

The magnetic–electrical coupled problem can be solved if an imposed terminal voltage is given for each conductor of the problem. If the total current, I_i, is given instead of the terminal voltage, this becomes one more unknown and one more relation is necessary to relate it to the magnetic degrees of freedom. Since the tangential field H_t is involved as an unknown in the F.E.M.–B.E.M. coupling, the following expression of Ampere's law gives such a relation :

$$I_i = \int_{\Gamma_i} H_t \, d\Gamma_i \qquad (12)$$

A special treatment is possible to model coils where eddy currents are prevented by the smallness of the wire section. In that case, the following relation is used instead of (12) [1]:

$$N_i I_i = -\sigma_i \int_{D_i} \frac{dA}{dt} dD_i - \sigma S_i U_i \qquad (13)$$

where N_i is the number of turns and S_i the section of one turn in the subdomain D_i. The current density is given by $J_i = I_i/S_i$ instead of (11) and there is no more skin effect, only a global inductive effect is allowed.

From a more general point of view, we can systematically relate to every conductor of the problem an equation such as (12) or (13) and two external degrees of freedom: the total current in the conductor, I_i, and the terminal voltage, U_i . One circuit equation is necessary for each conductor to solve the system. The simplest case corresponds to imposing U_i or I_i for each individual conductor but we are more interested in the general case of N_c independent conductors connected with any external R.L.C. circuit according to any topology.

If that external circuit has N_e state variables (the number of inductances and capacitors), the circuit theory gives N_c+N_e relations or first order differential equations which are added to the magnetic model ones and solved in the same time.

IV EXAMPLE : MODELLING OF THE RELAY

a. Movement equations

In this problem, only one direction of movement is free and there is only one movement equation. The moving core (mass=0.2 kg) is fixed to a spring (k=2500. N/m) tending to keep the relay open while the magnetic force tends to make the closure (Fig. 2). The movement is 7mm long; the relay is 50mm deep; friction is neglected.

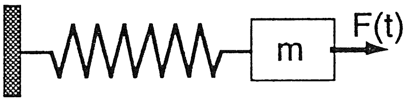

Fig. 2. Mechanical diagram of the relay.

b. Circuit equations

The excitation circuit (Fig. 3) consists of an initially charged capacitor C (V_c = 10V) connected with an external resistance R and with the two conductors in series representing the coil in the model (N_c = 2). The capacitor voltage has to be chosen as a state variable (N_e = 1). The circuit theory gives the following three equations :

$$\begin{aligned} &V_c - R\, I_1 + U_1 - U_2 = 0 \\ &I_1 + I_2 = 0 \\ &I_1 + C \frac{dV_c}{dt} = 0 \end{aligned} \qquad (14)$$

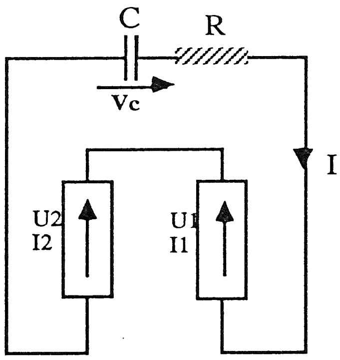

Fig. 3. Electrical diagram of the relay. The conductors are represented by their external degrees of freedom U_i and I_i.

V RESULTS

As a deliberate choice for this modelling, the electrical and mechanical diagrams are extremely simple when considered separately (Fig. 2 and 3). They present elementary and well known individual behaviours.

In spite of this, time evolutions become quite intricate as soon as the coupled problem is considered and a global analysis of the results is not of interest. We rather apply ourselves in this paper to demonstrate successively that expected interplays appear actually in computed results.

Two figures are presented showing time evolutions of the total applied force on the moving core (including the magnetic force and the spring force), the position of the core and the current in the coil. All curves are scaled down for comparison, scaling values are given in Table 1.

Fig. 4 and 5 correspond respectively to the cases where the magnetic circuit is made of a linear or a nonlinear material. Both materials are identical at low fields.

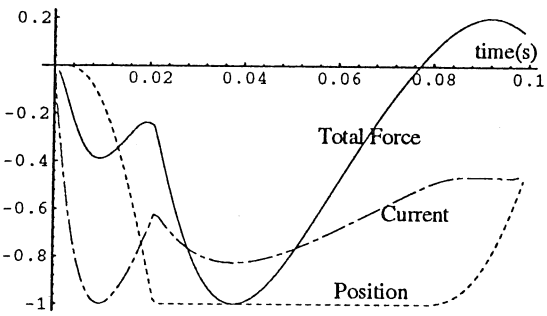

Fig. 4. Time evolutions of the total force, the position of the moving core and the current in the linear case

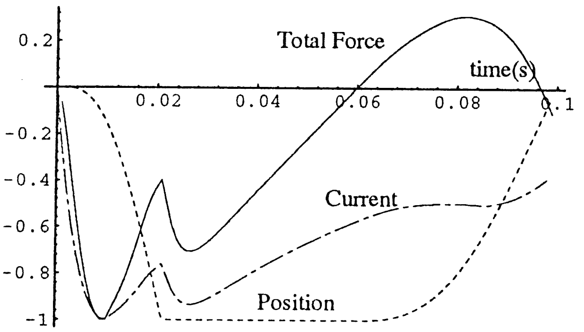

Fig. 5. Time evolutions in the nonlinear case.

	linear	nonlinear
F_{max} (N)	27.32	10.92
I_{max} (A)	6.186	6.293

Table 1. Scaling values of the total force and the current.

a. Influence of current on force

The magnetic force is closely related to the square value of the flux crossing the air gap which is itself proportional to the current in the coil in the linear case. On Fig. 4 and 5, it can be checked that the current and force curves exhibit similar shapes. They both present the characteristic retrogression point when the closure of the relay occurs, i.e. when the motion induced current vanishes.

b. Influence of the electric circuit on the magnetic model

As pointed out before, the magnetic model can be seen as an RL device if U_i or I_i is imposed on each conductor. Resulting time evolutions are combinations of exponential functions.

The situation is completely different when the conductors are inserted in an electric circuit, especially if this one contains capacitors. In this case, the system must rather be seen as an RLC device with a kind of damped oscillating behaviour.

Because the inductance depends on the motion, the circuit is a true RLC circuit as long as the moving core remains motionless. One can check in Fig. 4 that the current curve is a piece of a damped sinusoidal function in that case. On the other hand this is no longer true in Fig. 5 because of the nonlinearity of the material.

c. Influence of movement on current

The downward motion of the moving core decreases the length of the air gap and increases its reluctivity in the magnetic circuit. An inverse current is then induced in the coil (in accordance with the Lenz law) which opposes the setting of the direct current. That inverse current reaches a maximum when the speed does and it vanishes when the closure is done.

CONCLUSION

It is obvious that the correct waveform for current in a relay results from a complex interplay between the electric circuit and the variation of inductance due to displacement of moving parts.

A numerical implementation has been done with the F.E.M.–B.E.M. coupling method. This method presents several advantages for that kind of modelling :

- There is no mesh deformation during the movement.
- There is no need for an explicit term for the motional induced electromotive force.
- The method makes it easy to evaluate force and torque by the Maxwell stress tensor method.
- The method makes easy it to define the electrical external degrees of freedom of the conductors of the model and to make the connection with an electrical circuit.

Analysis of the computed results shows that they are in good agreement with the different physical phenomena involved in the problem.

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