# PERFORMANCE OF CODED TIME-DIVERSITY FREE-SPACE OPTICAL LINKS

Fang Xu, Mohammad-Ali Khalighi, Patrice Caussé, Salah Bourennane

Institut Fresnel, École Centrale Marseille, Marseille, France

# ABSTRACT

Depending on the tolerable delay latency, we can benefit from some degree of time diversity in free-space optical links, by employing channel coding and interleaving. We present a comparative study of the performance of different channel coding techniques at the presence of time diversity. We show that turbo-codes are particularly efficient under strong turbulence conditions. For relatively weak turbulence, however, a simple convolutional code makes a good compromise between decoding complexity and performance. We also address the problem of channel estimation at the receiver, and show that only few pilot symbols are sufficient to provide a performance close to the case of perfect channel knowledge.

# 1. INTRODUCTION

Free-space optics (FSO) can provide a very high rate and secure data transmission along a line of sight. Due to their cost-effective and rapid deployment, FSO systems have attracted growing attention since a few years for a variety of applications, e.g. last mile connectivity, entreprise connectivity, optical-fiber backup, etc. [1]. However, solar heating and wind cause inhomogeneities in the temperature and pressure of the atmosphere, which lead to the variations of the air refractive index. These variations, usually referred to as scintillation, deteriorate the transmitted signal in both amplitude and phase [2]. Mitigating the resulting *channel fading* constitutes a real challenge in FSO transmission systems.

In this paper, we consider fading reduction by making use of channel time diversity. In fact, when the channel coherence time is small, compared to the data frame length, we can exploit the corresponding inherent time diversity by employing channel coding and interleaving. We assume that the transmitter and the receiver are perfectly aligned. Also, we consider a system working at a single wavelength, and assume one single lens at both the transmitter and receiver. This means that we do not have any source of spatial or frequency diversity available. We also assume that the lenses are small, so that we can not benefit from aperture averaging.

Our aim is to provide a comparative study of different channel codes under different turbulence conditions. We consider a typical FSO system employing intensity modulation with direct detection (IM/DD), and using On-Off Keying (OOK) modulation. Four channel coding approaches that are usually employed in communication systems are studied: convolutional codes, Reed-Solomon (RS) codes, concatenated convolutional and RS (CCRS) codes, and turbo-codes (TC). We model the atmospheric turbulences by the gamma-gamma distribution. We show that under strong turbulence conditions and when enough diversity is available, TCs bring a significant performance improvement. However, for relatively weak turbulences, a simple convolutional code provides a performance close to that of a TC, while having a lower decoding complexity. We also consider the case where the channel is estimated at the receiver based on some training symbols.

Note that, a similar study is done in [3], where the channel is assumed to change over the duration of each symbol. In practice, however, channel time variations are not so fast and we have far less time diversity available. In our study, we consider typical channel coherence times and discuss the implied delay latency for a given time diversity order.

The organization of the paper is as follows. In Section 2, we explain the atmospheric channel model. We describe the signal detection and decoding tasks in Section 3. Numerical results are provided in Section 4 to compare the system performance with different coding solutions under different time-diversity conditions. Section 5 concludes the paper.

### 2. ATMOSPHERIC TURBULENCE MODEL

We use the recently-proposed gamma-gamma statistical channel model, in which small-scale (diffractive) intensity fluctuations are considered to be multiplicatively modulated by large-scale (refractive) intensity fluctuations. Let us denote by  $I_x$  and  $I_y$  the small- and large-scale irradiance fluctuations, respectively.  $I_x$  and  $I_y$  are assumed to be statistically independent and described by the gamma distribution. Based on these assumptions, the received optical intensity  $I = I_x I_y$  follows a gamma-gamma distribution with the probability density function (PDF) as follows [2, 4].

$$P(I) = \frac{2 \left(\alpha\beta\right)^{(\alpha+\beta)/2}}{\Gamma(\alpha) \Gamma(\beta)} I^{(\alpha+\beta)/2-1} K_{\alpha-\beta} \left[ 2(\alpha\beta I)^{1/2} \right], \ I > 0$$
<sup>(1)</sup>

Here,  $\alpha$  and  $\beta$  are the effective numbers of small scale and large scale eddies of the scattering environment, and  $K_{\alpha}(x)$ is the modified Bessel function of the second kind and order  $\alpha$ . Assuming propagation by plane wave and turbulent eddies of zero inner scale, these parameters can be directly related to the atmospheric conditions according to:

$$\alpha = \left[ \exp\left(\frac{0.49\,\chi^2}{\left(1 + 1.11\,\chi^{12/5}\right)^{7/6}}\right) - 1 \right]^{-1} \qquad (2)$$

$$\beta = \left[ \exp\left(\frac{0.51\,\chi^2}{\left(1 + 0.69\,\chi^{12/5}\right)^{5/6}}\right) - 1 \right]^{-1} \qquad (3)$$

where  $\chi^2 = 1.23 C_n^2 k^{7/6} L^{11/6}$  is the Rytov variance. Here,  $k = 2\pi/\lambda$  is the wave number with  $\lambda$  the wavelength, L is the link distance, and  $C_n^2$  stands for the index of refraction structure parameter. The interest of this model is that, by setting the parameters  $\alpha$  and  $\beta$  appropriately, we can use it to describe either weak or strong turbulences.

For the channel time variations, we use the theoretical blockfading model, according to which the channel fades remain almost constant during a block of several consecutive symbols (corresponding to the channel coherence time), and change to new independent values from one block to next. That is usually recognized as the *frozen* channel model.

### 3. SYMBOL DETECTION AND CHANNEL DECODING AT THE RECEIVER

At the receiver, the electrical signal after the optical/electrical conversion is [5]:

$$r_e = \eta \left( I_s + I_a \right) + n \tag{4}$$

where  $I_s$  is the received signal light intensity, which can be considered as a product of  $I_0$ , the emitted light intensity, and h, the channel atmospheric turbulence with the PDF given in (1):  $I_s = I_0 h$ . Also,  $I_a$  is the ambient light intensity,  $\eta$  is the optical/electrical conversion efficiency, and n is the sum of thermal, dark, and shot noise. We suppose that the receiver is thermal noise limited, and consider n as a Gaussian additive noise, independent of the signal, and of zero mean and variance  $\sigma_n^2$ . Assuming that  $I_a$  is known and can be perfectly cancelled, the received signal before demodulation will be:  $r = \eta I_0 h + n$ . Without loss of generality, let us assume that  $\eta = 1$ . As mentioned in Section 1, we consider the OOK modulation, where the presence of  $I_0$  represents a symbol s = 1, and its absence, a symbol s = 0. We have, hence:

$$r = sh + n. \tag{5}$$

# 3.1. Signal detection

From (5), we consider the maximum a posteriori (MAP) criterion to detect s from r. The MAP detector provides the detected symbol  $\hat{s}$  as:

$$\hat{s} = \arg\max P(r|s)P(s), \tag{6}$$

where P(r|s) denotes the PDF of r conditioned to s. Considering equiprobable symbols, i.e., P(s) = 1/2, (6) reduces to:  $\hat{s} = \arg \max P(r|s)$ . Given that n is assumed to be Gaussian distributed, to obtain  $\hat{s}$ , we calculate the logarithmic likelihood ratio (LLR) as follows:

LLR = 
$$\log \frac{P(r|s=1)}{P(r|s=0)} = \frac{2hr - h^2}{2\sigma_n^2}.$$
 (7)

In the case of hard signal detection,  $\hat{s}$  is simply obtained by using the sign of the LLRs: if LLR < 0, we make the decision  $\hat{s} = 0$ , and  $\hat{s} = 1$  otherwise. For soft signal detection, however, we keep the LLR on each transmitted symbol.

#### 3.2. Channel decoding

As mentioned in Section 1, we consider four channel coding approaches: convolutional, RS, CCRS, and TC. Except for the RS scheme, where we perform hard-decoding following hard signal detection, we perform soft signal detection and soft channel decoding. This way, the decoder takes LLRs at its input and provides at its output LLRs on the information bits, what is called soft-input soft-output (SISO) decoding. For SISO decoding convolutional codes, we use the softoutput Viterbi algorithm (SOVA). For CCRS coding, we perform outer encoding using an RS code, followed by outer interleaving and inner convolutional encoding. The decoding consists in SISO decoding of the inner code, followed by outer de-interleaving and hard decoding of the outer RS code. On the other hand, the TC scheme considered here consists in parallel concatenation of two convolutional codes. At the receiver, SOVA-based iterative decoding is done, where two SISO decoders exchange extrinsic LLRs (see [6] for details).

### 4. NUMERICAL RESULTS

We present here some simulation results to compare the performances of the four coding schemes under different channel turbulence conditions. The system performance is evaluated in terms of bit error rate (BER). Signal-to-noise ratio (SNR) is considered as  $\mathbb{E}\{I_s^2\}/\sigma_n^2$  and is converted to  $E_b/N_0$ in the results to be presented. We consider communication by burst where frames of  $N_F$  OOK modulated symbols, are transmitted through the channel. Concerning the channel coding schemes, we consider the RS code (255,239) of code rate  $R_c = 0.94$ , and the recursive systematic convolutional (RSC) code  $(1, 133/171)_8$  of constraint length 7 and rate  $R_c = 1/2$ . For CCRS coding, we perform outer RS (255,239) encoding, followed by pseudo-random outer interleaving and inner RSC  $(1, 133/171)_8$  encoding; this corresponds to the scheme proposed in the DVB-S standard-2004 [7]. At last, for the TC case, we consider the parallel concatenation of two identical RSC  $(1, 15/17)_8$  codes of constraint length 4. Also, we consider a pseudo-random channel interleaver.

To model the fading statistics according to the gamma-gamma model, we set the Rytov variance  $\chi^2$ . We consider two typical cases of weak and relatively strong atmospheric turbulences, for which we set  $\chi$  to 0.2 and 3, respectively. Although in general, there is no direct relationship between the turbulence strength and the channel time correlations, we consider two



Fig. 1. Weak turbulence conditions, ( $\alpha = 51.9$ ,  $\beta = 49.1$ ). Cases of no diversity, TDO=2, and TDO=4; perfect channel knowledge.

channel coherence times  $\tau_c$  of 1ms, and  $20\mu s$ , for the two cases of  $\chi = 0.2$  and 3, respectively.

## 4.1. Weak turbulence conditions

Setting  $\chi$  to 0.2, results in  $\alpha = 51.9$  and  $\beta = 49.1$  from (2) and (3). We consider a typical data rate of 1 Gbps that corresponds to the symbol duration of  $T_s = 1ns$ . Considering the channel coherence time of  $\tau_c = 1ms$  and our block fading model, the channel remains constant over the blocks of  $N_B = \tau_c/T_s = 10^6$  symbols. We consider the case where we have no time diversity available, as well as the cases where we have a potential time diversity order (TDO) of 2 and 4. For these three cases, we set the frame length  $N_F$  respectively to 4080,<sup>1</sup> 2 × 10<sup>6</sup>, and 4 × 10<sup>6</sup>. For the two latter cases, we undergo a delay-latency of 2ms and 4ms, respectively.

In Fig. 1, we have shown the curves of BER versus  $E_b/N_0$  for different coding schemes, as well as for the case of no channel coding. Since for this latter case, we cannot benefit from time diversity, it applies to the three cases of TDO.

First consider the case where we have no time diversity available. We see from Fig. 1 that the RS code is not efficient. The three other codes provide interesting and almost identical performance improvements, so, we have only presented the BER for RSC here. For instance, at BER= $10^{-5}$ , compared to the no-coding case, we obtain a gain of 1 dB in SNR by using the RS, and a gain of about 3.5 dB for the three other schemes.

For TDO=2, we notice again that the RS code is not efficient. However, we notice a considerable performance gain by employing the other three coding schemes. We notice a negligible difference between the performances of RSC, CCRS and TC; the performance of CCRS is between these of RSC and TC. Here at BER= $10^{-5}$ , compared to the no-coding case, we



**Fig. 2.** Strong turbulence conditions, ( $\alpha = 5.49$ ,  $\beta = 1.12$ ), Cases of no diversity, TDO=2, 4, and 8; perfect channel knowledge.

have a gain of 5.2 dB in SNR by the RSC code. At last, for TDO=4, we have only presented the performance for RSC, as the performances of CCRS and TC are almost identical to that of RSC. Here, we have a gain of 6.5 dB in SNR at BER= $10^{-5}$ , compared to the no-coding case.

### 4.2. Strong turbulence conditions

For this case, we set  $\chi = 3$  that results in  $\alpha = 5.49$  and  $\beta = 1.12$ . Here we consider the data rate of 100 Mbps that corresponds to the symbol duration of  $T_s = 10ns$ , and the channel coherence time of  $20\mu s$ . According to our block-fading channel model, the channel changes over the block of 2000 symbols. We consider three cases of  $N_F = 4080, 8160$ , and 16320, that result in TDOs about 2, 4, and 8, and impose delay-latencies of about  $40\mu s, 80\mu s$ , and  $160\mu s$ , respectively.

In Fig. 2, we have shown the BER curves for RSC and TC coding schemes, as well as for the no-coding case. We have observed that, when we do not have any time diversity available, channel coding is not efficient at all (results are not shown). In other words, the BER for the no-coding case corresponds also to the case of no-diversity. We do not consider the the case of simple RS nor the CCRS encoding; the latter provides a performance very close to that of RSC, and hence, is not interesting. From Fig. 2, we notice a considerable performance improvement with channel coding in the presence of time diversity. For instance, for TDO=2, we obtain a gain of 10 dB and 31 dB in SNR at BER= $10^{-5}$ , by using RSC and TC, respectively, compared to the no-diversity case. The corresponding gains are about 41.5 dB and 47.5 dB for TDO=4. For TDO=8, we have only shown the BER corresponding to TC; we notice a gain of 58.5 dB in SNR at BER= $10^{-5}$ .

#### 4.3. Pilot assisted channel estimation

As we notice from (7), we require the knowledge of the instantaneous channel fading h for signal detection. Up to now,

<sup>&</sup>lt;sup>1</sup>We chose this  $N_F$  because it is the minimum frame length for the case of CCRS coding: After RS coding, we have blocks of  $255 \times 8 = 2040$  bits; after RSC coding of rate 1/2, we obtain about 4080 bits.



**Fig. 3**. Sensitivity to channel estimation errors, weak and strong turbulences, TDO=2.

we have assumed that we have perfect channel knowledge at the receiver. In a practical system, however, the channel fading h is estimated based on some pilots symbols. It is interesting to see the effect of channel estimation errors on the BER performance. Let us denote by  $N_P$  the number of the pilots in each block of symbols corresponding to  $\tau_c$ . We perform maximum likelihood channel estimation [8] based on these pilot symbols prior to signal detection. In Fig. 3 we have shown the curves of BER versus  $N_P$  for the two cases of weak and strong turbulences, TDO=2, and the two coding schemes of RSC and TC. For each scheme, we have set the SNR to that results in BER= $10^{-5}$  in the case of perfect channel knowledge. We have then evaluated the BER for different  $N_P$ . We can see from Fig. 3 that under weak turbulence conditions, only few pilots symbols are sufficient to provide results close to the perfect channel knowledge case. This means that, we undergo a negligible loss in the data rate due to pilot insertion. In addition, we notice that the TC is more sensitive than RSC to channel estimation errors. On the other hand, for the case of strong turbulences, we notice that we obtain a performance almost identical to that of the perfect channel knowledge case by using only 2 pilots. As a matter of fact, since  $E_b/N_0$  is much higher for strong turbulences, (57 dB for TC), in the cases of relatively low instantaneous fadings, we obtain a good channel estimate. In the cases of relatively deep fadings, however, we are likely to lose the entire data frame irrespective of the quality of channel estimate.

### 5. CONCLUSION

For an FSO transmission link submitted to atmospheric turbulences, we investigated how much we can benefit from the available time diversity by employing channel coding and interleaving in order to improve the link performance. We showed that substantial gains in SNR can be obtained, especially under strong turbulence conditions. From the presented results we can conclude that the convolutional codes are appropriate coding solutions for weak turbulence conditions, taking into account the achieved performance improvement and the decoding complexity. Also, turbo-codes appear to be an appropriate and quite efficient coding solution under strong turbulences, conditioned to the presence of time diversity. We also considered the channel estimation issue and showed that few pilot symbols are sufficient to provide a performance close to the case of perfect channel knowledge.

In order to benefit from time diversity, we have to provide a large enough memory at the receiver, as well as to tolerate some delay latency in data detection. For too long channel coherence intervals, the receiver may become too complex due to the memory requirement, and also, the delay latency may be intolerable. At last, note that, as BER has little dependence on the frame length, the results we presented can be practically used for different channel coherence times, and the only factor to modify is the implied delay latency.

#### Acknowledgment

This work was supported in part by the French PACA (Provence-Alpes-Côte d'Azur) Regional Council. The authors wish to thank F. Chazalet from Shaktiware Co., Marseille, France, N. Schwartz from ONERA, Chatillon, France, and J.A. Anguita from University of Arizona, Tucson, AZ, for their fruitful discussions.

#### 6. REFERENCES

- [1] V. W. S. Chan, "Free-space optical communications," *J. Lightwave Technol.*, vol. 24, no. 12, pp. 4750–4762, Dec 2006.
- [2] L. C. Andrews and R. L. Phillips, *Laser Beam Propagation through Random Media*, SPIE Press, Bellingham, Washington, 2nd edition, 2005.
- [3] J.A. Anguitam, I.B. Djordjevic, M.A. Neifeld, and B.V. Vasic, "Shannon capacities and error-correction codes for optical atmospheric turbulent channels," *J. Opt. Network.*, vol. 4, no. 9, pp. 586–601, Sept. 2005.
- [4] L. C. Andrews M. A. Al-Habash and R. L. Philips, "Mathematical model for the irradiance probability density function of a laser beam propatating through turbulent media," *Opt. Eng.*, vol. 40, no. 8, pp. 1554–1562, Aug. 2001.
- [5] X.M. Zhu and J.M. Kahn, "Free-space optical communication through atomospheric turbulence channels," *IEEE Trans. Commun.*, vol. 50, no. 8, pp. 1293–1330, Aug. 2002.
- [6] C. Berrou and A. Glavieux, "Near optimum error correcting coding and decoding: turbo-codes," *IEEE Trans. Commun.*, vol. 44, no. 10, pp. 1261–1271, Oct 1996.
- [7] Digital Video Broadcasting (DVB); Framing Structure, Channel Coding and Modulation for Digital Terrestrial Television, ETSI EN 300 744 V1.5.1, European Standard (Telecommunications series), Nov. 2004.
- [8] M. Cole and K. Kiasaleh, "Signal intensity estimators for freespace optical communications through turbulent atmosphere," *IEEE Phot. Technol. Lett.*, vol. 16, no. 10, pp. 2395–2397, Oct. 2004.