Double-Laser Differential Signaling for Reducing the Effect of Background Radiation in Free-Space Optical Systems

Mohammad-Ali Khalighi, Fang Xu, Yacine Jaafar, and Salah Bourennane

Abstract—In order to reduce the impact of background radiation on the performance of terrestrial free-space optical systems, we propose to use two laser wavelengths and to perform the data detection at the receiver in a differential mode. We consider first the case of simple on–off keying modulation and show the performance improvement by using the proposed technique when the background noise dominates. We also extend our study to the case of pulse position modulation while proposing special signaling schemes that allow an increase in the data transmission rate at the same time as reducing the background noise effect.

Index Terms—Background noise; Background radiation; Differential signaling; Free-space optics; Optical communication.

I. INTRODUCTION

Free-space optical (FSO) systems are well-known for their potential of providing very high rate and secure data transmission along a line of sight. One of the impairments that degrade the performance of FSO links in practice is background radiation [1]. Background radiation can be reduced by means of narrow spectral bandpass and spatial filtering. However, under special conditions that occur frequently in practice, filtered radiation is still significant and can limit the performance of the system by causing an (variable) offset in the converted electrical signal. This, in turn, results in a reduced signal-to-noise ratio (SNR) [2] that can also be considered as a reduction in the effective receiver sensitivity [3]. The degradation is more significant for higher sensitivity receivers.

We focus in this work on the situations where the background noise is relatively significant and dominates the other noise sources. Such situations mostly occur when a background source (temporarily) enters the receiver’s field of view. For instance, it may consist of direct sunlight (typically during sunrise or dawn), reflected sunlight, or scattered sunlight from hydrometeors or other objects [4]. For the case of direct sunlight, the background radiation can even push the system out of operation by causing the saturation of the receiver.

Experimental measurements indicate that while the received optical signal power is typically about tens to hundreds of µW, the background radiation power $P_B$ is in the range of several µW for scattered sunlight by clouds or fog, about hundreds of µW for reflected sunlight, and up to about 10 mW for direct sun [4]. This latter (worst) case can statistically occur less than 1 hour per year, however.

In order to reduce the impact of the background noise, we propose here to use two laser wavelengths at the transmitter and to work in a differential mode at the receiver. We show that using two close wavelengths (with a difference of several tens of nanometers), the background noise component can be reduced significantly by differential detection, taking into account practical system parameters. We illustrate the system performance improvement for the case of the traditional on–off keying (OOK) modulation. To go farther, we consider the case of pulse position modulation (PPM) and propose some special signaling schemes that permit us to increase the transmission rate while at the same time reducing the background noise.

To illustrate the performance improvement by the proposed technique, we mostly consider the no-turbulence case. To show the interest in a practical system, we will also consider the more realistic weak-turbulence conditions, which are valid in short-range communications or when the receiver uses a relatively large lens for aperture averaging [5]. Moreover, to illustrate the need for our proposed method, we will show that channel coding alone is not efficient in reducing the destructive effect of (relatively significant) background radiation.

The paper is organized as follows. In Section II, we specify the receiver model that we consider and our general assumptions. The proposed double-wavelength transmission scheme and differential detection are described in Section III, whereas the proof of background noise rejection is presented in Appendix A. Differential signaling for the case of simple OOK modulation is considered in Section IV where some numerical results are presented to study the resulting system performance. The extension to the case of PPM is discussed in Section V. Next, in Sections VI and VII we apply the differential signaling scheme to quaternary PPM (4PPM) and binary PPM (BPPM), respectively. Therein, we consider signaling schemes that permit an increase in the data transmission rate in addition to reducing the background noise effect. Finally, conclusions and some discussions concerning system implementation are provided in Section VIII.
II. Receiver Model and General Assumptions

We assume perfect alignment between the transmitter and the receiver. Also, we assume perfect time synchronization of the system and the absence of intersymbol interference. Let \( r_e \) be the received signal after optical/electrical conversion:

\[
r_e = \eta I_r + n,
\]

where \( I_r \) is the received signal light intensity; \( \eta \) is the optical/electrical conversion efficiency; and \( n \) is the noise of the transimpedance circuitry and can be modeled as a Poisson random process. Here, we mostly consider situations where background radiation, on the other hand, originates from background radiation converted to photoelectrons and can be modeled as a Poisson random process. Here, we mostly consider situations where background radiation is relatively important. Under such conditions, the average number of received photons from background radiation is large enough to allow us to approximate the Poisson distribution by a Gaussian. Furthermore, the mean value of the background noise is rejected by the ac-coupled receiver circuitry. As a result, we can consider \( n \) as a sum of two zero-mean Gaussian random processes \( n_{th} \) and \( n_{bk} \), which represent thermal and background noise components with variances \( \sigma_{th}^2 \) and \( \sigma_{bk}^2 \), respectively: \( \sigma_{n}^2 = \sigma_{th}^2 + \sigma_{bk}^2 \). We further define for later use the parameter \( k = \sigma_{bk}^2/\sigma_{th}^2 \) that signifies the importance of background with respect to the thermal noise.

III. Double-Laser Transmission and Differential Detection

When background radiation is important, it can cause a considerable degradation in the receiver performance. We propose here to work with two laser wavelengths \( \lambda_1 \) and \( \lambda_2 \) and to perform signal detection at the receiver in a differential manner. We refer to this method as differential signaling. The block diagram of the receiver front end is shown in Fig. 1, which we describe in the following.

Let us denote by \( s_1 \) and \( s_2 \) the transmitted symbols on \( \lambda_1 \) and \( \lambda_2 \), respectively; \( s_1, s_2 \in \{0, 1\} \). At the receiver, we use equal-bandwidth narrow optical bandpass filters (OBPFs) on \( \lambda_1 \) and \( \lambda_2 \) to separate the signals of the two lasers. In practice, we can accomplish this filtering with more than 40 dB cross-talk attenuation, so, almost perfectly. Let us denote the corresponding received signals after optical/electrical conversion by \( r_1 \) and \( r_2 \). We have

\[
\begin{align*}
    r_1 &= hI_0s_1 + n_1 \rightarrow \lambda_1 \\
    r_2 &= hI_0s_2 + n_2 \rightarrow \lambda_2.
\end{align*}
\]

Note that we have assumed almost the same channel fading coefficient \( h \) for \( \lambda_1 \) and \( \lambda_2 \). This hypothesis is quite realistic as in practice we use wavelengths with a difference of about tens of nanometers only. (Note that in the weak turbulence regime, the atmospheric scintillation index is proportional to the \( 7/6 \) power of the wavelength; for wavelengths around 1550 nm, for instance, the difference in scintillation index for the two wavelengths is very small.)

We then perform the detection in a differential mode, that is, we use \( r_d = r_1 - r_2 \) for the detection of the transmitted bits. Let \( s_d = s_1 - s_2 \) and \( n_d = n_1 - n_2 \). We have

\[
r_d = hI_0s_d + n_d.
\]

The signal \( r_d \) is then sampled and digitized to be used by the signal detection part (see Fig. 1). The noise term \( n_d \) in Eq. (3) includes a thermal noise component \( n_{d,th} \) and a background noise component \( n_{d,bk} \). The variance \( \sigma_{d,th}^2 \) of \( n_{d,th} \) is given by

\[
\sigma_{d,th}^2 = 2\sigma_{th}^2.
\]

On the other hand, the background noise components in \( n_1 \) and \( n_2 \) that we denote by \( n_{1,bk} \) and \( n_{2,bk} \), respectively, are highly correlated and can be considered as almost identical (see Appendix A for the proof). In other words, the variance of \( n_{d,bk} \) is very small and we can practically neglect \( n_{d,bk} \) in \( r_d \), which gives

\[
r_d \approx hI_0s_d + n_{d,th}.
\]

IV. Differential OOK Signaling

Let us consider differential signaling for the classical case of OOK modulation. At the transmitter, corresponding to a bit “1” we send an impulse on \( \lambda_1 \) (i.e., \( s_1 = 1 \) and \( s_2 = 0 \)) and corresponding to a bit “0” we send an impulse on \( \lambda_2 \) (i.e., \( s_1 = 0 \) and \( s_2 = 1 \)). In other words, the laser with \( \lambda_1 \) uses OOK modulation and the other laser uses complemented OOK with
respect to that on \( \lambda_1 \). We will call this scheme D-OOK, standing for differential OOK.

For instance, if in Eq. (2) we consider \( s_1, s_2 \in (0, 1) \), from Eq. (4) \( s_d \) takes the values of \( \pm 1 \) depending on whether a bit “1” or “0” has been sent. As we will explain later, we set \( I_0 \) (transmit signal intensity corresponding to ON slots) depending on the average transmit optical power.

A. Signal Demodulation at the Receiver

For signal demodulation, we can perform hard detection simply by comparing \( r_d \) with a zero threshold. Making the assumptions of equiprobable binary bits at the transmitter and the absence of turbulence, i.e., \( h = 1 \), the probability of error for D-OOK is given by

\[
P_e = \frac{1}{2} \text{erfc} \left[ I_0/(\sigma_d \text{th} \sqrt{2}) \right] = \frac{1}{2} \text{erfc} \left[ I_0/(2\sigma_{\text{th}}) \right],
\]

where the second equality is due to the fact that \( \sigma_d^{2, \text{th}} = 2\sigma_{\text{th}}^2 \).

Let us also recall the expression of \( P_e \) for the case of OOK:

\[
P_e = \frac{1}{2} \text{erfc} \left[ I_0/(2\sigma_{\text{th}} \sqrt{2}) \right] = \frac{1}{2} \text{erfc} \left[ I_0/(2\sigma_{\text{th}} \sqrt{2(1+h)}) \right].
\]

B. Receiver Performance for D-OOK

To quantify the receiver performance, we consider the average bit error rate (BER) versus the electrical SNR. This latter is considered in the form of \( E_b/N_0 \), where \( E_b \) is the average received energy per information bit, and \( N_0 \) is the unilateral power spectral density (PSD) of the reference thermal noise, that is, the thermal noise in either channel, which is assumed to be white. We denote by \( \sigma_n^2 \) the variance of the noise intervening in signal detection. To do a fair comparison between OOK and D-OOK, we should fix for both modulation schemes the average transmit optical power, denoted by \( \bar{P}_t \). Note that the information transmission rate \( R_b \) is the same for both cases. We also set the reference receiver thermal noise variance \( \sigma_{\text{th}}^2 \) according to the low-pass filter (LPF) bandwidth (BW) that we consider as \( 1/2T_s \), where \( T_s = 1/R_b \) is the symbol duration. We have for both cases \( \sigma_{\text{th}}^2 = N_0/T_sR_b \).

- For OOK, \( \sigma_n^2 = \sigma_{\text{th}}^2(1+k) \). Then,

\[
I_0 = 2\bar{P}_t; \quad E_b = \frac{\bar{P}_t^2}{R_b}; \quad \sigma_n^2 = \frac{\bar{P}_t^2}{E_b/N_0}(1+k).
\]

- For D-OOK: \( \sigma_n^2 = 2\sigma_{\text{th}}^2 \). Then,

\[
I_0 = \bar{P}_t; \quad E_b = \frac{\bar{P}_t^2}{R_b}; \quad \sigma_n^2 = \frac{\bar{P}_t^2}{E_b/N_0}.
\]

(To avoid any confusion on parameters’ units, notice that, for instance, in the expression of \( E_b \), we have in fact \( \eta \bar{P}_t^2 \) that is simplified to \( \bar{P}_t^2 \) as we supposed \( \eta = 1 \).)

Taking fixed \( R_b \) and \( \bar{P}_t \) into account, we set the transmit beam intensity \( I_0 \) (in ON slots) for each modulation. We also set the receiver total noise variance \( \sigma_n^2 \) depending on \( E_b/N_0 \).

Note that for the performance study of different signaling schemes, we will provide BER curves as a function of \( E_b/N_0 \), as it is done classically. To compare OOK and D-OOK schemes, we first evaluate the loss in \( E_b/N_0 \) for D-OOK with respect to OOK for a desired BER that we denote by \( \Delta E_b/N_0 \). In other words, we need \( \Delta E_b/N_0 \) as more SNR for D-OOK to attain the same BER as for OOK:

\[
\Delta E_b/N_0 = (E_b/N_0)_{\text{D-OOK}} - (E_b/N_0)_{\text{OOK}} \quad \text{(in dB)}.
\]

However, we should convert \( \Delta E_b/N_0 \) to the loss in \( \bar{P}_t \) because this latter is our main design parameter. Using Eqs. (7) and (8), the effective loss in \( \bar{P}_t \) (that is, the additional average optical power we need for D-OOK to attain the same performance as OOK), which we denote by \( \Delta \bar{P}_t \), can be calculated as follows:

\[
\Delta \bar{P}_t = 1.5 + \frac{1}{2} \Delta E_b/N_0 \quad \text{(in dB)}.
\]

1) Case of No Turbulence: Consider first the case of a no-turbulence channel (\( h = 1 \)) where no channel coding is used. Figure 2 shows BER curves for different values of \( k = \sigma_{\text{th}}^2/\sigma_n^2 \).

We have used the expressions of the error probability (or the BER) from Eqs. (6) and (5) and the definitions of \( E_b/N_0 \) from Eqs. (7) and (8) for OOK and D-OOK, respectively.

For \( k = 0 \), we have the same \( E_b/N_0 \) for OOK and D-OOK, that is, \( \Delta E_b/N_0 = 0 \) dB. This results in a \( \Delta \bar{P}_t \) of 1.5 dB from Eq. (10). In other words, we have a penalty of 1.5 dB in the average transmit optical power for D-OOK to attain the same performance as OOK under the conditions of negligible background radiation. By comparing the expressions of error probabilities for D-OOK and OOK from Eqs. (5) and (6), it can be shown that the former outperforms the latter (in the sense of \( \bar{P}_t \)) for \( k > 1 \). This is confirmed in Fig. 2: for \( k = 1 \), we have a \( \Delta E_b/N_0 \) of –3 dB, which turns to \( \Delta \bar{P}_t = 0 \) dB from Eq. (10).
Fig. 2. Performance of D-OOK compared with the classical OOK for different ratios of background to thermal noise power $k$, no turbulence and no channel coding, and the BER curve for OOK with $k = 0$ the same as that of D-OOK.

Notice that the comparison we made in Fig. 2 cannot be extended to too large $k$ values. As a matter of fact, too significant background radiation can result in the saturation of the photodetection circuitry (for both OOK and D-OOK).

2) Weak Turbulence Regime: Let us now consider the more realistic conditions of weak turbulence in order to confirm the interest of the proposed differential signaling in practice. As mentioned previously, the conditions of weak turbulence hold in short-range communications or when the receiver uses a relatively large aperture [5]. We also consider the use of channel coding to see its efficiency against the background noise effect. In other words, we want to see whether or not coding can be an alternate solution to our proposed scheme in the case of relatively significant background radiation. For this, we use a simple convolutional code because it makes a good compromise between complexity and performance in optical communication systems [9]. We consider a simple rate-1/2 recursive systematic convolutional (RSC) code of constraint length $K = 3$, with the octal representation $(1, 5/7)$ [10].

Then, at the receiver, we perform soft signal demodulation based on maximum a posteriori (MAP) [9] followed by soft channel decoding based on the soft-output Viterbi algorithm (SOVA) [11]. Notice that the benefit of soft decoding is that it provides better performance, compared with hard decoding [10]. For soft signal demodulation, we calculate logarithmic likelihood ratios (LLRs) on transmitted bits, which are then fed to the soft decoder [9]. It can be shown that for the received signal $r_d$ from Eq. (4), the LLR is $L = \frac{M_f}{\sigma_{th}^2} r_d$.

Figure 3 shows BER curves for the two cases of no-turbulence and weak-turbulence channels. For the weak-turbulence regime, we have used the IT model by setting the Rytov variance to $\sigma_R^2 = 0.04$, assuming a point receiver and plane wave propagation [7]. These curves are obtained through Monte Carlo simulations: For a given $E_b/N_0$, we have generated as many symbol frames as necessary to obtain at least 1000 frame errors and 5000 bit errors, and the frame length is set to 2000. Thermal and background noise samples are generated randomly for each time slot. For the weak turbulence regime, we have generated randomly the channel fading coefficient $h$ for each frame of symbols according to the $\Gamma$ pdf. $h$ is kept constant for the duration of a frame according to the quasi-static channel model described in Section II.

We notice that the advantage of D-OOK over OOK is preserved when channel coding is used. In other words, coding alone is not efficient enough against background noise (otherwise, we would observe an improvement of the performance of OOK compared with D-OOK). Also, the performance improvement by differential signaling is still significant for the weak turbulence case. This confirms the efficacy of our proposed differential scheme in the presence of turbulence and when coding is employed.

V. APPLYING DIFFERENTIAL SIGNALING TO THE CASE OF PPM

Here we apply the idea of differential signaling to the case of PPM. The reason that we consider PPM here is that it is average-energy efficient [8,12]. As in the case of OOK, here, we can practically reject the background noise by differential signaling.

But before applying the differential signaling idea to the case of PPM, we should clarify an important point. Consider the case of BPPM. One may ask, why not use a single laser and work in differential mode at the receiver by subtracting the signals of the two slots, hence removing the contribution of the background noise? The answer is that such a cancellation can be done only after analog-to-digital (A/D) conversion. As a result, it cannot be done efficiently in practice: when background radiation is relatively significant, the A/D input would already be saturated, that is, the signal component will effectively be lost. Note that if we want to subtract the signals of two slots in analog, we have to use an analog delay
line, which will cause considerable practical implementation difficulties. By differential signaling as proposed in this paper, background noise cancellation is done on the analog (electrical) signal and before A/D conversion (see Fig. 1). Thus, in order to remove background radiation for the case of PPM, we again use two laser wavelengths as we described for the case of OOK.

To go farther, we can benefit from differential signaling to increase the data transmission rate at the same time as reducing the background noise. This was not possible for D-OOK where the differential signaling alphabet remains of size 2, as in classical OOK. Data rate increase becomes possible for the PPM, however.

We first consider the constraint that there is always one ON slot per transmitted symbol from each laser. Considering this constraint, data rate increase is not possible for differential BPPM because the signal alphabet remains of size 2: For BPPM, we have two time slots per symbol duration. By the differential scheme, for example, for a bit “0” we transmit a pulse in the first time slot on \( \lambda_1 \) and in the second time slot on \( \lambda_2 \). We do the inverse for a bit “1”. After subtracting the signals on \( \lambda_1 \) and \( \lambda_2 \) at the receiver, the “signal” part in the two time slots will be \((+1, -1)\) or \((-1, +1)\) for a transmitted “0” or “1” bit, respectively. The signal alphabet hence remains of size 2.

Rate increase becomes possible for higher-order PPM modulations, however. For the sake of simplicity, we focus on 4PPM. It could be considered as a good compromise between energy efficiency and implementation complexity (regarding time synchronization and the switching speed requirements of the electronic circuitry). We will study this case in the following section and show how we can increase the transmission rate from 2 to 3 bits per symbol. We will also specify the signaling scheme and signal detection and will study the system performance. Later, in a second step, we will relax the constraint on signal transmission and allow that one laser transmits an ON slot and the other transmits nothing. This way, we can obtain a larger signal alphabet size and hence increase the number of transmitted bits per symbol. We will apply this idea to the case of BPPM in Section VII and show how we can increase the transmission rate from 1 to 2 bits per symbol. We will specify the signaling scheme and study the system performance as well.

**VI. DIFFERENTIAL 4PPM**

In 4PPM we have four time slots per symbol duration. First, consider the classical 4PPM by which a pair of information bits \((b_2b_1)\) is mapped to a set of four possible signals. Let us denote each symbol by \((x_1,x_2,x_3,x_4)\), where \(x_i\) denotes the transmitted signal on the time slot \(i\) and takes the values \(0,1\).

Table I shows an example of bit-symbol mapping.

<table>
<thead>
<tr>
<th>Information Bits ((b_2b_1))</th>
<th>Signal ((x_1,x_2,x_3,x_4))</th>
</tr>
</thead>
<tbody>
<tr>
<td>00</td>
<td>((1,0,0,0))</td>
</tr>
<tr>
<td>01</td>
<td>((0,1,0,0))</td>
</tr>
<tr>
<td>10</td>
<td>((0,0,1,0))</td>
</tr>
<tr>
<td>11</td>
<td>((0,0,0,1))</td>
</tr>
</tbody>
</table>

Table I shows an example of bit-symbol mapping.

For double-wavelength signaling, as explained in the previous section, we consider the constraint that each laser sends one ON slot per symbol. In other words, we impose that the same energy is transmitted from the two lasers during a symbol duration. Note that we should not transmit an impulse on \( \lambda_1 \) and \( \lambda_2 \) at the same time slot; otherwise, after their subtraction at the receiver, the “signal” components will be eliminated. Thus, we send one pulse on \( \lambda_1 \) and \( \lambda_2 \) each, in two nonoverlapping slots. It can be easily shown that we have, in this way, a total of \( P^2 = 12 \) possibilities for the transmit signals. Using a set of 8 symbols among these 12, we can hence transmit 3 bits per symbol instead of 2 for the classical 4PPM.

### A. Bit-Symbol Mapping

Let us denote the subtraction of the transmitted signals from the two lasers by \( x_d = (x_{d1},x_{d2},x_{d3},x_{d4}) \), where \( x_{di} \) takes the values \( -1,0,1 \). Setting ON slot positions in the symbols of the two lasers for a set of three input bits that we denote by \((b_3b_2b_1)\) can be considered as mapping these bits to \( x_d \). We have shown in Table II an example of bit-symbol mapping that we will refer to as D-4PPM (for differential 4PPM). This mapping has the particularity that it maximizes the average Euclidean distance between symbols among other possible mappings and can be considered as an optimized mapping in this sense. Notice that maximizing this average distance (which equals 2.17 here) is equivalent to minimizing the symbol error probability. Furthermore, the mapping has been optimized to result in minimum bit error probability: the maximum Euclidean distance of \(\sqrt{8} \) between two symbols is taken when the number of corresponding different bits is 3.

<table>
<thead>
<tr>
<th>Information Bits ((b_3b_2b_1))</th>
<th>Signal (x_d) ((x_{d1},x_{d2},x_{d3},x_{d4}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>000</td>
<td>((1, -1, 0, 0))</td>
</tr>
<tr>
<td>001</td>
<td>((1, 0, -1, 0))</td>
</tr>
<tr>
<td>010</td>
<td>((0, 0, 1, -1))</td>
</tr>
<tr>
<td>011</td>
<td>((0, 1, 0, -1))</td>
</tr>
<tr>
<td>100</td>
<td>((0, -1, 0, 1))</td>
</tr>
<tr>
<td>101</td>
<td>((0, 0, -1, 1))</td>
</tr>
<tr>
<td>110</td>
<td>((-1, 0, 1, 0))</td>
</tr>
<tr>
<td>111</td>
<td>((-1, 1, 0, 0))</td>
</tr>
</tbody>
</table>

### B. Signal Detection at the Receiver

Let us denote the received signal corresponding to \( \lambda_i \) in the slot \( j \) by \( r_{ij} \), where \( r_{ij} = h_i x_{ij} + n_{ij} \), \( i = 1,2 \) and \( j = 1,2,3,4 \). Also, we denote by \( r_d \) the received signal corresponding to slot \( j \) after the subtraction of the signals on \( \lambda_1 \) and \( \lambda_2 \). We have \( r_{dj} = h_0 x_{dj} + n_{dj} \), where \( n_{dj} \) is the thermal noise component with the variance twice that of \( n_{ij} \). To find the transmitted bits \( b = (b_3b_2b_1) \) from \( r_d = (r_{d1},r_{d2},r_{d3},r_{d4}) \), we consider signal
We can set the information transmission rates accordingly and bits per symbol for 4PPM. Thus, to compare their performance, we can calculate LLRs on bits from Eq. (13) while using the approximation of \( \log(e^m + e^n) \approx m(n, n) \) in order to simplify the calculations. Let us denote the LLR on bit \( b_1 \) by \( L_1 \). After straightforward calculations we obtain

\[
L_1 = \frac{hI_0}{\sigma^2_{d,th}} \left[ \max(r_{d,2} - r_{d,1}, r_{d,4} - r_{d,3}, r_{d,1} - r_{d,3}, r_{d,2} - r_{d,4}) - \max(r_{d,1} - r_{d,2}, r_{d,3} - r_{d,4}, r_{d,1} - r_{d,4}) \right].
\]

Remember that LLRs are useful when soft signal detection and soft decoding are to be done at the receiver. If that is not the case, hard decisions on transmitted bits can simply be obtained by taking the sign of the LLRs.

C. Performance Comparison With 4PPM

We again consider the BER performance as a function of \( E_b/N_0 \). Note that we have 3 bits per symbol for 4-PPM and 2 bits per symbol for 4PPM. Thus, to compare their performance, we can set the information transmission rates accordingly and see the degradation in \( E_b/N_0 \) by using 4-PPM that would be the price paid for the increase in the transmission rate. Alternatively, we can consider the same transmission rate \( R_b \) for both schemes and see the improvement in \( E_b/N_0 \) for D-4PPM. We have decided to consider the second approach that would be more interesting in practice. Notice that the occupied bandwidth and the required speed for the electronic circuitry are the same for these two modulation schemes.

Thus, we again fix the transmission rate \( R_b \) and the average transmit optical power \( P_t \), and set the transmitted beam intensity \( I_0 \) in ON slots for each modulation. Here, \( \sigma^2_{th} \) is calculated based on the LPF BW, 1/2T, where \( T \) is the slot duration: \( \sigma^2_{th} = N_0/2T \). For calculating \( E_b \), notice that each symbol corresponds to 2 bits for 4PPM and to 3 bits for D-4PPM. For the sake of completeness, we also consider the classical 8PPM, which has the same number of bits per symbol as D-4PPM. We specify below the calculation of \( I_0 \) and the total receiver noise variance \( \sigma^2_n \) for a given \( E_b/N_0 \):

- For 4PPM, \( T = T_s/4, R_b = 2/T_s = 1/2T \), and hence, \( \sigma^2_n = \sigma^2_{th}(1 + k) = N_0 R_b(1 + k) \). Then,
  \[
  I_0 = 4P_t; \quad E_b = \frac{4\bar{P}_t^2}{R_b}; \quad \sigma^2_n = \frac{4\bar{P}_t^2}{E_b/N_0}(1 + k).
  \]

- For 8PPM, we have \( T = T_s/8, R_b = 3/T_s = 3/8T \), and hence, \( \sigma^2_n = \sigma^2_{th}(1 + k) = \frac{9}{2} N_0 R_b(1 + k) \). Then,
  \[
  I_0 = 8P_t; \quad E_b = \frac{8\bar{P}_t^2}{R_b}; \quad \sigma^2_n = \frac{32\bar{P}_t^2}{3E_b/N_0}(1 + k).
  \]

- For D-4PPM, we have \( T = T_s/4, R_b = 3/T_s = 3/4T \), and hence, \( \sigma^2_n = \frac{2}{3} N_0 R_b \). Then,
  \[
  I_0 = 2P_t; \quad E_b = \frac{2\bar{P}_t^2}{R_b}; \quad \sigma^2_n = \frac{2\bar{P}_t^2}{3E_b/N_0}.
  \]

Figure 4 contrasts the BER curves for the three modulation schemes for the case of no turbulence where no channel coding is considered. Monte Carlo simulations are performed to obtain these results, as we did for the results in Fig. 3. For 4PPM and 8PPM, we have only shown the results for \( k = 0 \), that is, for negligible background radiation. Obviously, for these two modulation schemes, similar to the case of OOK, the receiver performance degrades in the presence of background noise.

Remember that we should compare the modulations in terms of \( P_t \). For instance, to compare D-4PPM and 4PPM, from Eqs. (19) and (17), it can be shown that the relationship between \( \Delta E_b/N_0 \) and \( \Delta P_t \) is again given by Eq. (10). From Fig. 4 \( (k = 0) \), we notice a difference of about 3.8 dB in \( E_b/N_0 \), and hence from Eq. (10), a penalty of 3.4 dB in \( P_t \) for D-4PPM compared with 4PPM at BER \( \approx 10^{-5} \). This \( P_t \) penalty is larger than that for the case of D-OOK. The reason is the increase in data rate and the more complex data detection for the case of D-4PPM. Note that the \( P_t \) penalty with respect to 8PPM is about 5.6 dB. Similar conclusions are derived for the case of the weak-turbulence channel (results are not shown).

Notice that, despite the \( P_t \) penalty for the case of \( k = 0 \), D-4PPM conserves its advantage over the classical modulations of 4PPM and 8PPM under the conditions of relatively significant background noise thanks to differential signaling (results are not shown for the sake of brevity).
respectively, are given by

\[
L_1 = \frac{hI_0}{\sigma_{th}^2} \max(x_{d1}, x_{d2}) - \max(-x_{d1}, -x_{d2}),
\]

\[
L_2 = \frac{hI_0}{\sigma_{th}^2} \max(x_{d1}, -x_{d2}) - \max(-x_{d1}, x_{d2}).
\]

B. Performance Comparison With BPPM

Here we compare the performance of D-BPPM with the classical BPPM as well as 4PPM that provides the same data rate as D-BPPM. Fixing the transmission rate \(R_b\), and the average transmit optical power \(\bar{P}_t\), we set the transmitted beam intensity \(I_0\) in \(ON\) slots for each modulation. We specify below the calculation of \(I_0\) and the total receiver noise variance \(\sigma_n^2\) for a given \(E_b/N_0\):

- For BPPM, \(T = T_s/2, R_b = 1/T_s = 1/2T\), and hence, \(\sigma_n^2 = \sigma_{th}^2(1 + k) = N_0R_b(1 + k)\). Then,

\[
I_0 = 2\bar{P}_t; \quad E_b = \frac{2\bar{P}_t^2}{R_b}; \quad \sigma_n^2 = \frac{2\bar{P}_t^2}{E_b/N_0}(1 + k).
\]

- For D-BPPM, we have \(T = T_s/2, R_b = 2/T_s = 1/T\), and hence, \(\sigma_{th}^2 = N_0R_b/2\). Then,

\[
I_0 = 2\bar{P}_t; \quad E_b = \frac{2\bar{P}_t^2}{R_b}; \quad \sigma_n^2 = \frac{2\bar{P}_t^2}{E_b/N_0}. \quad \text{(22)}
\]

The corresponding parameters for 4PPM have already been provided in Subsection VI.C. For later comparison of D-BPPM and BPPM, we notice that from Eqs. (21) and (22) we have

\[
\Delta\bar{P}_t = \frac{1}{2} \Delta E_b/N_0 \quad \text{(in dB)}. \quad \text{(23)}
\]

We have shown in Fig. 5 curves of BER versus \(E_b/N_0\) for the three modulation schemes assuming a negligible background noise level (i.e., \(k = 0\)), the absence of turbulence, and no channel coding. In the case of relatively significant background radiation, the benefit of differential signaling is obvious, as seen previously.

From Fig. 5 and according to Eq. (23), we notice, interestingly, D-BPPM performs almost as well as the classical BPPM. The case is analogous with the coherent binary phase-shift keying (BPSK) and quadrature phase-shift keying (QPSK) modulation schemes. Sending bits on \(I\) (in-phase) and \(Q\) (quadrature) channels in QPSK permits a rate increase of a factor 2 compared with BPSK where we send 1 bit per symbol and only the \(I\) channel comes into play. It is well known that the performance of QPSK is almost the same as BPSK [10]. Here, by the proposed differential signaling, we, in fact, send one bit on the \(I\) channel and another bit on the \(Q\) channel. Similar conclusions are derived for the case of the weak-turbulence channel (results are not shown).
was shown to be particularly interesting considering power D-BPPM scheme with 2 bits per symbol. This special example $\lambda$ conditions on signaling from the case of D-4PPM with 3 bits per symbol. By relaxing the rejecting the background noise. We studied as an example increase in data rate can be obtained at the same time as extended the idea to the case of PPM and showed that an situations where the background noise level is significant. We consider here. In particular, we should take into account the signal-dependent shot noise. When using an APD, in contrast to the case of the PIN diode, the variances of shot noises arising from signal and background radiation are multiplied by $G_F$, where $G$ is the APD gain and $F$ is its excess noise factor [6]. The shot noise can still be modeled as Gaussian distributed [13]. Thus, the same analysis applies to this case regarding background noise rejection by differential signaling. Similarly, the differential signaling scheme proposed here can be used in photon-counting receivers.

Notice that our hypothesis of perfect background noise rejection by differential signaling is obviously not satisfied in practice. This is due to the simplifying assumptions we made in this paper such as identical BPFs, perfect separation of the signals of two lasers at the receiver, and the other assumptions we made in Appendix A. Thus, the results presented in this paper (Fig. 2 in particular) may be considered as optimistic. However, in practice, we can effectively reject most of the background noise and the effect of the residual background noise can be reduced by employing error correcting codes.

Regarding system implementation, we have an increased transceiver cost because we should use two laser diodes at the transmitter and duplicate hardware for some parts of the receiver such as the optical filter, photodiode, and transimpedance circuitry (see Fig. 1).

Finally, notice that such differential signaling could also be done by using two different polarizations, like the scheme proposed in [14] in the context of time diversity: This way, we can transmit on two left/right circular bipolarizations, sending an ON slot on one polarization for a bit ”0” and an ON slot on the other one for a bit “1”. Compared with such a system, our proposed double-laser system has the advantage of reduced cost and simpler system implementation. In particular, by our proposed system, we can use the same optical amplifier at the transmitter (an erbium-doped fiber amplifier, for example), which is the expensive component. Also, the modulation laser is of relatively lower power and less costly: For the case of using two polarizations, the laser power is divided by 2 and we would need a higher-power laser. On the other hand, the corresponding devices for polarization generation at Gb/s rates are expensive and the receiver is also more complex.

### C. Comparison of Different Differential Schemes

For the sake of completeness, we compare the performances of the three differential schemes that we studied in this work, that is, D-OOK, D-4PPM, and D-BPPM. This can be done using the already-presented results in Figs. 2, 4, and 5 (for the case of a no-turbulence channel with no coding). Notice that the same information rate $R_b$ is considered for all schemes. Consider a desired BER of $10^{-5}$. We notice that there is a very small difference between the $E_b/N_0$ of D-OOK and D-4PPM. This turns to an improvement of $\Delta P_t \approx 1.5$ dB of D-4PPM, compared with D-OOK. On the other hand, we notice a difference of about 3 dB between the $E_b/N_0$ of D-OOK and D-BPPM. This again turns to an improvement of $\Delta P_t \approx 1.5$ dB of D-BPPM, compared with D-OOK (notice that the conversion equation of $\Delta E_b/N_0$ to $\Delta P_t$ is different from the previous case). As a result, D-BPPM has practically the same advantage as D-4PPM, compared with D-OOK, considering the criterion of average transmit power $P_t$. However, given the simpler mapping and demapping functions for D-BPPM, it should be preferred to D-4PPM.

Similar conclusions are derived for the case of the weak-turbulence regime with or without channel coding (results are not presented for the sake of brevity).

### VIII. Conclusions and Discussions

We have considered in this paper the use of double-wavelength transmission and differential mode data detection for FSO systems for the purpose of reducing the background noise effect. Such a scheme is especially interesting in the situations where the background noise level is significant. We extended the idea to the case of PPM and showed that an increase in data rate can be obtained at the same time as rejecting the background noise. We studied as an example the case of D-4PPM with 3 bits per symbol. By relaxing the conditions on signaling from $\lambda_1$ and $\lambda_2$, we proposed the D-BPPM scheme with 2 bits per symbol. This special example was shown to be particularly interesting considering power consumption at the transmitter as well as the mapping and demapping functions.

In our receiver model in Section II, we assumed that the noise component is independent of signal, which is valid when we use a PIN diode for photodetection. For large communication distances, however, an avalanche photodiode (APD) is usually employed at the receiver, which provides a current gain thanks to the process of impact ionization. In such a case, the receiver model will be different from what we considered here. In particular, we should take into account the signal-dependent shot noise. When using an APD, in contrast to the case of the PIN diode, the variances of shot noises arising from signal and background radiation are multiplied by $G_F$, where $G$ is the APD gain and $F$ is its excess noise factor [6]. The shot noise can still be modeled as Gaussian distributed [13]. Thus, the same analysis applies to this case regarding background noise rejection by differential signaling. Similarly, the differential signaling scheme proposed here can be used in photon-counting receivers.

### APPENDIX A: Proof of Negligible Background Noise Contribution After Differential Detection

In Section III we made a very important assumption on which the main part of this work is based, that is, we assumed that the background noise component after...
differential detection, $n_{d, bk}$, is very negligible. We explain here the rationality of this assumption.

As a matter of fact, the original background noise field, whose envelope we denote by $\theta$, is spread over a very large spectral band $B_{bk}$. Let us denote the impulse responses (IRs) of these filters by $h_1(t)$ and $h_2(t)$ and the corresponding transfer functions by $H_1(f)$ and $H_2(f)$, respectively. Also, denote the central frequencies of the two bandpass optical filters by $f_1$ and $f_2$ and their bandwidth by $B_0$, as shown in Fig. 6. The envelopes of the filtered noises’ fields after $H_1$ and $H_2$ are denoted respectively by $\theta_1$ and $\theta_2$. Note that $\theta, \theta_1$, and $\theta_2$ are optical field envelopes, but we will simply call them “noise” in the sequel. Frequencies $f_1$ and $f_2$ correspond to the laser wavelengths $\lambda_1$ and $\lambda_2$, respectively (see Section III). Let us also define $\Delta f = f_2 - f_1$. The BW $B_0$ and $\Delta f$ are very small with respect to $B_{bk}$. Thus, we can consider almost flat and identical PSDs for the corresponding bandpass noises $\theta_1$ and $\theta_2$. Each one of $\theta_1$ and $\theta_2$ passes then through a photodetector (PD) and a LPF. For a typical data rate of 1 Gbps, the BWs of the LPFs are on the order of $10^9$ Hz. The BW of the optical BPFs corresponding to $\theta_1$ and $\theta_2$ are on the order of $10^{16}$ Hz (about 20 to 30 nm), on the other hand.

To take into account the function of the PD, we consider $\theta_1$ as the convolution of the input (original) noise $\theta$ and the filter IR, $h_1(t)$:

$$\theta_1(t) = \int_{-\infty}^{+\infty} h_1(\tau) \theta(t - \tau) \, d\tau. \quad (A.2)$$

where the integral is calculated over the symbol period $T_s$; we have used the notation $\int_{T_s}$ to indicate this. After subtracting the outputs of the two PDs, we obtain the noise component $n_{d,bk}$. We show here that $n_{d,bk}$ is practically very negligible.

Let us focus on $I_{\theta_1}$. We consider $\theta_1$ as the convolution of the input (original) noise $\theta$ and the filter IR, $h_1(t)$:

$$I_{\theta_1} = \int_{T_s} |\theta_1(t)|^2 \, dt, \quad (A.1)$$

where the integral is calculated over the symbol period $T_s$; we have used the notation $\int_{T_s}$ to indicate this. After subtracting the outputs of the two PDs, we obtain the noise component $n_{d,bk}$. We show here that $n_{d,bk}$ is practically very negligible.

As a result, from Eq. (A.1),

$$I_{\theta_1} = \int_{-\infty}^{+\infty} \left[ \int_{-\infty}^{+\infty} h_1(\tau_1) \theta(\tau_1) \, d\tau_1 \right] h_1(\tau_2) \theta(\tau_2) \, d\tau_2 \, d\tau_1 \, d\tau_1. \quad (A.4)$$

Let us focus first on the term within the parentheses in Eq. (A.4). As $\theta$ is spread on a very wide band $B_{bk}$, we have practically $T_s \gg 1/B_{bk}$. Assuming $\theta$ a stationary and ergodic random process, we can consider this integral in Eq. (A.4) as the statistical autocorrelation of $\theta$:

$$\int_{T_s} \theta(\tau_1) \theta(\tau_2) \, dt = \mathbb{E}[\theta(\tau_1) \theta(\tau_2)], \quad (A.5)$$

where $\mathbb{E}[\cdot]$ denotes the expected value. This expectation can be approximated as follows:

$$\mathbb{E}[\theta(\tau_1) \theta(\tau_2)] = \sigma_\theta^2 \delta(\tau_1 - \tau_2), \quad (A.6)$$

where $\delta(\cdot)$ is the Dirac delta function and $\sigma_\theta^2$ denotes the variance of the original background noise. (The variance $\sigma_\theta^2$ should not be confused with $\sigma_{bk}^2$, which we defined in Section II; the latter denotes the variance of the background noise after photodetection.) This is a very reasonable approximation for the calculation of $I_{\theta_1}$ in Eq. (A.4). Assuming that $H_1$ is an ideal BPF, its IR is $h_1(t) = \text{sinc}(2\pi B_{bk} t) \text{cos}(2\pi f_1 t)$ with $\text{sinc}(x) = \sin(x)/x$. The IR $h_1(t)$ is a sinc function with its first zero at $(1/\Delta f)$, modulated by $\cos(2\pi f_1 t)$. On the other hand, assuming a rectangular-shaped PSD for $\theta$ (just for the sake of demonstration simplicity), its autocorrelation function will be a sinc function with its first zero at $1/B_{bk}$. Given that $B_{bk} \gg \Delta f$ in practice, this autocorrelation function can relatively be considered as a Dirac delta function. In other words, $\theta$ can be considered as a white noise in the calculation of $I_{\theta_1}$. This justifies the approximation we made in Eq. (A.6). Note that all the hypotheses we made are quite logical and reasonable in practice.

Now, turning back to Eq. (A.4), we have

$$I_{\theta_1} = \sigma_\theta^2 \int_{-\infty}^{+\infty} h_1(\tau)^2 \, d\tau. \quad (A.7)$$

As a result, after the subtraction of the noises at the outputs of the PDs we have

$$I_{\theta_1} - I_{\theta_2} = \sigma_\theta^2 \int_{-\infty}^{+\infty} [h_1(\tau)^2 - h_2(\tau)^2] \, d\tau. \quad (A.8)$$

This is what we had denoted by $n_{d,bk}$ in Section III. Using the Parseval equality, we obtain

$$n_{d,bk} = \sigma_\theta^2 \int_{-\infty}^{+\infty} [H_1(f)^2 - H_2(f)^2] \, df = 0. \quad (A.9)$$
The equality to zero is due to the fact that the transfer functions of the two filters are assumed to be identical except the frequency shift of $\Delta f$. Thus, the background noise component is practically suppressed by differential detection.

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**REFERENCES**


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