Smooth contour coding with Minimal Description Length active grid segmentation

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Smooth Contour Coding with Minimal Description Length Active Grid Segmentation Techniques

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Abstract

We analyze the influence of the contour coding term in segmentation techniques based on active grids and on the Minimum Description Length (MDL) principle. These segmentation techniques have been developed up to now with a contour coding term adapted to polygonal objects. However, this approach can lead to degraded segmentation results for smooth contours of objects which can be observed for example in geoscience, medicine or microscopy. We demonstrate that an appropriate choice of the contour coding term can improve segmentation results with MDL active grid approaches in the presence of regions with smooth boundaries. This improvement opens a large class of application domains and still allows one to obtain low computational time.

Key words: Image segmentation, Statistical image processing, Minimum
1. Introduction

Image segmentation is a critical step in many image processing applications, particularly in the case of noisy images (Chan and Vese, 2001; Zhang et al., 2001; Hermes and Buhmann, 2004; Tison et al., 2004; Ben Ayed et al., 2005; Collins and Kopp, 2008). In general, segmentation algorithms require the determination by the user of different parameter values and may also lead to large computational times. Efficient segmentation techniques based on Minimum Description Length (MDL) principle (Rissanen, 1978, 1989) have been proposed (Leclerc, 1989; Kanungo et al., 1994; Pan, 1994; Nohre, 1995; Zhu and Yuille, 1996; Lee, 2000; Figueiredo et al., 2000; Ruch and Réfrégier, 2001; Galland et al., 2003; Martin et al., 2004) so that the optimal partition is the minimizer of a criterion with a reduced number of undetermined parameters. Furthermore, it has been shown in Galland et al. (2003) that a MDL segmentation technique based on a polygonal active grid (i.e. a set of nodes connected with segments in order to define the different regions of the image) leads to fast algorithms. Indeed, with this polygonal MDL based Active Grid (MDL-AG), initially devoted to Synthetic Aperture Radar (SAR) images in which the speckle noise is represented with gamma probability density function (PDF) (Galland et al., 2003), the number of homogeneous regions, their shapes and the regularity of their contours are estimated in less than 2 seconds on a standard personal computer (PC) for a 400 × 400 pixel image.

The polygonal MDL-AG has been generalized to different kinds of noise in
order to broaden the application domain of the method. For instance, in Galland et al. (2005), the polygonal MDL-AG has been adapted to various PDF that belong to the exponential family (Gaussian, Poisson or Bernoulli), and more recently, to Fisher PDF (Galland et al., 2009) to provide efficient segmentations of high resolution SAR images with textures and strong reflectors. A generalization to non-parametric PDF that are based on step functions has also been proposed in Delyon et al. (2006). In Morio et al. (2007), the polygonal MDL-AG has been adapted to polarimetric and interferometric SAR images (PolInSAR) for which each pixel value is a 6 component complex valued Gaussian vector with correlated components.

Although the polygonal description of the contour adopted in Galland et al. (2003) can be useful for a number of applications, poor segmentation results may be obtained with regions with smooth boundaries.

Schematically, two main MDL segmentation based approaches adapted to smooth contours have been developed. The first ones are based on contour description with parametric techniques such as spline descriptors (Figueiredo et al., 2000). The second ones are based on a non-parametric or chain code description of the contours (Leclerc, 1989; Kanungo et al., 1994; Lee, 2000), such as in the region competition approach (Zhu and Yuille, 1996), in level set based segmentation technique (Martin et al., 2004), or in other domains such as Markov Random Fields (Figueiredo and Leitao, 1997). However, up to now, no general solution has been implemented for segmentation techniques based on the MDL-AG developed in Galland et al. (2003) with a contour description adapted to regions with smooth boundaries, although such a possibility may be applied to a large class of application domains. It is
Thus the purpose of this paper to describe such an adaptation of the MDL-AG which relies on a simple theoretical approach that keeps the advantages of the method proposed in Galland et al. (2003), namely: a) the ability to be adapted to different noise models (Galland et al., 2005, 2009; Delyon et al., 2006; Morio et al., 2007); b) to define algorithms without undetermined parameters in the optimized criterion; c) to lead to low computational times.

In the next section the basic principle of the MDL active grid algorithm is summarized. Contour coding terms adapted to isotropic and to smooth region boundaries are presented in section 3. In section 4, the behavior of these contour coding terms are analyzed with synthetic images and illustrated with different noisy images. Section 5 is devoted to a conclusion and a discussion on the different possible perspectives.

2. Background on polygonal MDL active grid

Let \( s = \{s(x, y), (x, y) \in [1, N_x] \times [1, N_y]\} \) be the image grey levels and \( \Gamma \) be the contours of the partition of the image into different regions. The MDL segmentation technique developed in Galland et al. (2003) consists of minimizing the description length (i.e. code length) \( \Delta(s, \Gamma) \) of the image when \( \Gamma \) is defined with a polygonal grid. This description length is equal to the information quantity (measured in nats if natural logarithms are used) needed to describe the image. It can be written as the sum of three terms:

\[
\Delta(s, \Gamma) = \Delta_L(s|\theta, \Gamma) + \Delta_P(\theta|\Gamma) + \Delta_G(\Gamma) \tag{1}
\]

where \( \Delta_G(\Gamma) \) represents the code length required to encode the contour \( \Gamma \) of the partition, \( \Delta_P(\theta|\Gamma) \) is the description length needed to describe the
PDF of the grey levels in each region knowing $\Gamma$, and where $\Delta_L(s|\theta, \Gamma)$ is the description length of the image grey levels knowing $\Gamma$ and the PDF in each region.

Assuming spatial statistical independence of the fluctuations, $\Delta_L(s|\theta, \Gamma)$ is approximated by the sum of the negative of the generalized log-likelihood (Galland et al., 2003) or by the product of an approximation of the entropy by the number of pixels $N_r$ in the considered region (Galland et al., 2009).

The second term $\Delta_P(\theta|\Gamma)$ can be approximated with $\sum_{r=1}^{R} \alpha \log \sqrt{N_r}$, where $\alpha$ is the number of parameters needed to describe the PDF of the grey levels in each region, $R$ is the number of regions defined by $\Gamma$ and where $N_r$ is the number of pixels in the region number $r$ of the partition (Rissanen, 1989; Galland et al., 2003).

The polygonal description of $\Gamma$ implemented in Galland et al. (2003) is a graph on a discrete grid and is defined as nodes connected by segments. An approximation of the code length $\Delta_G(\Gamma)$ required to encode $\Gamma$ can be obtained with the decomposition of $\Gamma$ into Eulerian graphs, which correspond to graphs that can be drawn without taking the pen off the paper. The minimal number of Eulerian graphs $n$ is equal to $n_{ON}/2 + n_{SC}$ where $n_{ON}$ is the number of odd nodes, i.e. that are connected to an odd number of segments, and where $n_{SC}$ is the number of simply connected graphs without odd nodes (Bondy and Murty, 1976). Let $p$ be the number of segments of the polygonal grid $\Gamma$, then the code length approximation considered in Galland et al. (2003) to

\[^{1}\text{This discrete grid is the underlying pixel grid translated by 1/4 pixel along the horizontal axes and 1/2 along the vertical axes in order to avoid ambiguity on the region to which a pixel of the image belongs (Germain and Réfrégier, 2001).}\]
encode $\Gamma$ is:

$$\Delta_G(\Gamma) = n(\log N + \log p) + \Omega_G(\Gamma) \quad (2)$$

where $N = N_x \times N_y$ and where $\Omega_G(\Gamma)$ is the number of nats needed to encode the $p$ segments of $\Gamma$ assuming that the number of segments and the coordinates of the first node of each Eulerian contour have been coded. In Galland et al. (2003) a simple maximum entropy approach has lead to the following contour coding term

$$\Omega_{Gpoly}(\Gamma) = 2p + p \log(4 \hat{m}_x \hat{m}_y) + \log(p) \quad (3)$$

where $\hat{m}_x = \frac{1}{p} \sum_{i=1}^{p} |u_i|$ and $\hat{m}_y = \frac{1}{p} \sum_{i=1}^{p} |v_i|$ and where $u_i$ (respectively $v_i$) is the horizontal (resp. vertical) length of the segment number $i$ of the grid.

The approach developed in Galland et al. (2003) thus consists in minimizing Eq. 1 with the polygonal coding term provided by Eqs. 2 and 3. This optimization is initialized with a regular wall-like polygonal grid delimiting $8 \times 8$ pixel regions (see Fig. 1-b). The segmentation is then performed by alternatively merging regions, moving nodes, and removing nodes as detailed in Galland et al. (2003), in order to minimize $\Delta(s, \Gamma)$ of Eq. 1 until no further tested modification allows one to decrease $\Delta(s, \Gamma)$. For the sake of clarity, this technique will be denominated in the following the polygonal MDL-AG.

The segmentation result obtained on a synthetic image corrupted with Gamma noise is shown in figure 1. The image contains letters of different sizes and of grey level mean value equal to 1 while the background mean value is equal to 4. Clearly, the polygonal contour description leads to degraded boundary estimations of smooth objects. These poor results with smooth contours are due to $\Delta_G(\Gamma)$ which can be interpreted as a penalty term which
allows one to regularize $\Gamma$. In the next sections, contour coding terms $\Delta G(\Gamma)$ that improve the MDL-AG technique for smooth contours are presented and analyzed.

3. Global contour coding

Let us still define contours of the grid as a series of nodes connected with segments. However, different contour coding terms will now be determined with a class of a priori probability laws adapted to the description of regions with smooth boundaries.

Let us consider two neighbor nodes of $\Gamma$ with coordinates $(x_n, y_n)$ and $(x_m, y_m)$, respectively. The segment vector number $i$ that connects these nodes is $\mathbf{r}_i = (x_m - x_n, y_m - y_n)$ and the coding of the contour $\Gamma$ (assuming that the number of segments and the coordinates of the first node of each Eulerian contour have been coded) is equivalent to the coding of its $p$ vectors $\mathbf{r}_i$. The contour coding term $\Omega_G(\Gamma)$ can thus be approximated by $\Omega_G(\Gamma) \approx \sum_{i=1}^p Q_i$ where $Q_i$ is the Shannon information quantity needed to encode $\mathbf{r}_i$. Furthermore, if $P_{\mathbf{r}}(\mathbf{r})$ is the probability law of $\mathbf{r}$, i.e. of observing a segment vector $\mathbf{r}$, then $Q_i = -\log [P_{\mathbf{r}}(\mathbf{r}_i)]$.

Although a simple maximum entropy argument is developed in Galland et al. (2003) in order to determine $P_{\mathbf{r}}(\mathbf{r})$, this choice is not unique. In particular, maximizing the Shannon entropy under some different constraints can lead to a priori probability law $P_{\mathbf{r}}(\mathbf{r})$ that are adapted to describe smooth objects as shown in the following.
Figure 1: Segmentation of a $400 \times 400$ pixel synthetic image corrupted with gamma noise and which contains 13 regions. The shape parameter of the noise is $L = 1$, the mean grey level value inside the objects is equal to 1 while it is equal to 4 for the background. (a) Non noisy image; (b) Initialization of the polygonal active grid with a wall-like grid with $8 \times 8$ pixel regions; (c) Noisy image; (d) Segmentation result obtained with the polygonal approach proposed in Galland et al. (2003); Lower images (c,d) Zooms inside the three white squares shown in Fig. 1-a.
3.1. Local isotropic contour coding

Let us first consider the standard nearest neighbor chain-code representation (Jain, 1989, Sec. 9.6) classically used in MDL based segmentation techniques (Leclerc, 1989; Kanungo et al., 1994; Lee, 2000; Martin et al., 2004). In that case, the contour is a polygon with nodes that are neighbors to each other with neighborhood regions defined with the 8-connectivity on the square node grid. A segment vector can thus only belong to a set of 8 vectors as illustrated in figure 2. The a priori probability law of $r$ can then be written $P_r(r) = \alpha_U$ for a vector $r$ that corresponds to a nearest neighbor and where $U$ is one of the eight directions ($N, NE, E, SE, S, SW, W, NW$). In particular $P_r(r) = 0$ if the Euclidian length $|r|_{euc}$ of $r$ is not equal to 1 or $\sqrt{2}$. A simple maximum entropy argument obviously leads to $\alpha_U = 1/8$. The contour coding cost is thus in that case $\Omega_{G}^{pix}(\Gamma) = \log(8) \cdot |\Gamma|_{pix}$ where $|\Gamma|_{pix}$ is the length of the contour $\Gamma$ measured in pixel numbers. However, such a coding approach does not correspond to an isotropic cost as illustrated in figure 2.

Isotropic coding not only imposes $\alpha_N = \alpha_E = \alpha_S = \alpha_W = \alpha_{NE} = \alpha_{SW} = \alpha_{SE}$ but also that $\alpha_E$ and $\alpha_{SE}$ are determined so that the coding costs of an horizontal or a diagonal contour of the same Euclidian length $\ell$ are equal. In the first case the number of contour pixels is $n_\ell = \ell$ while in the second case it is $n'_\ell = \ell/\sqrt{2}$. The coding cost is thus respectively $-\ell \log(\alpha_E)$ and $-\ell \log(\alpha_{SE})/\sqrt{2}$ and the isotropy requirement leads to $\alpha_E = \alpha_{SE}^{1/\sqrt{2}}$. The constraint that $\sum_U \alpha_U = 1$ corresponds to $\alpha_E + \alpha_{SE}^{1/\sqrt{2}} = 1/4$ and a numerical resolution leads to $\alpha_E \simeq 0.169046$ and $\alpha_{SE} \simeq 0.0809535$. The contour coding term of the $p$ segments of $\Gamma$ with this local isotropic coding
is

$$\Omega_G^{\text{pix}}(\Gamma) = -\log(\alpha_E) \ |\Gamma|_{\text{euc}} \simeq 1.77 \ |\Gamma|_{\text{euc}}$$  \hspace{1cm} (4)

where $|\Gamma|_{\text{euc}}$ is the Euclidian length of $\Gamma$.

3.2. Non local contour coding

The previous discussed coding term (Eq. 4) considers that two nodes are connected with a segment of length 1 or $\sqrt{2}$. Imposing such a locality of
the contour coding can lead to an over estimation of the needed information
to coding a straight line and then to fluctuations in the estimated
contour segment. A possibility to relax the constraint of locality of the
contour coding can be obtained when the average value of the distance $|r|_{euc}$
between successive nodes is fixed. The isotropic a priori probability law $P_r(r)$
which maximizes the Shannon entropy with this constraint leads to

$$P_r(r) = A(m) \exp \left(-2 \frac{|r|_{euc}}{m} \right)$$

where $A(m)$ is a normalization constant. Since the pixel surface is set equal
to one (i.e. $dx dy = 1$), an approximation of $A(m)$ can be obtained with its
continuous PDF. The following approximation of the probability law

$$P_r(r) = \frac{2}{\pi m^2} \exp \left(-2 \frac{|r|_{euc}}{m} \right)$$

will thus be considered in the following. In that case, the a priori mean
value of the Euclidian distance between nodes is $m \simeq \sum_r |r|_{euc} P_r(r)$. The
information quantity of the contour $\Gamma$ with $p$ segments, assuming the number
of segments and the coordinates of the first node of each Eulerian graph of
$\Gamma$ has been coded, is thus

$$\Omega_{G,m}(\Gamma) \simeq 2 \frac{|\Gamma|_{euc}}{m} + p \log(\pi m^2 / 2)$$

The precise choice of $m$ (and thus of $P_r(r)$) is a way to introduce a priori
knowledge and thus to favor or to penalize some contour $\Gamma$.

3.3. Smooth isotropic coding

Strong polygonal shapes are obtained with the approach developed in
Galland et al. (2003) since it allows one to get a very low information quan-
tity needed to describe the contour $\Gamma$ when a polygonal contour is chosen
with a large mean distance between its connected nodes. Smooth shapes are obtained in general with isotropic local coding since it allows one to describe the contour $\Gamma$ with a larger information quantity (i.e. a longer description length) than with the approach developed in Galland et al. (2003). Thus we propose in this subsection to determine $m$ so that the information quantities needed to describe a contour $\Gamma$ are equal if the local isotropic coding $\Omega_G^{\text{euc}}(\Gamma)$ or the previous non local coding $\Omega_{G,m}(\Gamma)$ is implemented. With this approach, it is expected that smoother contours than with the approach of Galland et al. (2003) can be obtained without strong fluctuations on straight lines.

When a contour of length $|\Gamma|_{\text{euc}}$ is coded with a polygon with a mean distance between nodes equal to $m_0$, its number of nodes is $p \simeq |\Gamma|_{\text{euc}}/m_0$. Eq. 7 thus leads to $\Omega_{G,m_0}(\Gamma) = 2 \frac{|\Gamma|_{\text{euc}}}{m_0} + \frac{|\Gamma|_{\text{euc}}}{m_0} \log(\pi m_0^2/2)$ while Eq. 4 is simply $\Omega_G^{\text{euc}}(\Gamma) = 1.77 |\Gamma|_{\text{euc}}$. Thus $m_0$ satisfies $1.77 m_0 = 2 + \log(\pi m_0^2/2)$. A numerical solution of this equation leads to $m_0 \simeq 2.35$ and thus to the smooth isotropic coding term

$$\Omega_G^{\text{sic}}(\Gamma) = 0.85 |\Gamma|_{\text{euc}} + 2.16 p$$ (8)

Let us remark that fixing $m = m_0$ does not impose that the average distance between nodes measured on $\Gamma$ has to be equal to $m_0$ on the segmentation results. It only imposes that the information quantity to describe a contour is determined with the law that assumes a mean Euclidian distance equal to $m_0$. In other words, $m_0$ is only the a priori mean Euclidian distance between nodes which can be different to the a posteriori Euclidian mean distance, i.e. actually obtained with a segmentation. Allowing such a difference between a priori and a posteriori means is a standard property in Bayesian techniques.
3.4. Isotropic maximum likelihood coding

The information quantity $\Omega_{G,m}(\Gamma)$ of Eq. 7 is the negative of a log-likelihood function that allows one to estimate $m$ with a maximum likelihood approach which leads to $\hat{m}_{ML} = |\Gamma|_{euc}/p$. Plugging-in $\hat{m}_{ML}$ into Eq. 7 provides the coding term

$$\Omega_{G}^{isoML}(\Gamma) = p \log(\pi |\Gamma|_{euc}^2/(2p^2)) + 2p + \frac{1}{2} \log(p)$$

(9)

where the last term $\frac{1}{2} \log(p)$ has been added since it corresponds to the coding of $\hat{m}_{ML}$ which requires $\log(\sqrt{p})$ nats (Rissanen, 1989). This solution is analogous to the one developed in Galland et al. (2003), and thus of Eq. 3, but it now corresponds to an isotropic coding.

It thus appears that $\Omega_{G}^{isoML}(\Gamma)$ penalizes essentially the number of nodes, that $\Omega_{G}^{euc}(\Gamma)$ penalizes the length of the contour, while $\Omega_{G}^{sic}(\Gamma)$ corresponds to a trade off between these two approaches.

4. Illustration of the proposed approaches on different examples

4.1. Optimization strategy

An analogous optimization strategy to the one implemented for polygonal MDL-AG described in Galland et al. (2003) has been considered. However, as described below, a slower optimization process that tests a larger number of configurations is used in an attempt to reach a lower local minimum of the criterion. Furthermore, in order to allow one to obtain smooth contours, nodes are progressively added during each node moving step until connected nodes are neighbors with the 8-connectivity neighborhood.

The optimization strategies implemented in this paper can be defined with the following steps:
Step 1: Regions are merged using a generalized likelihood ratio test with a threshold fixed to 3, see Galland et al. (2003) for details;

Step 2: Nodes are progressively added to each segment and moved in order to minimize $\Delta(s, \Gamma)$. This operation is performed until connected nodes are neighbor with the 8-connectivity neighborhood.

Step 3: Nodes are progressively removed one by one, in order to minimize $\Delta(s, \Gamma)$, and each time a node has been removed, its connected nodes can be moved to decrease $\Delta(s, \Gamma)$, see (Ruch and Réfrégier, 2001; Galland et al., 2003) for details;

Step 4: Region are merged in order to minimize $\Delta(s, \Gamma)$.

The implemented optimization process for the non local isotropic coding terms ($\Omega_{isoML}^{\text{G}}(\Gamma)$ and $\Omega_{isoML}^{\text{G}}(\Gamma)$) is the succession of steps [1,2,3,4,2,3,4], while for local coding ($\Omega_{euc}^{\text{GML}}(\Gamma)$) it is [1,2,3,4,2,4] since the distance between nodes has to be equal to one with this latter approach.

4.2. Analysis of the different contour coding terms with synthetic images

The segmentation results obtained with the contour coding terms and the optimization strategy described above are reported in Fig. 1, 3, 4 and 5 for the Gamma noisy image depicted in Fig. 1-c.

A first observation is that similar segmentation results are obtained with both $\Omega_{isoML}^{\text{G}}(\Gamma)$ and $\Omega_{isoML}^{\text{G}}(\Gamma)$, see Fig. 1-d and 4, and thus this last criterion will not be discussed anymore in the following.

The segmentation results obtained with $\Omega_{isoML}^{\text{G}}(\Gamma)$ and $\Omega_{isoML}^{\text{G}}(\Gamma)$ show that the first one is preferable with polygonal objects while the second one leads
Figure 3: Segmentation result obtained on the image shown in Fig. 1-c using the contour coding term $\Omega_G^{\text{euc}}(\Gamma)$. Below the segmented image: zooms of the segmentation in the three white squares shown in Fig. 1-a.
Figure 4: The same as in Fig. 3 but with the contour coding term $\Omega^{isoML}_G(\Gamma)$.

Figure 5: The same as in Fig. 3 but with the contour coding term $\Omega^{sic}_G(\Gamma)$. 

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to better results with smooth objects. Finally, the results obtained with
\( \Omega_{G}^{\text{ic}}(\Gamma) \) exhibit a trade-off between the previous approaches. In particular,
it recovers straight lines and the smooth parts of the objects with reduced
oscillations of the contours.

It is interesting to investigate these results more quantitatively. Two main
properties are relevant in order to characterize the segmentation quality. The
first one corresponds to the precision of the contour locations which can be
characterized with the number of misclassified pixels (NMCP). This quant-
tity can be estimated on the analysed image with the number of pixels of the
different objects that are classified in the background region plus the number
of pixels of the background region that are not classified in the background
region. The second property which is relevant in order to characterize seg-
mentation quality corresponds to the ability to detect regions in the image.
This ability can be quantified by measuring the number of regions obtained
in the segmented image (NROSI).

In order to analyze the precision of the contour locations and the ability
to detect small regions with the different discussed coding terms, we analyze
the segmentation results obtained with the following criterion:

\[
\Delta_{\lambda}(s, \Gamma) = \Delta_{L}(s|\theta, \Gamma) + \Delta_{P}(\theta|\Gamma) + \lambda \Delta_{G}(\Gamma) \quad (10)
\]

This criterion is equal to the description length defined in Eq. 1 when \( \lambda = 1 \),
(i.e. \( \Delta(s, \Gamma) = \Delta_{\lambda=1}(s, \Gamma) \)) and allows us to discuss the relevance of the above
considered MDL approaches. In particular, even if the criterion \( \Delta(s, \Gamma) \) does
not contain any tuning parameter, it is interesting to find out whether or not
adding such parameter \( \lambda \) could improve segmentation results.

The NMCP for the different coding terms are respectively represented
Figure 6: Segmentation quality criteria as a function of $\lambda$ obtained with $\Delta'_\lambda(s, \Gamma)$ (Eq. 10) on the image in Fig. 1-a averaged on 25 independent noise realizations (with the same noise conditions as for Fig. 1). (a) Number of misclassified pixels (NMCP) obtained on the letter S appearing on the first line of the image; (b) The same but on the vertical bar of letter I; (c) Idem but on the letter C; (d,e) The same but results obtained on the whole image and with a final merging step 4. Curves (a,b,c,d) have been obtained when the segmentation algorithm has been initialized with the true contours while those of curve (e) corresponds to the segmentation with the wall-like grid shown in Fig. 1-b. (f) Number of regions obtained in the segmented image (NROSI) obtained with the same initial wall-like grid as for (e). (The curves have different vertical scales for visual purposes).
as a function \( \lambda \) in Fig. 6-(a,b,c,d,e). In Fig. 6-(a,b,c) the NMCP obtained respectively for the letter S, the vertical bar of the letter I and the letter C appearing on the first line of Fig. 1-a are reported when the segmentation algorithm has been initialized with the true contours of the object. In that case, since it is assumed that the number of regions does not have to be estimated, only the optimization steps 2 and 3 have been used with the coding terms \( \Omega_{G}^{isoML}(\Gamma) \) and \( \Omega_{G}^{sic}(\Gamma) \). For the coding term \( \Omega_{G}^{euc}(\Gamma) \), only the optimization step 2 has been used in this case since the nodes have to be 8-connected. In Fig. 6-d the NMCP obtained on the global image is reported when a final region merging step 4 has been added in order to estimate the number of regions, and when the segmentation algorithm has still been initialized with the true contours of the object. In Fig. 6-e the NMCP obtained on the global image is finally reported with the global optimization strategy defined in the previous subsection when the segmentation algorithm has been initialized with the wall-like grid shown in Fig. 1-b.

These curves show that the optimal value of the parameter \( \lambda \) that minimizes the NMCP is around \( \lambda = 1 \) for \( \Omega_{G}^{euc}(\Gamma) \) for the different analyzed letters while this optimal value depends on the considered letter for the other coding terms \( \Omega_{G}^{euc}(\Gamma) \) and \( \Omega_{G}^{isoML}(\Gamma) \). In particular, \( \Omega_{G}^{euc}(\Gamma) \) leads to the smallest NMCP for the smooth object S when \( \lambda \simeq 2 \) (the optimal value with the letter C is \( \lambda \simeq 1.5 \) and \( \lambda \simeq 1 \) on the bar of the letter I). Furthermore, the smallest NMCP is obtained with \( \lambda < 1 \) with \( \Omega_{G}^{isoML}(\Gamma) \) and particularly with smooth objects such as S or C. Finally, when \( \lambda \simeq 1 \), the obtained NMCP with \( \Omega_{G}^{sic}(\Gamma) \) is always among the best results that can be obtained with the other coding terms \( \Omega_{G}^{euc}(\Gamma) \) and \( \Omega_{G}^{isoML}(\Gamma) \).
The NMCP of the segmentation results shown in Fig. 6-e demonstrates that the implemented optimization strategy is better adapted for the coding terms $\Omega_{G}^{isoML}(\Gamma)$ and $\Omega_{G}^{sic}(\Gamma)$ than for $\Omega_{G}^{euc}(\Gamma)$. Indeed, only small differences are observed between Fig. 6-d and Fig. 6-e for $\Omega_{G}^{isoML}(\Gamma)$ and $\Omega_{G}^{sic}(\Gamma)$ while a larger difference is observed with $\Omega_{G}^{euc}(\Gamma)$. Although it could be interesting to define a better optimization strategy for $\Omega_{G}^{euc}(\Gamma)$, the results of Figs. 6-a, 6-b, 6-c and 6-e show that $\Omega_{G}^{euc}(\Gamma)$ provides an interesting trade-off. This result is confirmed with Fig. 6-f which represents the NROSI as a function of $\lambda$. In particular, this figure clearly shows that although increasing the value of $\lambda$ for $\Omega_{G}^{euc}(\Gamma)$ may decrease the NMCP for some objects, it will also decrease the ability to detect regions with this contour coding term. Indeed, even if a NROSI equal to the true number of regions does not necessarily mean that all the regions have been recovered (for example an object which have been splitted into two regions can compensate a small object which has not been detected), a NROSI lower than the true number of regions means that some regions have not been detected.

4.3. Illustration of the results with real images

In this section, segmentation results obtained with the above discussed contour coding terms are illustrated with different kinds of images. Indeed, the above approaches can be directly applied to the different MDL segmentation techniques based on the active grid that have been developed up to now.

Let us first analyze results obtained on a medical ultrasound image of a tissue-mimicking phantom (breast phantom custom-made by Computerized Imaging Reference Systems Inc. (CIRS, Norfolk, VA, USA)). It shows a hy-
poechochogenic cystic liquid inclusion within a uniform background. The image was acquired by the clinical ultrasound imaging scanner Aixplorer® (Super-Sonic Imagine, Aix-en-Provence, France). This ultrasound image exhibits speckle noise and we assume that the grey level fluctuations are distributed with a Gamma PDF with shape parameter $L = 1$. As a consequence, the terms $\Delta_P$ and $\Delta_L$ involved in the description length of the polygonal MDL-AG (Eq. 1) are those provided in Galland et al. (2003). The different segmentation results obtained using the different contour coding terms discussed in the previous section are shown in fig. 7.b, c and d. These results qualitatively confirm the behaviors observed in the previous section on synthetic images. In particular, the contour coding term $\Omega_{isoML}^G(\Gamma)$ leads to a strong polygonal behavior of the contour shapes and it produces a larger number of regions than with $\Omega_{sic}^G(\Gamma)$ and $\Omega_{euc}^G(\Gamma)$. Furthermore, $\Omega_{euc}^G(\Gamma)$ leads to contours with more fluctuations when it is compared to $\Omega_{isoML}^G(\Gamma)$ which establishes a trade off between $\Omega_{isoML}^G(\Gamma)$ and $\Omega_{euc}^G(\Gamma)$.

There exist situations for which it can be interesting to implement a non parametric PDF estimation technique. Such an approach was implemented with the polygonal MDL-AG in Delyon et al. (2006). With this approach, the unknown PDF are approximated with step functions (i.e. piecewise constant functions) with irregular bin lengths, their number and size being estimated by minimizing the description length of the image. This non parametric version of the polygonal MDL-AG can be implemented with the various contour coding terms defined in Sec. 3. The results are illustrated with a scanning electron microscopy image (provided by P. Ferrand, obtained at university of Wuppertal) of self assembled photonic crystals (Ferrand et al., 2003, 2004).
Figure 7: (a) Extract (152 × 146 pixels) of an ultrasonic image of a phantom provided by SuperSonic Imagine, Aix-en-Provence, France. The segmentation technique is adapted to fluctuations with gamma PDF of shape parameter $L = 1$. (b) Results with $\Omega_{G}^{isoML}(\Gamma)$; (c) With $\Omega_{G}^{euc}(\Gamma)$; (d) With $\Omega_{G}^{sic}(\Gamma)$. The grey level values have been modified for better visualization.
The segmentation results obtained with the different contour coding terms are shown in Fig. 8-(b,c,d). Basically, the above conclusions still hold with this non parametric version of the polygonal MDL-AG. The image being mainly a pattern of quasi-circular objects, it is not a surprise that $\Omega_{G}^{isoML}(\Gamma)$ provides a somewhat coarse description of the expected boundaries in the image. Conversely, $\Omega_{G}^{sic}(\Gamma)$ and $\Omega_{G}^{euc}(\Gamma)$ lead to better segmentation results, at least from a qualitative evaluation viewpoint. Here again, $\Omega_{G}^{sic}(\Gamma)$ provides an interesting trade-off.

The SAR image shown in Fig. 9-a is an extract of a single look high resolution intensity SAR image of an urban area acquired by the Onera SAR airborne sensor RAMSES (Dubois-Fernandez et al., 2002). Since this image presents important textures and strong reflectors, the PDF in each region is described by a Fisher PDF (Tison et al., 2004). The terms $\Delta_P$ and $\Delta_L$ involved in Eq. 1 are thus those described in Galland et al. (2009). The segmentation results shown in Fig. 9-(b,c,d) suggest that the regions with the lowest contrast are best detected with $\Omega_{G}^{isoML}(\Gamma)$. The coding terms $\Omega_{G}^{euc}(\Gamma)$ provides the simplest partitions whereas $\Omega_{G}^{sic}(\Gamma)$ leads to an intermediate number of regions which is in agreement with the results previously discussed on synthetic images.

The different coding techniques introduced in this paper lead approximately to the same computational times. For example, for a scalar image of size 400 × 400 pixels, the segmentation is obtained in approximately 13 s (using monothreaded C programming on a standard PC equiped with a 3.73 GHz processor) with the optimization strategy described in the previous section and a criterion adapted to Gamma PDF. If the same optimization strategy
Figure 8: (a) Scanning electron microscopy image (712 × 421 pixels) of self assembled photonic crystals, provided by P. Ferrand, obtained at university of Wuppertal. The segmentation technique uses the non-parametric PDF approach presented in Delyon et al. (2006) to model the fluctuations. Segmentation results obtained with: (b) $\Omega_{isoML}^G(\Gamma)$; (c) $\Omega_{euc}^G(\Gamma)$; (d) $\Omega_{sic}^G(\Gamma)$. Below each segmented image, zooms of the segmentation results obtained inside the two white squares of (a).
Figure 9: (a): Extract (501 × 377 pixels) of a single look high resolution SAR image of an urban area, acquired by the Onera SAR airborne sensor RAMSES (X band, HH polarization, resolution < 50 cm). The segmentation technique is adapted to Fisher PDF. (b) Results with \( \Omega_{isoML}^G(\Gamma) \); (c) With \( \Omega_{euc}^G(\Gamma) \) and (d) with \( \Omega_{sic}^G(\Gamma) \). Below each segmented image: zooms of the segmentation inside the two white squares (101 × 101 pixels and 131 × 131 pixels) shown on (a). The displayed grey level values have been modified for visual purposes.
as the one implemented\textsuperscript{2} in Galland et al. (2003) is used, then the segmentation result is obtained in less than 2 s with Gamma PDF without strong degradation on the considered examples with $\Omega_{G}^{\text{isoML}}(\Gamma)$ and $\Omega_{G}^{\text{sc}}(\Gamma)$.

5. Conclusion and perspectives

The segmentation technique based on the Minimum Description Length active grid initially developed in Galland et al. (2003) has been extended to different models of noisy images (see for example Galland et al. (2005); Delyon et al. (2006); Galland et al. (2009); Morio et al. (2007)) but, up to now, using a coding term adapted to polygonal contours. It has been shown in this paper that segmentation results in the presence of regions with smooth boundaries can be improved with an appropriate choice of the contour coding term. In particular, based on the information theory framework, a simple method can be implemented without significant increase of the computational time. This improvement opens a large class of application domains since it can be applied to all the various situations for which the polygonal MDL active grid has been developed.

There exists several interesting perspectives to this work. It will be interesting to obtain theoretical results on the implemented optimization strategy and to improve it. This will be particularly useful when the Euclidian length contour coding term is implemented since it has been shown that the proposed optimization strategy is not perfectly adapted to that coding. The

\textsuperscript{2}In Galland et al. (2003), the addition of nodes to each segment was not considered in the optimization strategy, i.e., the optimization step 2 reduces to a moving step of the nodes in order to reduce $\Delta(s, \Gamma)$; see Galland et al. (2003) for details.
considered choice for the a priori probability law \( P_r(r) \) of the distance between contour nodes have been determined with simple approaches and with some rough approximations. It will be interesting to investigate if other theoretical approaches can still improve the segmentation quality of smooth object without significantly increasing the computational time. Coupling segmentation with a priori contour description of particular objects will also be very challenging. It will, for example, allow one to apply techniques analogous to the ones proposed in (Staib and Duncan, 1992) and (Foulonneau et al., 2009). Such an approach can indeed correspond to the choice of the a priori probability law \( P_r(r) \) adapted to some expected objects in the image. Analogous situations can also appear when active grid segmentation techniques are applied to image sequences (Le Men et al., 2008), which is another interesting and challenging problem.

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