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Filtering of unpolarized light by gratings

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Abstract
The filtering effect we are dealing with in the present paper is obtained by using a diffraction grating made by perturbing a planar waveguide. A phenomenological theory allows us to deduce important properties of the sharp filtering phenomena generated by this kind of structure when the incident light excites guided waves. It is shown that the resonance phenomenon occurring when a single guided mode is excited acts on a single eigenvalue of the Hermitian reflection matrix. As a consequence, we demonstrate that a high efficiency filtering of unpolarized light requires the simultaneous excitation of two uncoupled guided waves. This criterion is reached when these guided waves propagate in orthogonal directions. Numerical examples are given.

Keywords: Grating, filter, resonance, polarization, waveguide, pole

1. Introduction

Curves of efficiency of dielectric diffraction gratings versus the wavelength may present sharp peaks (classified as anomalies) generated by the excitation of guided waves. These anomalies occur when the space and time frequencies imposed by the incident beam are close to those of an eigenmode of the structure. They have been widely studied in the case of classical one-dimensional (1D) gratings illuminated with in-plane mountings, in particular when only one order is reflected or transmitted by the grating [1, 2]. It has been shown that the reflectivity of these gratings is, in general, close to that of the planar structure except for a sharp peak culminating at 100%, corresponding to the excitation of a guided mode, at least for 1D symmetrical shallow surfaces. This remarkable property may be valuable for light filtering, but unfortunately it is limited to polarized light [3, 4]. In order to use this property for many technological applications of filtering, it has been suggested to use off-plane (conical) mounting or two-dimensional (2D) gratings in order to design filters for unpolarized light [5–7]. However, the main difficulty is in handling the large number of parameters describing these gratings, since the behaviour of the reflectivity versus the wavelength, angle of incidence and incident polarization in the general vectorial case is still little understood.

In order to overcome this difficulty, we have developed a phenomenological theory of filtering properties of 1D or 2D gratings deposited on planar waveguides. Our phenomenological study makes use of the notion of analytic continuation of complex functions of a real variable in the complex plane. Poles and roots of the eigenvalues of Hermitian matrices derived from scattering matrices allow us to predict the filtering phenomena for polarized and unpolarized light.

Our first result is that high efficiency filtering properties for unpolarized light are quite impossible to obtain if the incident light cannot excite a couple of guided modes at the same wavelength. The second result is that this goal is reached when the two guided modes propagate in almost orthogonal directions. This interesting property is interpreted in terms of the Brewster effect. Rigorous numerical results will confirm these theoretical predictions for some kinds of gratings.

2. The physical problem

The periodic guiding structure presented in figure 1 in a Cartesian coordinate system of axes xyz is limited on top ($z = 0$) by air and at the bottom ($z = -e$) by a substrate of real relative permittivity ε_s. Its relative permittivity $\varepsilon(x, y, z)$ for $-e < z < 0$ is real and periodic along one or two different, possibly non-orthogonal, directions of the xy plane, vectors $\vec{d}_1$ and $\vec{d}_2$ symbolizing the two periodicities ($\vec{d}_2$ is infinite for a 1D grating). Figure 2 shows examples of such structures. The permittivity $\varepsilon(x, y, z)$ is supposed to be obtained by perturbing slightly a permittivity $\varepsilon'(z)$ in a periodic manner, the non-perturbed structure being a waveguide (for example, $\varepsilon'(z)$ may be constant and greater than ε_s).

The incident plane wave with wavevector $\vec{k}^{i+}$ (with $|\vec{k}^{i+}| = k = 2\pi/\lambda$, λ wavelength in vacuum) illuminates the grating

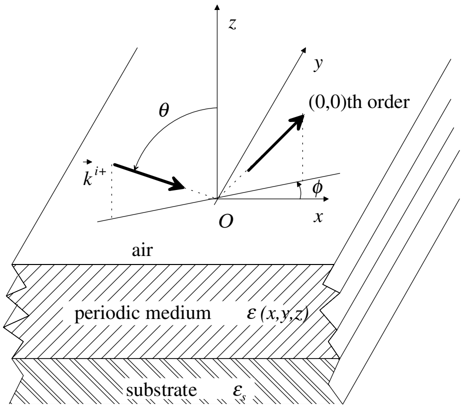

Figure 1. The periodic guiding structure.

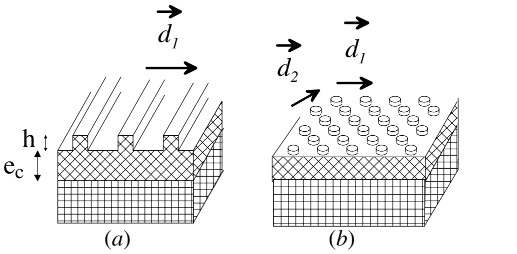

Figure 2. Examples of periodic guiding structures. (*a*) Classical lamellar grating used in conical mounting. (*b*) Crossed grating with circular bumps and hexagonal symmetry.

with an incidence characterized by angles ϕ (angle between the x axis and the projection of $\vec{k}^{i+}$ on the xy plane) and θ (angle between the z axis and $\vec{k}^{i+}$). To define properly the scattering matrix of the structure, it is also necessary to introduce an incident plane wave coming from the substrate, with wavevector $\vec{k}^{i-}$ (with $|\vec{k}^{i-}| = k\sqrt{\varepsilon_s}$) that has the same tangential component $\underline{\underline{\vec{k}}}^i$ on the (xy) plane as $\vec{k}^{i+}$. Since these two wavevectors have the same projection $\underline{\underline{\vec{k}}}^i$ on the xy plane, they generate reflected and transmitted waves in the same directions.

Defining the reciprocal grating by the vectors $(\vec{a}_1, \vec{a}_2)$ given by $\langle \vec{a}_i, \vec{d}_j \rangle = 2\pi\delta_{i,j}$, with δ the Kronecker symbol (for 1D gratings $\vec{a}_2$ vanishes) and using a time dependence in $\exp(-\mathrm{i}\omega t)$, the total electric field outside the inhomogeneous region at point $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z} = \underline{\underline{\vec{r}}} + z\hat{z}$ can be expressed by

$$\vec{E}^{\pm} = \vec{E}^{i\pm} + \vec{E}^{s\pm} = \vec{P}^{i\pm}\exp(\mathrm{i}\underline{\underline{\vec{k}}}^i \cdot \underline{\underline{\vec{r}}} \mp \mathrm{i}\gamma^{\pm} z) + \sum_{n=-\infty}^{+\infty}\sum_{m=-\infty}^{+\infty} \vec{P}^{\pm}_{n,m}\exp(\mathrm{i}(\underline{\underline{\vec{k}}}^i + n\vec{a}_1 + m\vec{a}_2)\cdot \underline{\underline{\vec{r}}} \pm \mathrm{i}\gamma^{\pm}_{n,m} z). \tag{1}$$

The symbol $\pm$ indicates whether we are placed in the air or in the substrate:

$$\gamma^{+}_{n,m} = \sqrt{k^2 - |\underline{\underline{\vec{k}}}^i + n\vec{a}_1 + m\vec{a}_2|^2}$$

and

$$\gamma^{-}_{n,m} = \sqrt{k^2\varepsilon_s - |\underline{\underline{\vec{k}}}^i + n\vec{a}_1 + m\vec{a}_2|^2}$$

with $\mathrm{Re}(\gamma^{\pm}_{n,m}) + \mathrm{Im}(\gamma^{\pm}_{n,m}) \geqslant 0$. Throughout the paper, it is assumed that only one scattered order (the zeroth order, corresponding to $n = m = 0$ in equation (1)) propagates towards $z = \pm\infty$ in the air or in the substrate, all the others being evanescent.

In order to define the polarizations of the incident and scattered waves, the amplitudes $\vec{P}^{i\pm}$ and $\vec{P}^{\pm}_{0,0}$ of the incident electric field and of the two propagating orders are projected on two unit vectors $\hat{s}^{i\pm}$, $\hat{p}^{i\pm}$, $\hat{s}^{d\pm}$, $\hat{p}^{d\pm}$ orthogonal to the corresponding wavevectors, vectors $\hat{p}$ lying in the plane of incidence, with positive or null component on the z axis, the trihedrons $(\vec{k}, \hat{p}, \hat{s})$ being direct for waves propagating towards $z = -\infty$ and inverse otherwise. We call $P^{i\pm,s}$, $P^{i\pm,p}$, $P^{d\pm,s}$ and $P^{d\pm,p}$ the projections of the polarization vectors on these unit vectors.

3. Scattering matrices: definitions and properties

Four incident and diffracted column matrices of two elements are defined by

$$\begin{aligned} I^{\pm} &= \left(P^{i\pm,s}\sqrt{\gamma^{\pm}_{0,0}},\, P^{i\pm,p}\sqrt{\gamma^{\pm}_{0,0}}\right), \\ D^{\pm} &= \left(P^{d\pm,s}\sqrt{\gamma^{\pm}_{0,0}},\, P^{d\pm,p}\sqrt{\gamma^{\pm}_{0,0}}\right) \end{aligned} \tag{2}$$

and from the linearity of Maxwell's equations, the diffracted matrices can be expressed linearly from the incident ones through square reflection matrices R_1 and R_2 and transmission matrices T_1 and T_2 of size 2×2:

$$D^+ = R_1 I^+ + T_2 I^- \tag{3}$$

$$D^- = T_1 I^+ + R_2 I^-. \tag{4}$$

Incident and diffracted column matrices I and D having four components are respectively equal to

$$\left(P^{i+,s}\sqrt{\gamma^{+}_{0,0}},\, P^{i+,p}\sqrt{\gamma^{+}_{0,0}},\, P^{i-,s}\sqrt{\gamma^{-}_{0,0}},\, P^{i-,p}\sqrt{\gamma^{-}_{0,0}}\right)$$

and

$$\left(P^{d+,s}\sqrt{\gamma^{+}_{0,0}},\, P^{d+,p}\sqrt{\gamma^{+}_{0,0}},\, P^{d-,s}\sqrt{\gamma^{-}_{0,0}},\, P^{d-,p}\sqrt{\gamma^{-}_{0,0}}\right)$$

in such a way that equations (3) and (4) can be condensed into a single one:

$$D = SI, \qquad \text{with } S = \begin{pmatrix} R_1 & T_2 \\ T_1 & R_2 \end{pmatrix}. \tag{5}$$

Since the materials are lossless, the energy balance entails that the scattered intensity is equal to the incident one, thus $|D| = |I|$. This property shows that the S matrix is unitary:

$$S^* S = 1 \tag{6}$$

1 denoting here the unit diagonal matrix and S^* the adjoint of S.

Secondly, the reciprocity theorem [8] allows one to show from tedious but straightforward calculations that, when the diffracting structure is symmetrical with respect to the z axis, the S matrix is symmetrical. Other symmetry properties of the S matrix can be derived from other kinds of symmetries of the structure.

4. Singularities and roots of the S matrix in the complex plane

We have assumed that the non-perturbed structure is a waveguide. After introducing a perturbation, it remains a waveguide and it is well known that a guided wave propagating with a propagation constant or wavevector $\underline{\underline{\vec{k}}}^g$ in the xy plane can be expressed from equation (1) by removing the incident terms:

$$\vec{E}^{\pm} = \sum_{n'=-\infty}^{+\infty} \sum_{m'=-\infty}^{+\infty} \vec{P}^{g\pm}_{n',m'} \times \exp(\mathrm{i}(\underline{\underline{\vec{k}}}^g + n'\vec{a}_1 + m'\vec{a}_2) \cdot \underline{\underline{\vec{r}}} \pm \mathrm{i}\gamma^{g\pm}_{n',m'} z) \quad (7)$$

$$\gamma^{g+}_{n',m'} = \sqrt{(k^g)^2 - |\underline{\underline{\vec{k}}}^g + n'\vec{a}_1 + m'\vec{a}_2|^2},$$

$$\gamma^{g-}_{n',m'} = \sqrt{(k^g)^2\varepsilon_s - |\underline{\underline{\vec{k}}}^g + n'\vec{a}_1 + m'\vec{a}_2|^2}.$$

For the unperturbed planar waveguide, the expression of the guided wave reduces to a single term, which corresponds to $n' = m' = 0$. As a consequence,

$$|\underline{\underline{\vec{k}^g}}| > k^g. \quad (8)$$

The question which arises is the following: the guided wave expressed in equation (7) can be considered as a scattered field which exists without any incident wave (see equation (1)). If an incident plane wave can excite this guided wave, it will generate a resonance phenomenon (grating anomaly), resulting in rapid variations of the scattered amplitudes. The double conditions to reach this goal are the following:

(a) to use an incident wave with a wavenumber k equal to the wavenumber k^g of the guided wave,
(b) to match the propagation constants $\underline{\underline{\vec{k}}}^i + m\vec{a}_1 + n\vec{a}_2$ in the xy plane of the incident wave to the propagation constants $\underline{\underline{\vec{k}}}^g + m'\vec{a}_1 + n'\vec{a}_2$ of the guided wave, thus

$$\underline{\underline{\vec{k}^i}} = \underline{\underline{\vec{k}^g}} + n_t\vec{a}_1 + m_t\vec{a}_2. \quad (9)$$

The first remark to be made on equation (9) is that n_t or m_t or both must be different from zero. Indeed, $|\underline{\underline{\vec{k}^i}}|$, the constant of propagation of a plane wave in the xy plane, is less than the wavenumber $k = k^g$, in contrast to $\underline{\underline{\vec{k}^g}}$, according to equation (8).

The second remark is that the wavenumber k^g must be complex; thus condition (a) cannot be fulfilled exactly. Indeed, if both conditions are satisfied, the propagation constants of the terms placed in the summation of equation (7) are identical to those corresponding to the scattered field in equation (1). This means that it contains at least one scattered plane wave on both sides of the periodic region, or in other words that the structure presents losses, due to these radiations. Since there is no incident wave in equation (7), these losses cannot be offset by any incident energy. An obvious consequence is that the field must decrease with time, and since it behaves in $\exp(-\mathrm{i}\omega t)$ the frequency $\omega = k^g c$ is complex, with a negative imaginary part.

On the other hand, when the perturbed waveguide tends to a planar structure, the amplitudes of the lossy terms vanish in equation (7) and k^g becomes real and equal to the wavenumber $k^{g,planar}$ of the guided wave of the planar structure with propagation vector $\underline{\underline{\vec{k}^g}}$ in the xy plane. However, we know that this guided wave cannot be excited by an incident plane wave since the single term with propagation vector $\underline{\underline{\vec{k}^g}}$ in equation (7) cannot match the single term with propagation vector $\underline{\underline{\vec{k}^i}}$ in the scattered field contained in equation (1).

It is worth noting that condition (a) can be satisfied if we renounce condition (b), in other words if we assume that the propagation vector $\underline{\underline{\vec{k}^g}}$ is complex. In that case, the field decreases as it propagates but does not decrease with time. In the frame of the present paper, we are interested in the behaviour of the scattered field when the wavelength is varied, and the choice of using complex values of the wavenumber is much more adequate, as we will see later.

Now, from a mathematical point of view, we are led to the following conclusion. The guided mode given by equation (7) has the same expression as the scattered field of equation (1), which would exist without any incident field for propagation vector $\underline{\underline{\vec{k}^g}}$. In other words, under the conditions where the guided wave exists, the diffracted column matrix D of equation (5) is different from zero while the incident column matrix I vanishes: thus the scattering matrix S has a pole. Since this pole corresponds to a complex value k^g of the wavenumber, it can be concluded that k^g is the pole of the analytic continuation of the S matrix (which is unique) in the complex plane of k, when the propagation constant $\underline{\underline{\vec{k}^i}}$ is fixed and equal to that of the guided wave $\underline{\underline{\vec{k}^g}}$. Of course, the wavenumber k of a plane wave is real but the existence of the pole outside the real axis has a vital importance on the behaviour on the real axis: a resonance phenomenon occurs as soon as the real wavenumber k of the incident plane wave comes close to the location of the pole, especially if the imaginary part of the pole is small.

In general, the fact that the determinant of S has a pole entails that all the coefficients of the S matrix, and hence all the sub-matrices R_1, R_2, T_1, T_2, have the same pole, as shown in the study of grating anomalies [1, 2]. Furthermore, the unitarity of the S matrix on the real axis and the extinction of the resonance phenomenon when the structure becomes planar lead to the conclusion that the determinant of the S matrix has a root k^r_S, called the root of the S matrix in the following. When the waveguide becomes planar, this root becomes equal to the pole:

$$k_S^{r,planar} = k^{g,planar}. \quad (10)$$

In contrast with the pole, the root of $\det(S)$ is not the same as the roots $k^r_{R_1}, k^r_{R_2}, k^r_{T_1}, k^r_{T_2}$ of the sub-matrices. However, all these roots have the same limit given by equation (10). Furthermore, it can be shown that the root and the pole of a given sub-matrix are the root and the pole of a single eigenvalue of this matrix, the other one having no pole and no root in the vicinity. Now, let us consider the matrix T_1. Its first eigenvalue can be expressed in the form

$$l^{(1)}_{T_1}(k) = u(k)\frac{k - k^r_{T_1}}{k - k^g}. \quad (11)$$

Since $u(k)$ has no pole and no root in the vicinity of the resonance, it can be considered as an analytic function and if the structure is slightly perturbed, the pole and root are very close to each other and close to the real axis, and $u(k)$ can be considered as constant in the resonance region, as well as the second eigenvalue $l^{(2)}_{T_1}(k)$. Equation (10) shows that, for the planar structure, the root identifies with the pole in such a way that the anomalous behaviour of the first eigenvalue does not hold.

5. Vital influence of the symmetries of the waveguide on the roots

Symmetries of the structure entail very important properties of the roots of the eigenvalues, which may become real. Using elementary theorems on analytic functions of the complex variable, it can be shown that the extension of equation (6) to the complex plane, in the vicinity of the real axis, can be written as

$$S^*(\bar{k})S(k) = 1. \qquad (12)$$

Developing this equation using the sub-matrices R_1, R_2, T_1, T_2, it emerges that the conjugate $\bar{k}^r_{T_2}$ of the root of T_2 is equal to the root $k^r_{T_1}$ of T_1. Now, if the structure is symmetrical with respect to the z axis, T_2 is the transpose of T_1 since S is symmetrical. Thus the root of T_2 is the root of T_1 as well and $\bar{k}^r_{T_1}$ is a root of T_1. Since T_1 has a single root in the vicinity of the resonance region, these two roots are equal and $k^r_{T_1}$ is real.

This interesting result can be expressed in the following way: when the diffractive structure is symmetrical with respect to the z axis, there exists a real wavelength $\lambda = 2\pi/k^r_{T_1}$ and a polarization I^+ of the incident wave propagating in air (equal to the value of the first eigenvector $V^{(1)}_{T_1}$ of T_1 at $k = k^r_{T_1}$) such that the transmitted energy is rigorously equal to zero. As a consequence, the reflected energy is equal to the incident one.

It can be shown that in these conditions, called the 'total reflection configuration', the diffracted column matrix D^+ is equal to $\bar{V}^{(1)}_{T_1}$.

It is worth noting that $I^+ = V^{(1)}_{T_1}$ does not correspond to a linearly polarized incident wave in general. Thus it is interesting to study what happens when, starting from the total reflection configuration, the polarization or the wavelength of the incident wave is changed. We suppose that the polarization is linear and that the incident energy $\langle I^+|I^+\rangle$ is equal to unity ($\langle V|U\rangle = \bar{V}_1U_1 + \bar{V}_2U_2$ is the Hermitian scalar product). Then the reflected energy ρ is given by

$$\rho = \langle D^+|D^+\rangle = \langle R_1I^+|R_1I^+\rangle = \langle R_1^*R_1I^+|I^+\rangle. \qquad (13)$$

Thus we are led to the study of the Hermitian reflection matrix $R_1^*R_1$ which has two real and positive eigenvalues $l^{(1)}_{R_1^*R_1}$ and $l^{(2)}_{R_1^*R_1}$ associated with two orthogonal eigenvectors $V^{(1)}_{R_1^*R_1}$ and $V^{(2)}_{R_1^*R_1}$. The $R_1^*R_1$ matrix has two poles ($k = k^g$ due to R_1 and $k = \bar{k}^g$ due to R_1^*) and two conjugate and complex roots. These poles and roots are present in the first eigenvalue $l^{(1)}_{R_1^*R_1}$ whilst the second one, $l^{(2)}_{R_1^*R_1}$, which has no pole and no root, is not sensitive to the resonance phenomenon when k is close to k^g.

Mathematically, the reflected energy can be written in the form

$$\rho = l^{(1)}_{R_1^*R_1}|\langle I^+|V^{(1)}_{R_1^*R_1}\rangle|^2 + l^{(2)}_{R_1^*R_1}|\langle I^+|V^{(2)}_{R_1^*R_1}\rangle|^2. \qquad (14)$$

Using the following notation:

$$V^{(1)}_{R_1^*R_1} = (\cos q, \sin q \exp(\mathrm{i}\varphi)) \qquad (15)$$

taking into account the orthogonality of $V^{(1)}_{R_1^*R_1}$ and $V^{(2)}_{R_1^*R_1}$, and bearing in mind that the incident wave is unitary and linearly polarized yields

$$I^+ = (\cos(\delta), \sin(\delta)) \qquad (16)$$

and a straightforward calculation shows from (14) that

$$\rho = \frac{l^{(1)}_{R_1^*R_1} + l^{(2)}_{R_1^*R_1}}{2} + \frac{l^{(1)}_{R_1^*R_1} - l^{(2)}_{R_1^*R_1}}{2}\tau\cos(2\delta - \psi) \qquad (17)$$

$$\tau = \sqrt{\cos(2q)^2 + \sin(2q)^2\cos(\varphi)^2} \qquad (18)$$

$$\tan(\psi) = \tan(2q)\cos(\varphi). \qquad (19)$$

When $k = k^r_{T_1}$, the reflected energy oscillates sinusoidally as the polarization angle δ is varied, the maximum value of $1 - \frac{1}{2}(1 - l^{(1)}_{R_1^*R_1})(1 - \tau)$ and a minimum value of $1 - \frac{1}{2}(1 - l^{(1)}_{R_1^*R_1})(1 + \tau)$ being obtained when $\delta = \frac{1}{2}\psi$ and $\frac{1}{2}(\psi + \pi)$, respectively.

Now, if the structure is close to the planar waveguide, we can conjecture that the eigenvalues of $R_1^*R_1$ (and therefore the reflectivity) are close to that of the planar structure when k is taken far enough from $k^r_{T_1}$. In the vicinity of $k^r_{T_1}$, the eigenvalue $l^{(1)}_{R_1^*R_1}$ has two poles and two roots. Its value increases from that of the planar waveguide to unity when k tends to $k^r_{T_1}$. *If the incident wave has this polarization, the reflected wave will have exactly the same behaviour. Thus, if the planar waveguide is a poor reflector, the perturbed waveguide will constitute a high efficiency filter for a given polarized light.* On the other hand, for the orthogonal polarization, the structure will reflect the incident light as the planar waveguide.

It can be deduced from these considerations that it is quite impossible to use the grating structure as a selective frequency filter with an efficiency close to unity for unpolarized light if one mode only is excited.

6. Numerical examples

Our first example illustrates the fact that if one mode only is excited, the reflectivity cannot reach 100% for any polarization of the incident beam. We consider a grating that supports one guided mode only, with wavevector $\underline{\underline{\vec{k}}}^g$, in the range of wavelength we are interested in. The parameters of the grating are chosen so that one order only is diffracted in the substrate and in the air. We study the reflectivity of the structure versus the wavelength for s and p polarization, under conical incidence as shown in figure 3(a). For a wavenumber close to the real part of k^g, sharp peaks appear in the reflectivity curves for both s and p polarizations (figure 3(a)). The peaks have different maximum values, and none of them reach 100%. Indeed, in that case, only one eigenvalue of the reflectivity matrix, $l^{(1)}_{R_1^*R_1}$, reaches one, while the other, $l^{(2)}_{R_1^*R_1}$, is not affected by the presence of the mode (figure 3(b)). As a result, the reflectivity oscillates between one and $l^{(2)}_{R_1^*R_1}$ as a function of the angle of polarization δ (figure 3(c)). In order to make both eigenvalues reach one, two modes have to be excited by the incident beam, for the same spatial frequencies and the same wavenumber. This amounts to saying that two dispersion curves, that relate the wavenumber to the wavevector of the modes, intersect. The

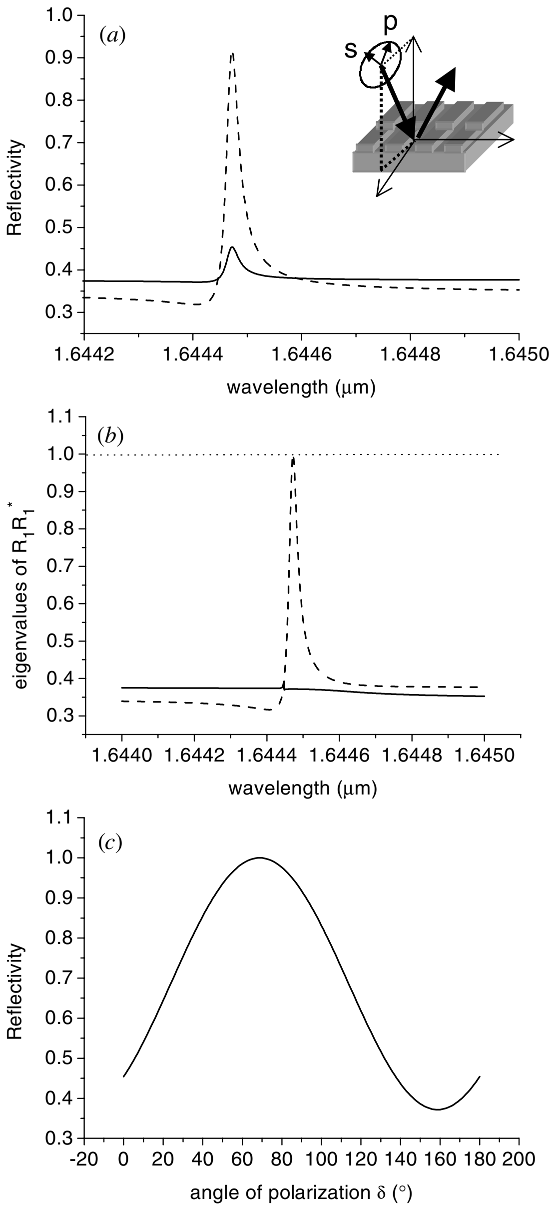

Figure 3. Reflection factor of a 2D grating with square bumps, refractive indices of the substrate $n_s = 1.5$, layer $n_c = 2.5$, thickness $e_c = 133$ nm, bump height $h = 7$ nm, period $d = 930$ nm and bump width $c = 465$ nm. The index of the bumps is equal to n_c. The incident parameters are $\theta = 15°$ and $\phi = 28°$. (*a*) Reflection factor versus wavelength for both s (full curve) and p (broken curve) polarizations. (*b*) First (full curve) and second (broken curve) eigenvalues versus wavelength. (*c*) Reflection factor versus the angle of polarization δ.

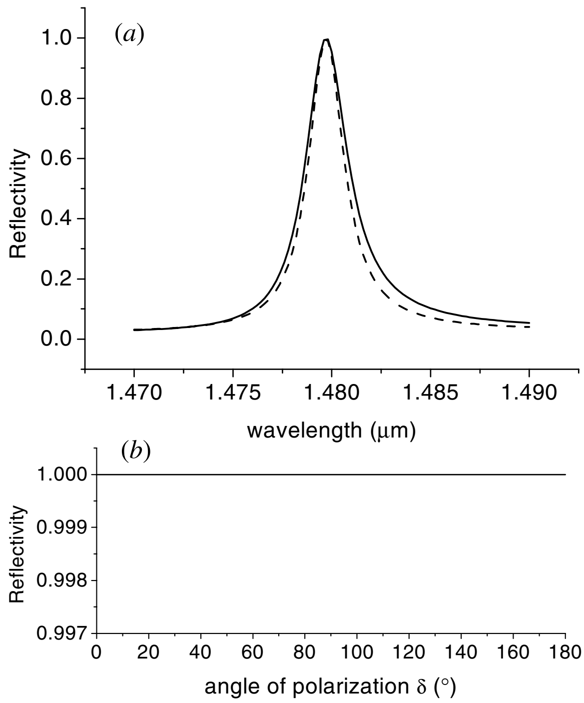

Figure 4. Reflection factor of a 1D lamellar grating, with refractive index of the substrate $n_s = 1.5$, layer $n_c = 2.0$, thickness $e_c = 300$ nm, groove depth $h = 87.5$ nm, period $d_x = 904$ nm and groove width $c = 226$ nm. The incident plane is perpendicular to the lines of the grating, and the angle of incidence is $\theta = 4°$. The guide supports a TE and a TM mode. (*a*) Reflection factor versus wavelength for both s (full curve) and p (broken curve) polarizations. (*b*) Reflection factor versus angle of polarization δ.

issue is thus to find gratings that support eigenmodes whose dispersion relations present points of degeneracy.

Intersection of the dispersion relations can be obtained easily with a structure that supports TE and TM modes. Under a classical mount these two dispersion relations are independent and can cross with well chosen grating parameters. In this case, one obtains peaks of reflectivity that reach one for all angles of polarization (figures 4(*a*) and (*b*)). It yields a filter for unpolarized light working under oblique incidence [7].

Degeneracy of the dispersion relation can also occur with structures supporting only one mode. When the height of the grating tends to zero, the dispersion relation tends toward that of the planar guide. At the boundaries of the Brillouin zone we observe degenerate points coming from the crossing of the various bands. The physical meaning of the intersection, at a given wavenumber $k^{g,planar}$, of band (0, 0) and band (m, n) is that the structure supports two guided waves propagating along different directions, with wavevector $\vec{\underline{\underline{k}}}^{g}$ and $\vec{\underline{\underline{k}}}^{g}_{R} = \vec{\underline{\underline{k}}}^{g} + m\vec{a}_1 + n\vec{a}_2$ with $|\vec{\underline{\underline{k}}}^{g}| = |\vec{\underline{\underline{k}}}^{g}_{R}|$ (see figure 5). When the height of the periodic perturbation is increased, these two guided waves are coupled to each other, through the grating. Indeed, one can regard the mode with wavevector $\vec{\underline{\underline{k}}}^{g}_{R}$ as the reflection of the incident mode, $\vec{\underline{\underline{k}}}^{g}$, on the (m, n) Bragg plane of the 2D grating. As a result, one obtains two pseudo-periodic eigenmodes, that are a combination of the guided waves, with different photonic energies [9]. Thus, the dispersion relation presents a band-gap and the point of degeneracy disappears. The energy gap is linked to the interaction strength between the two guided waves. In order to preserve the points of degeneracy, it is necessary to look for a configuration in which this interaction is null. This is possible by eliminating the reflection on the Bragg plane, for example by creating a Brewster effect. This Brewster effect is well known in a periodic multilayer used as a dielectric mirror. For a given angle of incidence, in p polarization, the incident beam can be totally transmitted by the mirror. It corresponds to a degenerate point of the dispersion relation of the Bloch waves of the infinite periodic system [10].

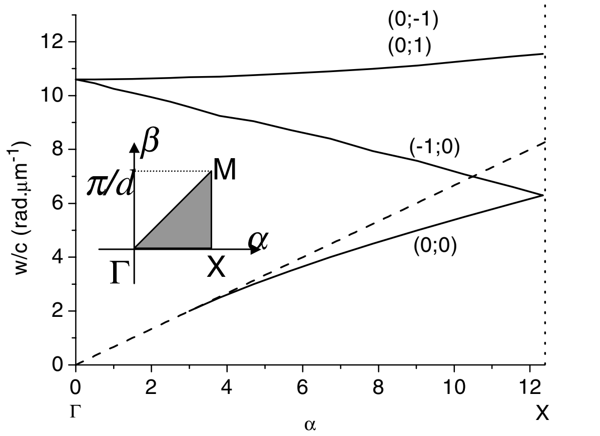

Figure 5. Dispersion relation of the first TE guided mode propagating in a layer with refractive index $n_c = 3$ and thickness $e_c = 80$ nm deposited on a substrate with refractive index $n_s = 1.5$. The period of the fictitious grating (the depth of which is zero) is $d = 507$ nm. The numbers of the bands are indicated in brackets.

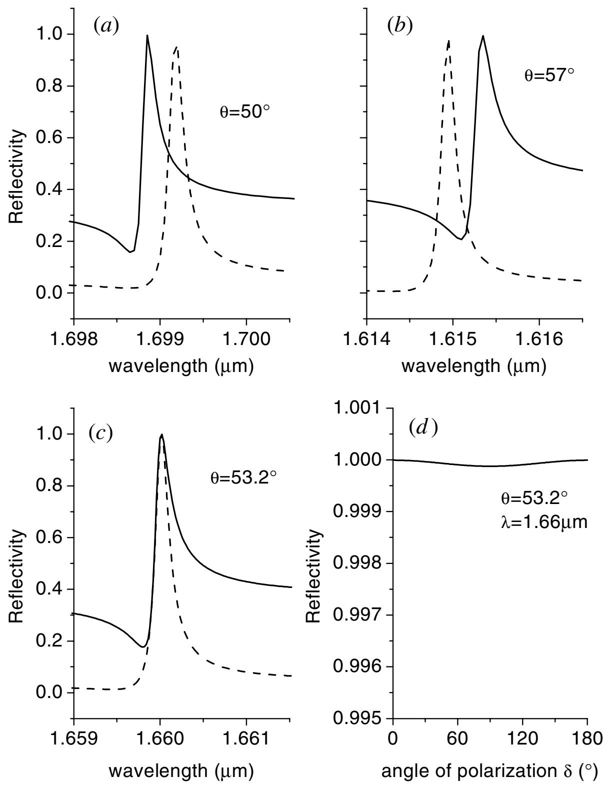

Figure 6. Reflection factor of a 1D lamellar grating, with refractive indices of the substrate $n_s = 1.2$, layer $n_c = 2.0$, thickness $e_c = 110$ nm, groove depth $h = 10$ nm, period $d_x = 1726$ nm and groove width $c = 1000$ nm. The incident plane is parallel to the lines of the grating. (*a*) Reflection factor versus the wavelength for both s (full curve) and p (broken curve) polarization when the angle of incidence is $\theta = 50°$. (*b*) The angle of incidence is $\theta = 57°$. (*c*) The angle of incidence is $\theta = 53.2°$. (*d*) Reflection factor versus the angle of polarization δ when the angle of incidence is $\theta = 53.2°$ and the wavelength $\lambda = 1.66$ μm.

One can find the same phenomenon with a TE guided wave propagating in a 1D lamellar grating along the $\underline{\underline{\vec{k}}}^{g}$ direction. Indeed these two configurations are similar from a conceptual point of view. The Brewster effect being a consequence of the anisotropy of the field radiated by electric dipoles, it is obtained when the angle Θ between the wavevector of the reflected mode, $\underline{\underline{\vec{k}}}^{g}_{R} = \underline{\underline{\vec{k}}}^{g} + m\vec{a}_1$ (with $|\underline{\underline{\vec{k}}}^{g}| = |\underline{\underline{\vec{k}}}^{g}_{R}|$) and that of the incident mode, $\underline{\underline{\vec{k}}}^{g}$, is close to 90°. A pseudo-Brewster effect is also observed with a 2D grating when the direction of propagation of the mode, viewed as a reflection on the (m, n) Bragg plane, is quasi-orthogonal to that of the incident guided wave. Actually, the angle Θ between the two directions of propagation of the guided waves that minimizes the Bragg plane 'reflection' depends, of course, on the various parameters of the structure, but, in general, it is about 90°. To obtain a two-times degenerate point in the dispersion relation (and a real Brewster effect), it is also necessary that the eigenmodes are independent, i.e. that their overlap integral is null. This condition is fulfilled when the structure is symmetrical with respect to the bisecting line $\hat{u}$ of $\underline{\underline{\vec{k}}}^{g}_{R}$ and $\underline{\underline{\vec{k}}}^{g}$, so that the eigenmodes are either symmetrical or antisymmetrical with respect to $\hat{u}$. In this case, one can find an angle Θ for which the energy gap is rigorously zero.

A 1D grating is the simplest grating with a plane of symmetry. The modes that are excited in the extreme conical mount ($\phi = 90°$) are symmetric and antisymmetric about this plane. By choosing the parameters of the structure so that the angle Θ between $\underline{\underline{\vec{k}}}^{g}_{R} = \underline{\underline{\vec{k}}}^{i} + \vec{a}_1$ and $\underline{\underline{\vec{k}}}^{g} = \underline{\underline{\vec{k}}}^{i} - \vec{a}_1$ is close to 90°, one can find an intersection point in the dispersion relations of the symmetric mode and the antisymmetric mode. In figures 6(*a*)–(*c*) we plot the reflectivity versus the wavelength before, after and on the point of intersection for which the s and p resonances occur for the same wavelength λ_0. Note that, with an asymmetric 1D grating with respect to the z axis, it is impossible to superpose the two peaks since the eigenmodes are not independent. The reflectivity versus δ for the wavelength λ_0 is plotted in figure 6(*d*). It illustrates that this structure can filter unpolarized light under non-normal incidence. Yet, the condition of orthogonality of the wavevectors of the modes that are excited with a 1D grating under conical incidence requires an effective index of the guided mode smaller than $\sqrt{2}$, when the superstrate is air. This is difficult to obtain in practice because the effective index of the guided mode is limited by the index of the substrate. Thus, we have also considered 2D gratings. The condition of orthogonality of the wavevectors of the two guided waves is strictly realized for a 2D grating with a square cell under normal incidence. Yet, by varying the parameters of the grating, degenerate points of the dispersion relation can be found for Θ slightly different from 90° and thus non-normal incidence can be used. A filter for unpolarized light has been obtained in this way in [7]. A triangular lattice seems to be a better means to excite modes with orthogonal wavevectors out of normal incidence since the reciprocal vectors of the lattice are not normal to each other. The triangular lattice is also interesting for its high order of symmetry. It has six planes of symmetry, which can be classified into two families: the planes which contain axes of the real lattice and the planes which contain axes of the reciprocal lattice. If the plane of incidence is along one axis of the reciprocal lattice, then the angle between the reciprocal vectors used to excite the modes is 120° (see figure 7(*a*)). A simple calculation gives the angle of incidence for which the wavevectors of the excited modes are

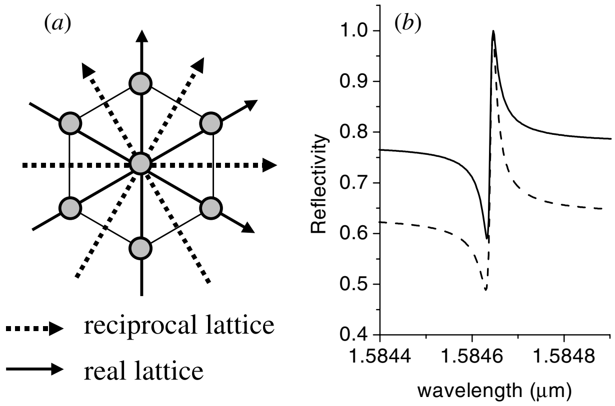

Figure 7. (*a*) Real and reciprocal lattice for a triangular cell. (*b*) Reflectivity factor versus the wavelength for both *s* (full curve) and *p* (broken curve) polarizations of a 2D grating with triangular cell. The parameters are: refractive index $n_s = 1.0$ for the substrate, $n_c = 3.5$ for the layer and n_c for the grating bumps, grating period $d = 1057.9$ nm, depth $h = 10$ nm, layer thickness $e_c = 90$ nm and radius of the circular bumps $r = 264$ nm.

perpendicular versus the period, the propagation constant of the guided mode and the wavelength. In figure 7(*b*) we managed to superpose the *s* and *p* peaks for an angle of incidence equal to 31.3°. If the incident plane is parallel to one axis of the real lattice, then the angle between the reciprocal vectors used to excite the modes is 60°. The *s* and *p* resonances can occur for the same wavelength for an angle of incidence close to 60°. Note that the superposition of the *s* and *p* resonances does not depend on the grating parameters (as long as the latter presents the required symmetry properties). It suffices to vary the angle of incidence so that Θ is modified about 90° to find the degenerate point.

7. Conclusion

We have presented a phenomenological study of the behaviour of the reflectivity of a resonant grating versus the polarization of the incident beam. In particular, we can now deduce the maximum and minimum values of the reflectivity from the response of the device to *s* and *p* polarization for a given incidence. One only has to calculate the eigenvalues of the energy reflection matrix expressed in the *s* and *p* vectors. The eigenvectors give the orthogonal polarizations (linear, circular or elliptic) for which the extremum values of the reflectivity are obtained.

When the grating supports one guided mode, we showed that one eigenvalue presents a complex root and a complex pole in the complex plane of the wavenumber. Moreover, if the structure is symmetric about the normal to the waveguide, this eigenvalue reaches unity at the resonance, while the other remains close to the reflectivity of the device outside the resonance. Thus, in order to design a high efficiency filter for all polarizations, both eigenvalues have to reach unity. With this aim, one has to find two modes that can be excited for the same incident wavevector and with equal real part of the wavenumber. This corresponds to a crossing point of intersection in the dispersion relations of the guided waves.

Pseudo-degenerate points of the relation dispersion appear, in particular, at the extremity of the Brillouin zone when the incident beam couples, via the grating, to two modes propagating in different directions. Unfortunately, the interaction between the two modes generally removes the degeneracy. We explained the relationship between the two modes in terms of reflection on a Bragg plane of the photonic crystal created by the grating. Then, we found that two TE guided modes are totally independent if they propagate along the Brewster directions. Yet, if we develop further the analogy with the reflection on a multilayer stack, it is obvious that the interaction (or Bragg plane reflection) can vanish in other configurations. In particular, for TM guided waves, independent modes can be found for a particular filling factor of the grating [6]. A perturbative approach [11] could be used to derive analytical expressions of the coupling between the two modes. Our work is in progress in that direction.

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