## Unpolarized narrow-band filtering with resonant gratings

Anne-Laure Fehrembach and Anne Sentenac

Institut Fresnel, CNRS UMR6133, Faculté de Saint Jérome, 13397 Marseille Cedex, France

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Periodically corrugated planar waveguides present sharp peaks in their reflectivity spectrum due to the excitation, by the incident beam, of an eigenmode of the structure. This property can be used to design free-space narrow-band filters. However, the extreme sensitivity of the resonances with respect to the angle and polarization under oblique incidence, has prevented, up to now, the development of efficient filters with this approach. We succeed in getting round these limitations by exiting four eigenmodes at the same time, and by modifying deeply the eigenmodes properties with an appropriate periodic corrugation. © 2005 American Institute of Physics.

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The presence of peaks in the reflectivity spectra of resonant gratings (RG), known as Wood anomalies, is due to the excitation by the incident beam of an electromagnetic eigenmode, typically a leaky guided wave, which was inaccessible prior to the imposition of the periodic perturbation on the waveguide.<sup>1</sup> An important theoretical and experimental work has been conducted to take advantage of this phenomenon to design a new type of free-space filters.<sup>2-4</sup> The potential of these structures, for filtering purpose, is all the more interesting than the maximum of reflectivity reaches 100% at the center of the peak<sup>2</sup> and the spectral width of the peak decreases with the etching depth of the periodic corrugation.<sup>5,6</sup> Unfortunately, it turns out that some characteristics of the resonance phenomenon are severe drawbacks for practical filtering applications. First, the resonance peak depends on the polarization of the incident beam. In general, an incident polarization always exists for which the excitation of the eigenmode is impossible.<sup>7</sup> Second, the angular width of the peak is usually proportional to the spectral width,<sup>5,6</sup> roughly 0.01° for 0.1 nm. Thus, the efficiency of the filter is dramatically disminished if one uses standard collimated incident beam with a typical divergence of  $0.2^{\circ}$ .<sup>8</sup> These limitations have severely hindered the development and use of RGs as efficient unpolarized filters. Yet, a notable exception has been observed with two-dimensional resonant square gratings illuminated under normal incidence. In this highly symmetrical configuration, the resonance occurs whatever the polarization of the incident beam<sup>9</sup> and the angular tolerance of the spectral peak can be improved with special design of the structure.<sup>6,8</sup> Unfortunately, in filtering optical devices, the normal incidence regime is usually avoided because it requires the use of beam separators which yields a loss of efficiency. In this work, we show how the properties of normal incidence can be recreated under oblique incidence.

We consider a reference planar structure consisting of a lossless dielectric waveguide deposited on a substrate and described by its permittivity  $\boldsymbol{\epsilon}_{ref}(z)$ . The slab supports a TE guided wave whose electric field can be expressed as  $\mathbf{E}(\mathbf{r},t) = \exp(i\mathbf{k}_{ref},\mathbf{r}_{\parallel} - i\omega_{ref}t)\mathbf{F}(z)$  where  $\mathbf{r} = (\mathbf{r}_{\parallel},z)$  and the real spatial and temporal frequencies of the mode  $(k_{ref}, \omega_{ref})$  are linked by their reference dispersion relation. To form a resonant grating, see Fig. 1, the structure is periodically perturbed in a region G of height *h* above the slab so that the permittivity inside G can be written as:

$$\boldsymbol{\epsilon}_{G}(\mathbf{r}) = \sum_{K \in \Omega} \boldsymbol{\epsilon}_{\mathbf{K}} \exp(i\mathbf{K} \cdot \mathbf{r}_{\parallel}), \qquad (1)$$

where  $\Omega$  is the reciprocal space of the periodic lattice. The eigenmodes of the perturbed structure are now pseudoperiodic and their electric field can be cast in the form

$$\mathbf{E}(\mathbf{r},t) = \exp(i\mathbf{k}_{\parallel} \cdot \mathbf{r}_{\parallel} - i\omega t) \sum_{\mathbf{K}\in\Omega} \mathbf{F}_{\mathbf{K}}(z) \, \exp(i\mathbf{K} \cdot \mathbf{r}_{\parallel}).$$
(2)

Hereafter, we consider eigenmodes whose spatial frequency  $\mathbf{k}_{\parallel}$  is real and inside the reduced Brillouin zone. Consequently, to ensure that the homogeneous Maxwell equations have non-null solutions, the temporal frequency  $\omega$  is a priori complex. When the perturbation is small, the pseudoperiodic eigenmode is very similar to the mode of the reference structure. In other words, reciprocal lattice vectors Q exist such that  $|\mathbf{k}_{\parallel} + \mathbf{Q}| \approx k_{\text{ref}}$  and  $|\omega| \approx \omega_{\text{ref}}$ . In this case, the Fourier components of the eigenmode are dominated by Fo. To excite the leaky eigenmode, the system is illuminated by a plane wave coming from the superstrate,  $\mathbf{E}_{inc}(\mathbf{r},t)$ = $\mathbf{E}_0 \exp(i\mathbf{k}_{inc},\mathbf{r}-i\omega_{inc}t)$ . The angle of incidence is chosen so that the projection of  $\mathbf{k}_{inc}$  onto the (0xy) plane is  $\mathbf{k}_{\parallel}$  while the incident frequency  $\omega_{inc}$  is close to  $\text{Re}(\omega)$  (the real part of  $\omega$ ). The resonance phenomenon yields a peak in the reflectivity spectrum that reaches 100% only if the polarization of the incident beam is similar to that of the propagative component of the mode,  $\mathbf{F}_0$ .<sup>10</sup> To obtain a resonance peak that reaches 100%, whatever the polarization of the incident beam, one needs to excite two degenerate modes (with necessarily orthogonal polarization).<sup>10,11</sup> The central frequency of the peak is close to  $\text{Re}(\omega)$  while the spectral width is linked to  $\text{Im}(\omega)$ (the imaginary part of  $\omega$ ). In the limit of small h, Im( $\omega$ ) is proportional to  $h^2 |\epsilon_0|^2$  where  $\epsilon_0$  is the grating coefficient responsible for the coupling of the incident beam to the eigenmode.<sup>10</sup> The angular sensitivity of the resonance can be

 $E_{inc}$   $(k_{inc}, \omega_{inc}) \wedge^{Z}$  (h) (h)

FIG. 1. Description of the resonant grating.

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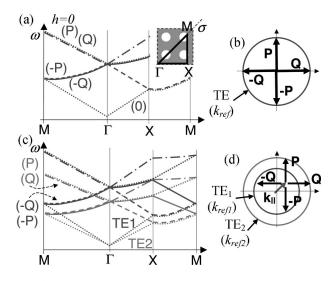
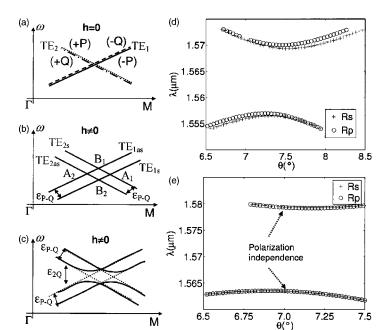


FIG. 2. (a) Dispersion relation of the pseudoperiodic mode in a virtual 2D monomode resonant grating; (b) excitation via  $+\mathbf{Q}$ ,  $-\mathbf{Q}$ , and  $+\mathbf{P}$ ,  $-\mathbf{P}$  of the TE reference mode, under normal incidence; (c) dispersion relation of the pseudoperiodic modes in a virtual 2D bi-mode resonant grating; (d) excitation via  $-\mathbf{Q}$  and  $-\mathbf{P}$  of the TE<sub>1</sub> mode and via  $+\mathbf{Q}$  and  $+\mathbf{P}$  of the TE<sub>2</sub> mode under oblique incidence.

determined by modifying the angle of incidence (and thus  $\mathbf{k}_{\parallel}$ ) and studying the variation of  $\omega(\mathbf{k}_{\parallel})$ . A good angular tolerance is obtained if the mode is locally dispersionless.<sup>6</sup>

With these theoretical considerations in mind, one can explain why a resonant square grating illuminated under normal incidence may present a narrow-band, polarizationindependent, angular-tolerant resonance peak. Figure 2(a) plots the dispersion relation of a TE guided mode in a virtual 2D grating (*h* tends to zero). We represented only the zero branch and the branches that are deduced with a translation vector **P**, **Q**,  $-\mathbf{P}$ , and  $-\mathbf{Q}$ . Along the  $\Gamma$ M direction, the (*P*) and (*Q*) branches and the (-P) and (-Q) branches are twofold degenerated. A normally incident beam will excite the mode at the  $\Gamma$  point of the dispersion relation,  $\mathbf{k}_{\parallel}=0$ . At this point, the reference mode is fourfold degenerated. It is basically a combination of four reference guided waves with wave vectors  $\pm \mathbf{Q}$  and  $\pm \mathbf{P}$ , where ( $\mathbf{Q}, \mathbf{P}$ ) are orthogonal recip-



rocal space wave vectors, such that  $|\mathbf{P}| = |\mathbf{Q}| \approx k_{\text{ref}}$  as illustrated in Fig. 2(b). When the grating height is increased, the counterpropagative guided waves, with parallel electric field, couple to each other via the grating coefficients  $\epsilon_{20} = \epsilon_{2P}$ . The dispersion relation is then flattened about the  $\Gamma$  point of the Brillouin zone and local frequency gaps are opened. Yet, the degeneracy is not totally removed. Indeed, if the structure is symmetric with respect to the z axis, the fourfold degenerate reference mode turns into four locally dispersionless eigenmodes among which two are degenerate.<sup>13</sup> The excitation of these dispersionless degenerate modes yields a peak in reflection that is polarization independent.<sup>9</sup> The spectral and angular widths of the resonance depending on  $\epsilon_0$  and  $\epsilon_{20}$ , respectively, they can be tuned independently by appropriate design of the grating pattern. Doubly periodic patterns<sup>6</sup> which satisfy  $|\epsilon_0| \ll |\epsilon_{20}|$  have been shown to exhibit narrowband resonances with good angular tolerance.

The issue now is to somehow retain these properties for  $\mathbf{k}_{\parallel} \neq 0.^{11}$  When the reference slab supports only one guided wave in the frequency range of interest, we observe that degenerate dispersionless modes appear solely at the highly symmetric points of the Brillouin zone  $\Gamma$ , M, and X which are of limited interest since they correspond either to normal incidence or to Littrow mounting (in this case, several orders are diffracted). Thus, to introduce new possibilities for the tailoring of the resonance peak, we have considered a reference slab that supports two TE guided waves denoted by TE<sub>1</sub> and TE<sub>2</sub> with spatiotemporal frequencies  $(k_{ref1}, \omega_{ref1})$  and  $(k_{\rm ref2}, \omega_{\rm ref2})$ . The periodic reference dispersion relation in this more complex configuration is plotted in Fig. 2(c). Along the  $\Gamma M$  line, a fourfold degenerate point exists at the crossing of the branches of TE<sub>1</sub>, (-Q)(-P), and of TE<sub>2</sub>, (+Q)(+P), when  $\mathbf{k}_{\parallel}$  satisfies simultaneously  $|\mathbf{k}_{\parallel} - \mathbf{Q}| = |\mathbf{k}_{\parallel} - \mathbf{P}| \approx k_{\text{ref1}}$  and  $|\mathbf{k}_{\parallel}|$  $+\mathbf{Q} = |\mathbf{k}_{\parallel} + \mathbf{P}| \approx k_{\text{ref2}}$  see Fig. 2(d). Using a perturbative analysis, it is possible to foresee the modification of this complex dispersion relation when h is increased, see Figs. 3(a)-3(c). First, the twofold degenerate branches of the TE<sub>1</sub> mode along the  $\Gamma M$  line separate into two curves corresponding to a symmetric  $TE_{1s}$  and antisymmetric  $TE_{1as}$  modes. The field of these modes are, respectively, symmetric and antisymmetric about the plane defined with the  $\Gamma M$  line and the z axis,

FIG. 3. (a), (b), and (c) modification of the dispersion relation of Fig. 2(c) along the  $\Gamma$ M line when *h* is increased; (d),(e) loci of the maximum of reflectivity calculated for 2D resonant gratings, as described in Fig. 4(a), and illuminated by s and p polarized incident waves with  $\mathbf{k}_{inc}$  fixed along the  $\Gamma$ M line; (d) width of the square holes are A=352.10 nm, B=175.56 nm, and C=276.28 nm.  $|\boldsymbol{\epsilon}_{2Q}|=0.507$ ,  $|\boldsymbol{\epsilon}_{\mathbf{P}-\mathbf{Q}}|=0.255$ ,  $|\boldsymbol{\epsilon}_{\mathbf{Q}}|=0.148$ ; (e) width of the square holes are now A=225.92 nm, B=175.56 nm, and C=276.28 nm.  $|\boldsymbol{\epsilon}_{2Q}|=0.506$ ,  $|\boldsymbol{\epsilon}_{\mathbf{P}-\mathbf{Q}}|=0.0, |\boldsymbol{\epsilon}_{\mathbf{Q}}|=0.148$ .

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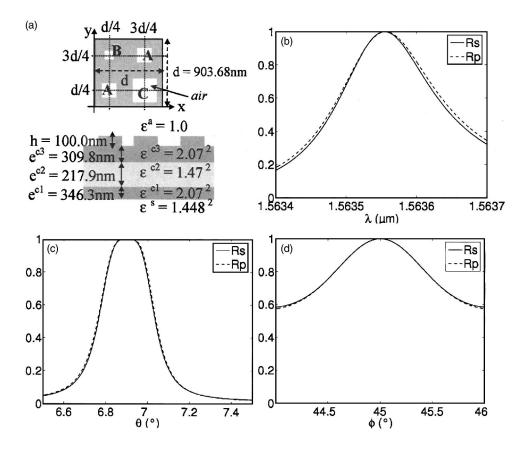


FIG. 4. (a) Description of a 2D resonant grating that supports two TE modes (top and side view). The grating cell is composed of four patterns denoted by A, A, B, C. The patterns are centered about (id/4, jd/4) with  $(i \in \{1,3\} \text{ and } j \in \{1,3\})$ . (c), (d), and (e) the pattern is the same as in Fig. 3(e), (b) reflectivity vs the incident wavelength for p and s polarized incident wave; (c) reflectivity vs the incident angle  $\theta$ ; (d) reflectivity vs the angle  $\phi$ .

which is a plane of symmetry of the structure. The frequency difference between these curves depends on  $\epsilon_{P-O}$  which is responsible for the coupling between the reference mode propagating along  $\mathbf{k}_{\parallel} - \mathbf{Q}$  and that propagating along  $\mathbf{k}_{\parallel} - \mathbf{P}^{10}$ . The same behavior is observed for the TE<sub>2</sub> mode. At the four intersection points A<sub>1</sub>, A<sub>2</sub>, B<sub>1</sub>, and B<sub>2</sub> of these nondegenerate curves, two different behaviors can be observed. If the modes have the same symmetry properties with respect to  $\Gamma M$  (points A<sub>1</sub> and A<sub>2</sub>), they couple to each other and the degeneracy is removed. The width of the frequency gap between the resulting modes is mainly governed by the grating permittivity coefficients which link together the counterpropagative guided waves with almost parallel electric fields. If  $\mathbf{k}_{\parallel}$  is not far from 0, this coefficient is the same as that obtained under normal incidence,  $\epsilon_{20}$ . On the other hand, if the modes have opposite symmetry properties (points B<sub>1</sub> and  $B_2$ ) they do not couple to each other and the degeneracy remains as seen in Fig. 3(c). To create a locally dispersionless almost degenerate mode, it is sufficient to disminish the frequency difference between the symmetric and antisymmetric branches while widening the local frequency gaps. This is done by imposing a periodic perturbation whose Fourier coefficients satisfy  $|\epsilon_{20}| \ge |\epsilon_{P-0}|$ . Moreover, to obtain a narrow spectral resonance peak one should take  $|\epsilon_0| \ll |\epsilon_{20}|$ . This analysis is confirmed in Figs. 3(d) and 3(e) where we plot a representation of the dispersion relation, along the  $\Gamma M$ line, of the antisymmetric and symmetric modes of two different resonant gratings. The loci of  $\omega$  is obtained by searching the maximum of reflectivity of the gratings illuminated by p and s polarized wave. The simulations are performed with a rigorous Fourier modal method.<sup>12</sup> In Fig. 3(d) the grating pattern has been chosen so that  $|\epsilon_{2\mathbf{Q}}| \approx 2|\epsilon_{\mathbf{P}-\mathbf{Q}}|$  while in Fig. 3(e)  $|\epsilon_{\mathbf{P}-\mathbf{Q}}|=0$  and  $|\epsilon_{2\mathbf{Q}}|$  is the same as in Fig. 3(d). As expected, we observe the presence of dispersionless branches that become almost degenerate when  $|\epsilon_{2\mathbf{Q}}| \gg |\epsilon_{\mathbf{P}-\mathbf{Q}}|$ , see Fig. 3(e). The relatively complex tri-atom pattern of the structure in Figs. 3(d) and 3(e) is depicted in Fig. 4(a).

Figure 4(b)–4(d) plot the reflectivity curves of the optimized tri-atom grating of Fig. 3(e) with respect to the wavelength,  $\theta$  and  $\phi$  for p and s polarized incident wave. We observe that the reflectivity spectrum presents a peak, whatever the incident polarization, with a very narrow spectral width and a relatively good angular tolerance. Note, that if there are fabrication errors on the period, slab thickness, or holes sizes, the angularly tolerant, polarization independent resonance will still exist, though at a different frequency and incidence angle. In our opinion, this result could yield a new kind of free-space narrow-band unpolarized filters, functionning under oblique incidence with a good efficiency even with standard collimated incident beams.

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