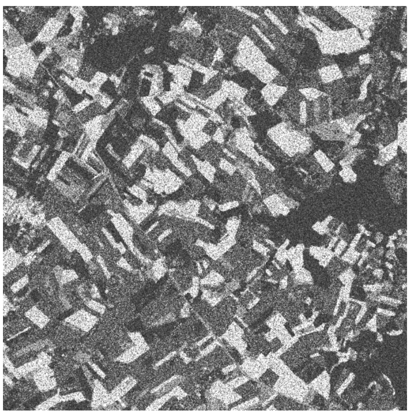
SAR Image Segmentation using Generalized Pairwise Markov Chains

Stéphane Derrode[#], Wojciech Pieczynski[*]

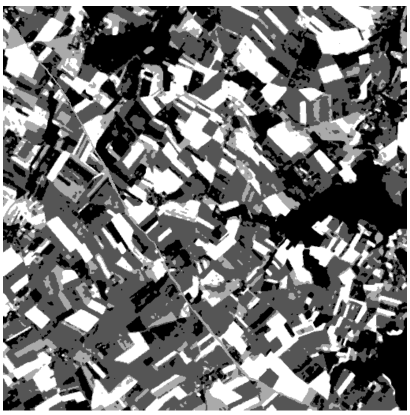
[#] Ecole National Supérieure de Physique de Marseille, GSM
stephane.derrode@enspm.u-3mrs.fr

[*] Institut National des Télécommunications, CITI
wojciech.pieczynski@int-evry.fr

Unsupervised classification of radar images



Radar image **Y**



Class image **X**

- Unknown distribution families
- Unknown radiometric parameters
- Unknown class image model
- Unknown spatial regularity parameters

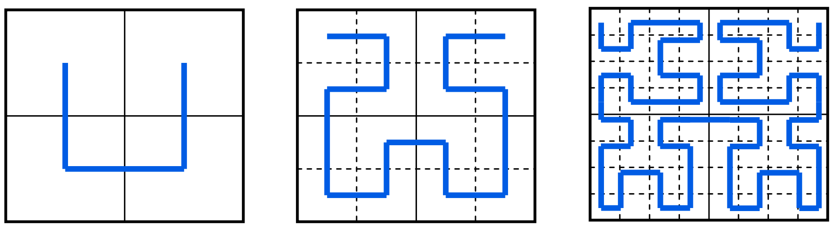
→ Application of the new “Pairwise Markov Chain” model to unsupervised SAR image segmentation.

Outline

1. PMC model
   - Introduction / Notation
   - Pairwise Markov Chain
   - Relation to Hidden Markov Chain
   - PMC parameters
2. Parameter estimation : the ICE method
   - Iterative Conditional Estimation (ICE)
   - Estimation of joint probabilities
   - Estimation of noise parameters (Gaussian and Generalized cases)
   - Pearson system of distributions
3. Segmentation of simulated and radar images
   - Segmentation of noisy simulated images
   - Segmentation of a JERS image
4. Conclusion and further work

Introduction / Notations

- 2D image -> 1D sequence



Hilbert-Peano scan

- Markov chain

$$P(X_n = x_n | X_1 = x_1, \ldots, X_{n-1} = x_{n-1}) = P(X_n = x_n | X_{n-1} = x_{n-1})$$

- Process

$X = (X_1, \ldots, X_N)$ Unknown class process $X_n \in \Omega = \{1, \ldots, K\}$

$Y = (Y_1, \ldots, Y_N)$ Observed image $Y_n \in R$

Pairwise Markov Chain (PMC)

- Pairwise process $Z = (Z_1, \ldots, Z_N)$ with $Z_n = (X_n, Y_n)$

- Distribution of a PMC $p(z) = p(z_1)\, p(z_2 | z_1) \ldots p(z_N | z_{N-1})$

- Stationary PMC $p(z_1, z_2) = p(i, j)\, f_{i,j}(y_1, y_2)$

$$\begin{cases} \text{Initial probabilities} & p(z_1) = \sum_{j=1}^{K} p(i,j) \int_R f_{i,j}(y_1, y_2)\, dy_2 = \sum_{j=1}^{K} p(i,j)\, f_{i,j}(y_1) \\ \text{Transition matrix} & p(z_2 | z_1) = \dfrac{p(z_1, z_2)}{p(z_1)} = \dfrac{p(i,j)\, f_{i,j}(y_1, y_2)}{\sum_{j=1}^{K} p(i,j)\, f_{i,j}(y_1)} \end{cases}$$

Relation to Hidden Markov Chain (HMC)

- Distribution of a HMC

$$p(z) = p(x, y) = p(x_1)\, f_{x_1}(y_1)\, p(x_2 | x_1)\, f_{x_2}(y_2) \ldots p(x_N | x_{N-1})\, f_{x_N}(y_N)$$

- Distribution of a stationary HMC

$$p(z_1, z_2) = p(i, j)\, f_i(y_1)\, f_j(y_2)$$

- PMC vs HMC

HMC can not take into account situations in which $f_{i,j}(y_1)$ does depend on j.

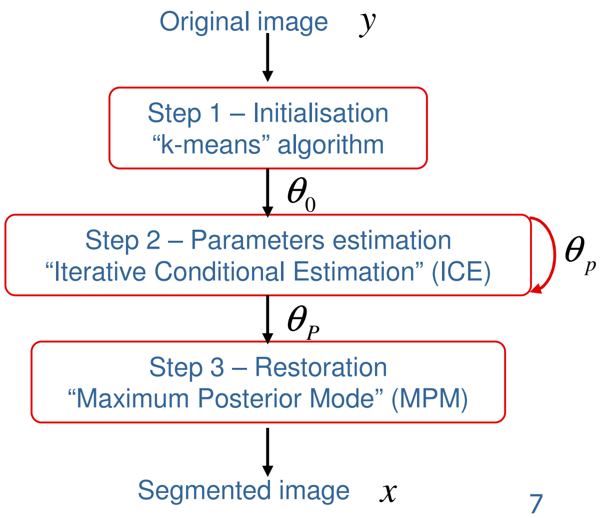
→ SAR image segmentation

PMC can take into account more complex spatially correlated speckle noise.

It has been shown that the PMC model is strictly more general than the HMC one.

PMC Parameters

- Parameters to be estimated
  - Joint probabilities (γ) $p(i,j)$
  - Noise parameters (δ) $f_{i,j}(y_1, y_2)$
    - "Gaussian" : $\mu^1_{i,j}, \mu^2_{i,j}, \sigma^1_{i,j}, \sigma^2_{i,j}, \rho$
    - "Generalized" : Pearson system
- ICE based segmentation algorithm $\theta = \{\delta, \gamma\}$



Iterative Conditional Estimation

- ICE assumptions
  1. There exists an estimator of θ from Z
  2. Or, the conditional expectation $E_\theta\left[\hat{\theta} \middle| Y = y\right]$ is computable or simulation of $X|Y = y$ is feasible.
- Iterative procedure
  1. Initialize $\theta = \theta_0$
  2. For $q > 0$
     - if the conditional expectation is computable :

$$\theta_{q+1} = E_\theta\left[\hat{\theta} \middle| Y = y\right]$$

→ Joint probabilities

     - else, simulate L realization $x_1, \ldots, x_L$ of $X|Y = y$, and

$$\theta_{q+1} = \frac{1}{L}\sum_{l=1}^{L} \hat{\theta}(x_l, y)$$

→ Mixture parameters

Estimation of joint probabilities

- «Forward» and «Backward» probabilities

$$\alpha_1(i) = p(x_1 = i, y_1)$$
$$\alpha_{n+1}(i) = \sum_{j=1}^{K} \alpha_n(j) p\left(x_{n+1} = i, y_{n+1} \middle| x_n = j, y_n\right)$$

$$\beta_N(i) = 1$$
$$\beta_n(i) = \sum_{j=1}^{K} \beta_{n+1}(j) p\left(x_{n+1} = j, y_{n+1} \middle| x_n = i, y_n\right)$$

- Joint *a posteriori* probabilities

$$\Psi_n(i,j) = p(x_n = i, x_{n+1} = j | y) = \frac{\alpha_n(i) p\left(x_{n+1} = j, y_{n+1} \middle| x_n = i, y_n\right)\beta_{i+1}(j)}{\sum_{(\omega_1,\omega_2)\in\Omega^2} \alpha_n(\omega_1) p\left(x_{n+1} = \omega_2, y_{n+1} \middle| x_n = \omega_1, y_n\right)\beta_{n+1}(\omega_2)}$$

- Marginal *a posteriori* probabilities

$$\Phi_n(i) = p(x_n = i | y) = \sum_{\omega\in\Omega} \Psi_n(i,\omega) = \frac{\alpha_n(i)\beta_n(i)}{\sum_{\omega\in\Omega} \alpha_n(\omega)\beta_n(\omega)}$$

→ Bayesian MPM restoration of X

- Conditional expectation of $p(i, j)$

$$p^{q+1}(i,j) = E_{\theta_q}\left[\hat{p}(i,j)(Z) \middle| Y = y\right] = \frac{1}{N-1}\sum_{n=1}^{N-1} \Psi_n^q(i,j)$$

Mixture parameters estimation

The parameters of the K^2 families $f_{i,j}(y_1, y_2)$ are not known and must be estimated from $Y = y$.

The conditional expectation of $\hat{\delta}(X,Y)$ are not computable, and estimation is performed using simulation of $X|Y = y$, according to :

$$p\left(x_{n+1} = i \middle| x_n = j, y\right) = \frac{\Psi_n(i,j)}{\Phi_n(i)}$$

- "Gaussian" PMC
  - $f_{i,j}(y_1, y_2)$ are bi-dimensional Gaussian distributions
  - Classical estimators for $\mu_{i,j}$ and $\Gamma_{i,j}$.
- "Generalized" PMC
  - Non Gaussian 2D densities are difficult to obtain explicitly.
  - Method : 2D -> 1D

$$f_{i,j}(y_1, y_2) = C\, g_1(u_1)\, g_2(u_2)$$

where u_1 and u_2 are linear combination of y_1 and y_2

Pearson system of distributions

- Densities verifying the differential equation :

$$\frac{1}{f(x)}\frac{df(x)}{dx} = -\frac{a+x}{c_0 + c_1 x + c_2 x^2}$$

Coefficients a, c_0, c_1, c_2 can be expressed as functions of

$$\begin{cases} \beta_1 = \dfrac{\mu_3^2}{\mu_2^3}, \sqrt{\beta_1} \text{ skweness} \\ \beta_2 = \dfrac{\mu_4}{\mu_2^2}, \text{kurtosis} \end{cases}$$

where μ_2, μ_3, μ_4 are the centered moments of order 2, 3 and 4

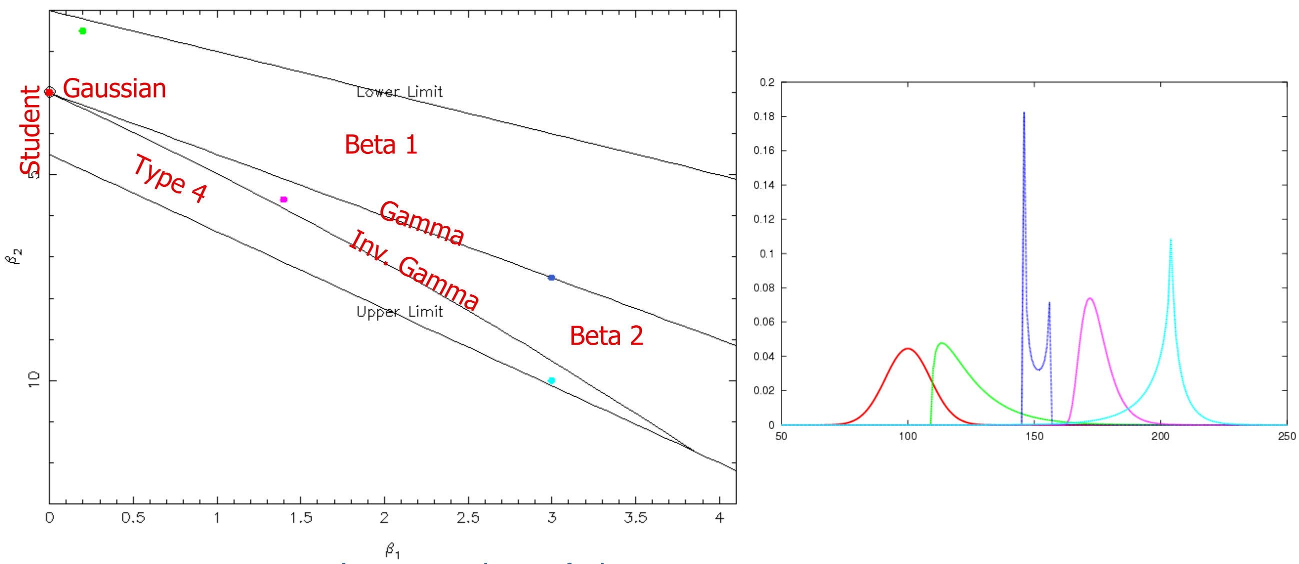
- Eight main families of distributions

Pearson system:

- Type 1,2 : Beta of the first kind
- Type 3 : Gamma
- Type 4 : No particular name
- Type 5 : Inverse Gamma
- Type 6 : Beta of the second kind
- Type 7 : Student't
- Type 8 : Gaussian

→ Pearson Diagram

Pearson diagram



Large variety of shapes

→ Symmetric and dissymmetric

→ Finite and semi-finite support

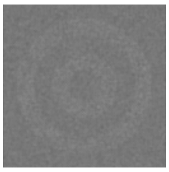
Noisy simulated images

- Original test image (Rings)

- Correlated noise
  1. Two white noises on the classes $\Omega = \{Black, White\}$
  2. Low-pass filtering with filter structure: $\frac{1}{3.8}\begin{pmatrix} 0 & 0.7 & 0 \\ 0.7 & 1 & 0.7 \\ 0 & 0.7 & 0 \end{pmatrix}$

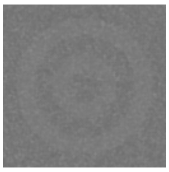
- Noisy simulated images

G-Rings



Law	μ_1	μ_2	μ_3	μ_4	β_1	β_2
Gaussian	120	49	0	7203	0	3
Gaussian	125	75	0	16875	0	3

P-Rings

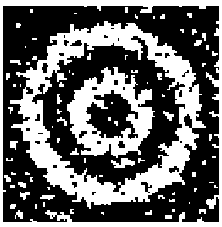


Law	μ_1	μ_2	μ_3	μ_4	β_1	β_2
Gamma	120	49	343	10805	1	4.5
Inv.Gamma	125	75	919	40159	2	7.1

Such images neither represent a HMC nor a PMC process.

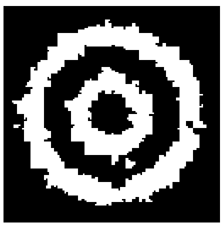
Segmentation results

- Gaussian PMC and HMC on the G-Rings image



HMC : ICE + MPM

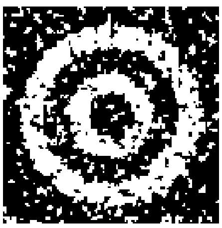
$\tau = 13.98\%$



PMC : ICE + MPM

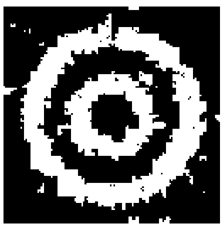
$\tau = 6.66\%$

- Generalized (Pearson) PMC and HMC on the P-Rings image



HMC : ICE + MPM

$\tau = 15.34\%$



PMC : ICE + MPM

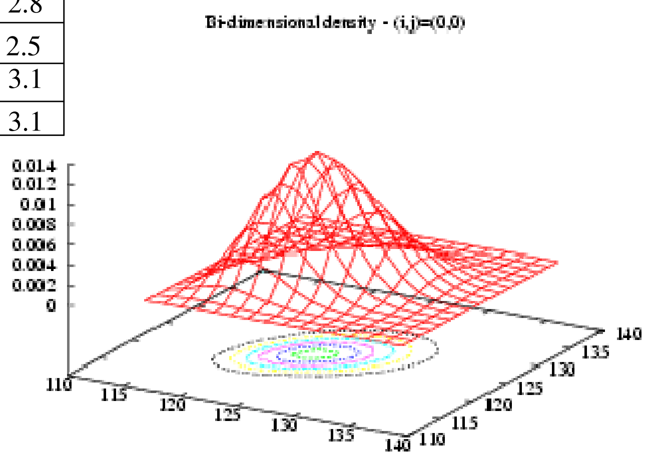
$\tau = 8.36\%$

Estimated parameters

- P-Rings images

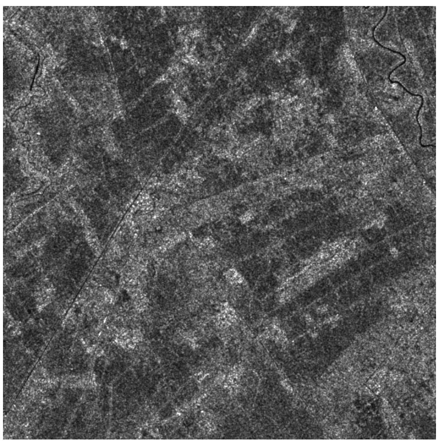
$f_{i,j}$	ρ	Law	μ_1	μ_2	μ_3	μ_4	β_1	β_2
$f_{0,0}$	0.53	Type IV	24.7	1.0	0.4	3.4	0.2	3.4
		Beta 1	26.5	1.0	0.4	3.2	0.1	3.2
$f_{0,1}$	0.60	Beta 1	37.2	1.0	0.9	2.9	0.7	3.0
		Beta 1	−22	1.0	0.7	3.1	0.5	3.2
$f_{1,0}$	0.35	Beta 1	24.9	1.0	0.7	2.7	0.5	2.8
		Beta 1	3.2	1.0	0.2	2.1	0.0	2.5
$f_{1,1}$	0.48	Beta 1	31.6	1.0	0.3	3.1	0.1	3.1
		Beta 1	31.6	1.0	0.3	3.1	0.1	3.1

$$\hat{p}(i,j) = \begin{pmatrix} 0.340 & 0.009 \\ 0.009 & 0.643 \end{pmatrix}$$



Segmentation of a Real SAR image

- JERS image of a rice plantation in Indonesia



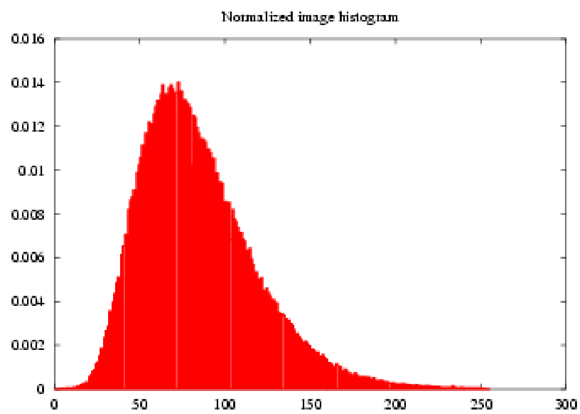


Image size : 512x512

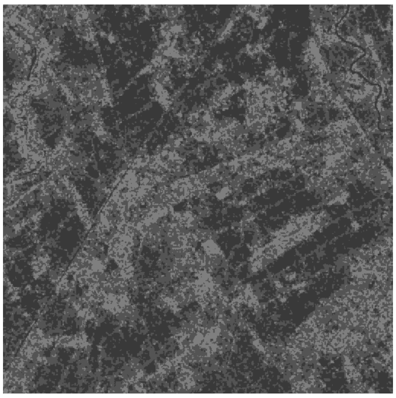
10 February, 1994

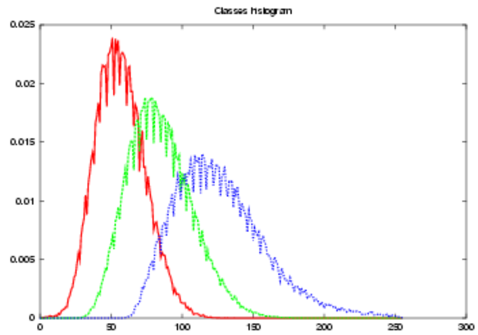
Early rice, Late rice and Other Cultivations

Segmentation into 4 classes

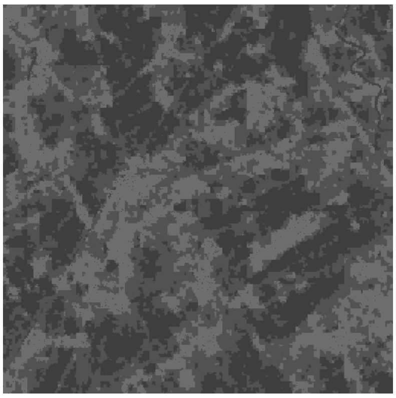
Segmentation results

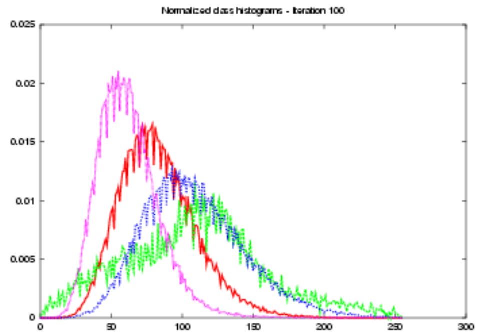
HMC : ICE + MPM
Time : 35 min.





PMC : ICE + MPM
Time : 2 h 40 min.





Conclusion and further work

1. Conclusion
   - Application of PMC to unsupervised radar image segmentation
   - The PMC model can take into account :
     - Non gaussian noise
     - Spatially correlated speckle noise
   - PMC based segmentation can significantly improve the HMC based one.
   - PMC-ICE is more time-consuming than HCM-ICE
2. Improvements and further work
   - Extension of PMC to multisensor image segmentation
   - Multiresolution extension of PMC (Pairwise Markov Tree)
   - New Markov model : "Triplet Markov Chain"