GEOMETRIC SHAPE PRIOR TO REGION-BASED ACTIVE CONTOURS USING FOURIER-BASED SHAPE ALIGNMENT

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Abstract—In this paper, we present a novel method to incorporate geometric shape prior into region-based active contours. Prior knowledge is obtained from a reference shape. This shape reference is used to define a new energy term obtained through a Fourier-based shape alignment. The new energy is invariant with respect to Euclidean transformations. Experimental results show the ability of the geometric shape prior to constrain an evolving curve to meet target shape and to show the new method benefits on segmentation results in presence of occlusion and noise.

Index Terms—region-based active contours, shape prior, alignment, geometric transformations.

I. INTRODUCTION

Active contour methods have been introduced by Kass & al. in 1988 [1]. The principle of active contours is to move a curve iteratively minimizing an energy functional. The minimum is reached at an object boundaries. Active contour methods can be classified into two families: parametric [1], [2] and geometric [3], [5], [4] active contours. The first family, called also snakes, uses an explicit representation of the contours. The second one adopts an implicit representation of the boundaries using level set methods [3]. Geometric active contour models have solved many problems that limited the parametric ones such as objects with deep concavities, poor initialization and multiple objects detection. However, active contour models fail to overcome clutter, occlusion and noise. One way to cope with those problems is to include shape prior information in the model.

In [6], Leventon & al. associate a statistical shape model to geodesic active contours. A set of training shapes is used to define a Gaussian distribution of shape. At each step of the surface evolution, the maximum a posteriori position and the template. Transformation parameters are estimated thanks to a method of shape alignment presented by Ghorbel in [10]. In section II, we start by describing the outlines of the used shape alignment method. The way a shape prior is introduced to the region-based active contours is presented in Section III. Experimental results are presented and commented in Section IV. Finally, we conclude the work and highlight some possible perspectives in Section V.

II. SHAPE ALIGNMENT USING FOURIER DESCRIPTORS

The closed contour of a planar object can be represented by a parametric equation:

$$\gamma : [0,2\pi] \rightarrow C$$
$$l \mapsto x(l) + iy(l),$$

(1)

for all integers $k$, the Fourier descriptors of $\gamma$ are given by:

$$C_k(\gamma) = \int_0^{2\pi} \gamma(l)e^{-ikl}dl, k \in \mathbb{Z},$$

(2)

Let $\gamma_1$ and $\gamma_2$ be centred (according to the center of mass) and arclength parameterizations of two closed planar curves having shapes $F_1$ and $F_2$. Suppose that $\gamma_1$ and $\gamma_2$ have a similar shape under rigid transformation. In shape space this is equivalent to have:

$$\gamma_1(l) = a e^{i\theta} \gamma_2(l + l_0),$$

(3)

where $a$ is the scaling factor, $\theta$ the rotation angle, $l_0$ is the difference between the two starting points of $\gamma_1$ and $\gamma_2$. 
Parameters estimation is obtained by minimizing the following quantity introduced by Persoon & al. in [12]:

\[ E_{rr}(\gamma_1, \gamma_2) = \min_{(\alpha, \theta, l_0)} ||\gamma_1(l) - \alpha e^{i\theta} \gamma_2(l + l_0)||_{L^2}, \]  

(4)

If \( \alpha \) is equal to 1, Ghorbel showed in [10] that the following quantity:

\[ E_{rr}(\gamma_1, \gamma_2) = \min_{(\alpha, \theta, l_0)} ||\gamma_1(l) - e^{i\theta} \gamma_2(l + l_0)||_{L^2}, \]  

(5)

is a metric in shape space and corresponds to the Hausdorff distance between the two shapes. In practice, this result implies the uniqueness of the motion parameters so obtained. Using Fourier descriptors \( C_k(\gamma_1) \) and \( C_k(\gamma_2) \), minimizing \( E_{rr} \) becomes equivalent to minimize \( f(\theta, l_0) \) in Fourier domain:

\[ f(\theta, l_0) = \sum_{k \in Z} |C_k(\gamma_1) - \alpha e^{i(k l_0 + \theta)} C_k(\gamma_2)|^2, \]  

(6)

Persoon & al. proposed in [12] an analytical solution to compute \( l_0 \) and \( \theta \). \( l_0 \) is one of the zeros of the function:

\[ g(l) = \sum_k \rho_k \sin(\psi_k + kl) \sum_k \rho_k \cos(\psi_k + kl) - \sum_k k \rho_k \sin(\psi_k + kl) \sum_k \rho_k \cos(\psi_k + kl), \]  

(7)

where \( \rho_k e^{i\psi_k} = C_k^*(\gamma_1) C_k(\gamma_2) \). \( \theta \) is chosen to satisfy 8 and minimize \( f(\theta, l_0) \) where \( l_0 \) is one of the roots of 7.

\[ \tan \theta = -\frac{\sum_k \rho_k \sin(\psi_k + kl_0)}{\sum_k \rho_k \cos(\psi_k + kl_0)}. \]  

(8)

Having the value of \( \theta \) and \( l_0 \) that minimize \( f(\theta, l_0) \), the scaling factor is given by:

\[ \alpha = \frac{\sum_k \rho_k \cos(\psi_k + kl_0 + \theta)}{\sum_k C_k^*(\gamma_2) C_k(\gamma_2)}. \]  

(9)

Based on Nyquist-Shannon theorem and B-spline approximation of the \( g(l) \) curve, Ghorbel & al. proposed in [11] the first implementation of this method of shape alignment and its application in rigid motion estimation for video compression.

### III. SHAPE PRIOR EMBEDDING IN REGION-BASED ACTIVE CONTOURS EQUATION

First, we start by recalling the principle of region-based active contours, then we describe the proposed shape prior function using contours alignment. Finally we give the entire evolving equation.

#### A. Region-based active contours model

The region-based active contours model [4] try to stop the evolution of the curve with an energy minimization approach rather than using an edge-stopping function. Consider a simple case where the image \( u_0 \) is formed by two regions of piecewise constant intensity. The proposed energy function is

\[ E(\gamma, c_1, c_2) = \mu \cdot \text{Length}(\gamma) + \nu \cdot \text{Area}(\text{inside}(\gamma)) + \lambda_1 \cdot \int_{\text{inside}(\gamma)} |u_0(x, y) - c_1|^2 \, dx \, dy \]

\[ + \lambda_2 \cdot \int_{\text{outside}(\gamma)} |u_0(x, y) - c_2|^2 \, dx \, dy, \]  

(10)

were \( \mu, \nu \geq 0 \) and \( \lambda_1, \lambda_2 \geq 0 \) are fixed parameters. The two real \( c_1 \) and \( c_2 \) are respectively the averages of grey level intensities inside and outside \( \gamma \). The first two terms help the curve be regular, and the two others correspond to the energy inside and outside the curve. The minimum of the previous energy is obtained when the curve is on the boundaries of the object to be detected. This energy can be formulated using level set methods: the evolving curve \( \gamma \) can be represented by the zero level set of a signed function \( \phi_0 \). Now consider the Heaviside function \( H \), and the Dirac measure \( \delta \).

If we consider \( z \) as a level of \( \phi \) we can write:

\[ H(z) = \begin{cases} 1, & i f \ z \geq 0 \\ 0, & i f \ z < 0 \end{cases}, \]

(11)

\[ \delta(z) = \frac{dH(z)}{dz}, \]

(12)

Let \( \Omega \) be the image domain, then the area of the region inside the curve is just the integral of the Heaviside function of \( -\phi \). The gradient of the Heaviside function defines the curve, so integrating over this region gives the length of the contour.

\[ \text{Length}(\phi = 0) = \int_{\Omega} \delta(\phi) \, |\nabla \phi(x, y)| \, dx \, dy \]  

(13)

Therefore the energy function \( E(\gamma, c_1, c_2) \) can be written as

\[ E(\gamma, c_1, c_2) = \mu \cdot \int_{\Omega} \delta(\phi) \, |\nabla \phi(x, y)| \, dx \, dy \]

\[ + \nu \cdot \int_{\Omega} H(-\phi(x, y)) \, dx \, dy \]

\[ + \lambda_1 \cdot \int_{\Omega} |u_0(x, y) - c_1|^2 H(-\phi(x, y)) \, dx \, dy \]

\[ + \lambda_2 \cdot \int_{\Omega} |u_0(x, y) - c_2|^2 H(\phi(x, y)) \, dx \, dy, \]  

(14)

where \( c_1 \) and \( c_2 \) are computed as:

\[ c_1 = \frac{\int_{\Omega} u_0(x, y) \cdot H(-\phi(x, y)) \, dx \, dy}{\int_{\Omega} H(-\phi(x, y)) \, dx \, dy}, \]  

(15)

and

\[ c_2 = \frac{\int_{\Omega} u_0(x, y) \cdot H(\phi(x, y)) \, dx \, dy}{\int_{\Omega} H(\phi(x, y)) \, dx \, dy}, \]  

(16)

The discrete and linear evolving equation obtained in the Euler-Lagrange framework by estimating the curvature \( K_{ij} \) from \( \phi_0_{ij} \) is:

\[ \frac{\phi_0_{ij}^{n+1} - \phi_0_{ij}^n}{\Delta t} = \delta_z(\phi_0^n)(-\mu \cdot K_{ij} + \nu + \lambda_1 \cdot |u_{ij} - c_1|^2 - \lambda_2 \cdot |u_{ij} - c_2|^2), \]  

(17)

where \( \delta_z(z) \) is a regular form of \( \delta(z) \) proposed in [4].
B. Shape prior formulation

The following proposed geometric shape prior deals essentially with the problem of occlusion and help Chan & Vese model to converge to the target object in case of noise with less iterations (see section ). Consider two curves \( \gamma \) and \( \gamma_{ref} \) and their corresponding level set functions \( \phi \) and \( \phi_{ref} \). If we consider the case where \( \phi \) is positive outside the curve \( \gamma \) and negative inside, so if we multiply \( \phi \) by \( \phi_{ref} \), shape difference between the evolving curve and the template may be represented with an area having negative values as shown in figure 2. Hence, the proposed energy function to be minimized is:

\[
E_{pr} = \lambda_3 \cdot \int_{\Omega} H(g(x,y))dxdy,
\]

(18)

where \( g(x,y) = -\phi(x,y) \cdot \text{sign}(\phi_{ref}(x,y)) \). As we can see, this energy corresponds to the area of occlusion weighted by \( \lambda_3 \). This energy is minimum only if the curve \( \gamma \) corresponds to the true contour of the object to be detected.

In other words, \( \lambda_2 \) will be proportional to \( 1/\text{iter} \) where \( \lambda_3 \) will be proportional to \( \text{iter} \) (\( \text{iter} \) represents the current iteration). Thus, the total evolution equation is:

\[
\frac{\phi_{ij}^{\text{iter}+1} - \phi_{ij}^{\text{iter}}}{\Delta t} = \delta_x (\phi_{ij}^n)(-\mu \cdot K_{i,j} + \nu + \lambda_1 \cdot |u_{i,j} - c_1|^2
- \lambda_2 \cdot |u_{i,j} - c_2|^2) + \lambda_3 \cdot \delta_x (g_{ij}^n),
\]

(20)

IV. EXPERIMENTAL RESULTS

In this section, we present a set of experimental results and discuss parameter effects. The alignment procedure to estimate parameters transformation between the evolving curve and the template is done only once. We fixed \( \lambda_3 = 100 \cdot \text{iter} \).

To minimize the region of occlusion, \( \phi \) should increase at the points of that region so that the curve moves towards the right contour, that is why \( \delta_x (g_{ij}^n) \) should be zero for every point which is not in that area. We start by applying the model on synthetic images, then we present some results obtained in medical images. The used templates are given in figure 3. We begin with a complex and partially occluded shape. We show in figure 4 how the new energy helps the model to deal with such a situation and detect the right contour. Figure 5 shows how the new added energy helps to complete the missed contour. We fixed \( \lambda_3 = -100 \cdot \text{iter} \). Figure 6 deals with a situation where the object to be detected is rotated and partially occluded. Figure 7 shows the robustness to noise. Without shape prior, the Chan & Vese model [4] will not be able to converge to the target object. The second part of experiments deals with medical imaging. We have experimented the presented model on two images to segment a specific objects which are the left ventricle of the heart and the femur. The used templates are presented in figure 8. With these two last examples, we show the proposed shape prior benefits to constraint the active contour model to be similar.
V. CONCLUSION

A new formulation of region-based active contours with shape prior has been presented. The proposed shape prior energy constraints the evolving curve to be similar to the template and to converge to the target shape in less iterations. A shape alignment method is used to estimate the rigid transformation (rotation angle, starting point and scale factor) between the region-based active contour and the template. Experimental results on synthetic and medical images show the ability of the proposed model to evolve toward the shape of interest in presence of noise and occlusion. Now, we plan to extend the shape prior to affine transformations [13] that are more general than the Euclidean ones.

REFERENCES