FOURIER-BASED INVARIANT SHAPE PRIOR FOR SNAKES

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ABSTRACT

A novel method of parametric active contours with geometric shape prior is presented in this paper. The main idea of the method consists in minimizing an energy functional that includes an additional information on a shape reference called “template” or “prototype”. Prior shape knowledge is introduced throw a complete family of Euclidean invariants, computed from the Fourier descriptors of the evolving contour and the prototype. It enhances the model robustness to noise and occlusion, and allows it to evolve in highly concave boundaries. The variational formulation of the proposed approach is described in details. Experimental results on both synthetic and real images are presented and discussed.

Key words : Snakes, active contours, shape prior, Fourier descriptors, Euclidean invariants.

1. INTRODUCTION

In this work, we introduce a prior information on the snake model by Kass & al. [1], by means of Euclidean invariant descriptors computed from the Fourier transform of closed-contours. During the last two decades, several approaches incorporating shape prior information have been presented.

First, Duncan and Staib [2] propose to determine the parameters of a Gaussian probability distribution that associates the object boundaries to a range of shapes. If the prior is not available, a uniform distribution is used. Prior distributions can be estimated from a sample shape by decomposing the model parameters and collecting statistics. The optimization problem is then performed by the maximum a posteriori using Bayesian rule. Zhong & al. present an affine-invariant deformable contour [3] in a Bayesian framework. They introduce a new internal energy to define the global and local shape deformation of the contours between the shape domain and the image domain. Diffusion-snakes presented in [4] use a modified Mumford-Shah functional to allow the incorporation of statistical shape prior in a single energy functional. A variational method is then used to minimize the snake energy.

As can be seen, most of existing works uses statistics to include shape prior in segmentation methods. Recently, a different approach [5] propose to introduce a function of distance between a region-based active contour and a shape reference using a set of descriptors based on Legendre moments. The minimization of that distance in an Eulerian formulation constrains the active contour convergence to the target template. In this work, we present a new method of snakes that integrate contour shape prior. A complete family of invariants [6] is used to constrain the curve evolution to a closed-contour of reference.

This paper is organized as follows. Section 2 recalls some basic facts about contour-based invariant description of shapes that are used later in the prior snake model detailed in Section 3. Experimental results on both synthetic and real images are presented and discussed in Section 4.

2. FOURIER INVARIANT FAMILY

Let Γ denotes a discrete arclength parametrization of a closed-curve with N points : $\Gamma(n) = x(n) + i\,y(n)$, $n = 1, \cdots, N$, where $x(n)$ and $y(n)$ are given according to the barycenter of Γ, see Fig. 1. Without loss of generality, the curve is normalized so that its length equals one.

Since the parametrization Γ is periodic, it can be expanded into Fourier series :

$$C_k(\Gamma) = \frac{1}{N} \sum_{n=0}^{N-1} \Gamma(n)\, e^{-j\frac{2\pi n k}{N}},$$

for $k = -\frac{N}{2}, \cdots, \frac{N}{2} - 1$. The set of complex coefficients

$$I_k(\Gamma) = C_{k_0}^{-1}(\Gamma)\, C_k(\Gamma), \quad k_0 \neq 0,\ C_{k_0} \neq 0, \qquad (1)$$

forms a complete family of shape descriptors invariant to translation, rotation and scale factor [6]. Invariance regarding the initial description point is not required by our model.

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[†] M. A. Charmi performed the work while at Institut Fresnel.

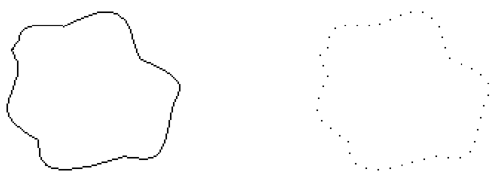


Fig. 1. A closed-curve and a discrete Euclidean parametrization by constant arclength ($N = 47$).

The *completeness* property means that it is possible to reconstruct the contour points from the invariant descriptors and the coefficient C_{k_0} using

$$C_k(\Gamma) = C_{k_0}(\Gamma)\, I_k(\Gamma), \quad k = -\frac{N}{2}, \cdots, \frac{N}{2} - 1, \tag{2}$$

and the inverse DFT

$$\Gamma(n) = \sum_{k=-N/2}^{N/2-1} C_k(\Gamma)\, e^{j\frac{2\pi nk}{N}}. \tag{3}$$

Remark 1 : The Fourier harmonic C_{k_0} contains the information about the orientation and size of Γ and acts as a normalization parameter on the Fourier spectrum. Hence, the set of invariants $\{I_k(\Gamma)\}$ can be interpreted as the Fourier spectrum of a canonical curve, with an orientation and size which serve as a reference.

Remark 2 : The simple family in Eq. (1) is not stable [7], i.e. *Stability* is the property which ensures that a small variation of contour points induces a small modification on invariant values. However, if C_{k_0} is big compared to other harmonics, numerical stability is ensured.

3. EMBEDDING SHAPE PRIOR INTO SNAKES

A snake is a discrete parameterized curve $\Gamma(n) = (x(n), y(n))$ which evolves under the influence of both internal energy (regularity and smoothness of the curve) and external energy (image data) : $E_{snake} = E_{int} + E_{ext}$. For more explanations about the snakes model, readers can refer to [1, 8, 9].

To introduce shape prior information, we propose to add a new energy term that guides the active contour in the image to a given template Γ_{ref}, independently of its pose, orientation and size. At each iteration t of the algorithm (see Fig. 2) :

(1) First, we compute a linear mixture of the snake invariants at time t and the template invariants according to

$$I_k(\Gamma'_t) = (1 - c_{k,t})\, I_k(\Gamma_t) + c_{k,t}\, I_k(\Gamma_{ref}), \tag{4}$$

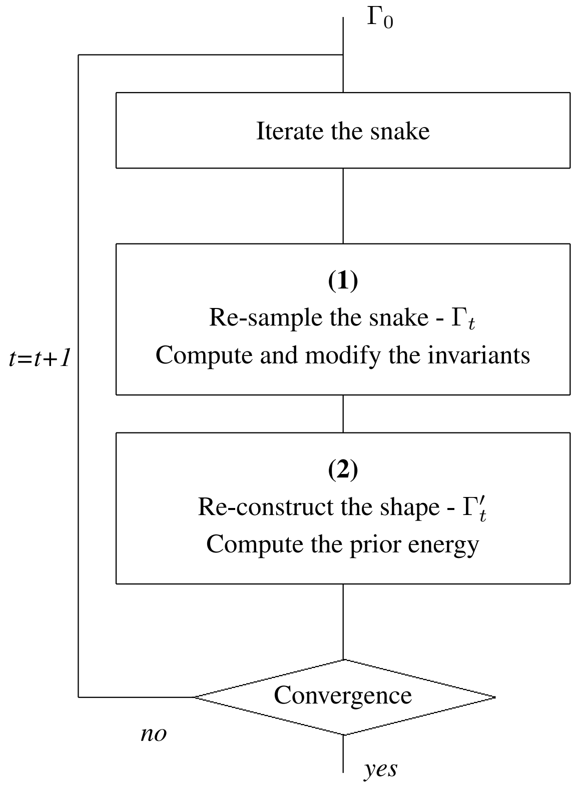


Fig. 2. Block-diagram of the proposed algorithm.

where $c_{k,t} \in [0, 1]$ is a weight function that depends on the harmonic order k and time t. That way, $I_k(\Gamma'_t)$ can be considered as the invariants set of a curve Γ'_t influenced by the classical snake evolution Γ_t and by the template contour Γ_{ref}.

For function $c_{k,.}$, we use a low-pass filter (e.g. Hamming window) in order to give more importance to low-order harmonics that represent the smooth shape of the template, and to reduce the influence of high-order Fourier coefficients that are generally noisy. We can also increase $c_{.,t}$ with t since more the snake moves under the image energy, more the estimation of transform parameters (rotation and scale factor) is better. And then, the information handled by the template invariants is more accurate.

(2) Second, we reconstruct Γ'_t using the completeness property of the invariant set, cf. Eq. (2)

$$C_k(\Gamma'_t) = C_{k_0}(\Gamma'_t)\, I_k(\Gamma'_t), \tag{5}$$

and the inverse Fourier transform from Eq. (3). Since we do not know harmonic C_{k_0} of curve Γ'_t, we put $C_{k_0}(\Gamma'_t) = C_{k_0}(\Gamma_t)$. Hence, according to the Remark 1 in section 2, Γ'_t is reconstructed with the same pose than Γ_t. According to Remark 2, the parameter k_0 is chosen, at each iteration, so that C_{k_0} is big compared to other harmonics.

We next define the prior shape energy as the difference between the snake Γ_t and the reconstructed shape Γ'_t after the invariants modification : $E_{prior} = \Gamma'_t - \Gamma_t$. This energy

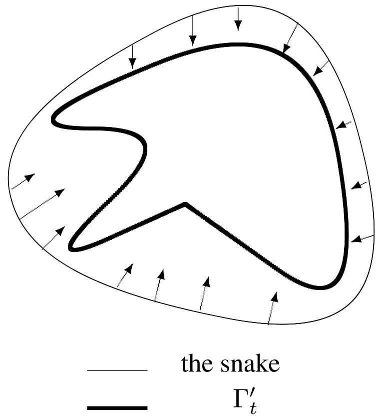


Fig. 3. Representation of the field of forces generated by the shape prior.

Table 1. Parameters used for experiments.

Image	c_1	c_2	N
4(b) & 4(e)	0.15	0.06	200
4(c) & 4(d)	0.15	0.06	300

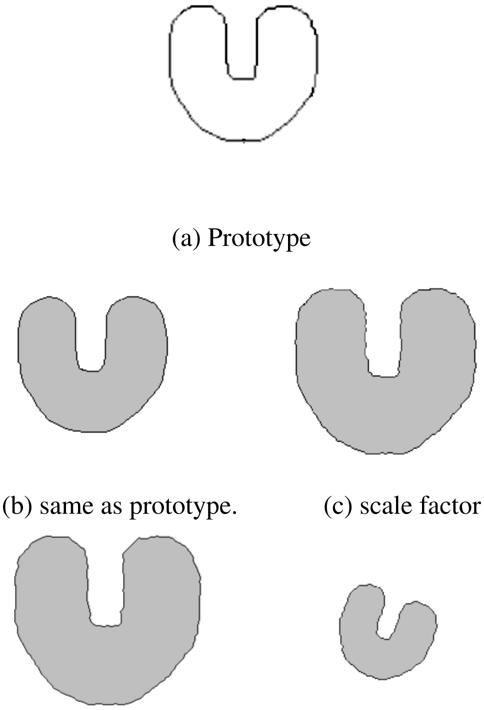
constrains the snake evolution to the reference shape, as illustrated in Fig. 3. The external energy becomes $E_{ext} = E_{image} + c_2\ E_{prior}$, where c_2 is a constant weight that is used to tune the impact of the shape prior forces with respect to the image energy, to find edges and lines of interest in the image.

4. EXPERIMENTAL RESULTS AND DISCUSSION

In the first example, we perform the segmentation of a "U" shape in many different positions as shown in Fig. 4(b)→(e), using Fig. 4(a) as a template. In each case, the snake fits the contour of the image perfectly. The snake overcomes the problem of evolution in concave boundaries whatever the pose of the contour and successes to find the good edges thanks to the additional external force coming from the prior shape.

In Fig. 5, we compare the result obtained with the new method with those from classical snakes [1], balloons model [8] and Gradient Vector Flow (GVF) [9]. As expected, Kass' snakes and the balloons model can not solve the problem of evolution in concave boundary while the proposed method and the GVF model give the same results.

Another experiment, presented in Fig. 6, shows a plier lying on a non uniform gray-level background. The plier is characterized by a very important concavity and results in Fig. 7 shows that the GVF model in unable to recover the correct boundaries, while the proposed algorithm successes. In fact, GVF has a large capture relatively to smoothed image gradient used in other snakes methods [1, 8] but can not reach deep concavity.



(a) Prototype

(b) same as prototype. (c) scale factor

(d) scale and translation (e) scale and rotation

Fig. 4. Results of the novel method on a "U" shape for different transformations using the prototype shown in (a). The black line shows the snakes after convergence. The parameters used for experiments are reported in Table 1.

Compared to Kass' original algorithm, the complexity of the novel algorithm increases because of the direct and inverse DFT computations, at each iteration. These DFT can be implemented using a Fast Fourier Transform (FFT) algorithm, by choosing an arclength parametrization with $N = 2^m$ contour points.

5. CONCLUSION

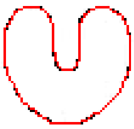
A new method of snakes with contour-based geometric shape prior has been presented in this paper. It uses information from a prototype and grey levels to guide the curve to desired features. The shape prior is incorporated as an additional external energy throw a complete and convergent family of rotation and scale invariants [6].

Experimental results on synthetic and real images show the model ability to evolve in deep concavities, where the GVF model fails. The experiments we have performed are not intended to conclude on the superiority of our method since we have compared a method which takes into account a prior

Kass [1].

Balloon [8].

GVF [9].



Proposed method.

Fig. 5. Comparison between the novel algorithm and other snake methods.

knowledge with others that do not. The goal is rather to show how contour-based prior information can be incorporated for segmentation and, by this way, to solve some difficult shape configurations for snakes.

As perspectives for further work, we plan to test robustness to initialization, noise and clutter, and especially when an approximative sketch is used as a template. The extension of the algorithms to more general geometrical transformations such as affine ones [10] is also an interesting perspective.

(a) The template.

(b) The image and the snake after convergence.

Fig. 6. The segmentation of a pliers using the proposed model of snakes.

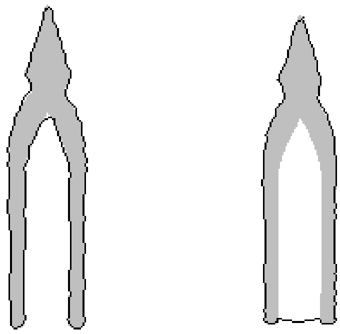


Fig. 7. The proposed method (left) successes to find the pliers edges while the GVF fails (right).

6. REFERENCES

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