Some details about a few copulas

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Table 1 gives details about a few copulas of interest (pdf, range for parameter and Kendall’s Tau). Among the numerous shapes, they have been selected as they are one-parameter ($\theta$) and since there exist a closed-form relation between $\theta$ and $\tau$, the Kendall’s tau. So that we write either $c_p(u_1, u_2; \theta)$ or $c_p(u_1, u_2; \tau)$. For the Student’s copula the degree of freedom $\nu$ is supposed to be known.

Figures 1 and 2 show pdf and cdf shapes for copulas #1 to #6 (table 1), all with a Kendall’s $\tau$ equal to 0.16 (this set of copulas is noted $\Pi_{0.16}$). Figures 3 and 4 report pdf and cdf shapes for copulas #1 to #3 and #6 to #8 (table 1), all with a Kendall’s $\tau$ equal to 0.7 (this set of copulas is noted $\Pi_{0.70}$).
\[
\frac{\frac{\beta^2}{2} - 1}{\log(1 + \frac{\theta^2}{2})} \left( \frac{1}{\log(1 + \frac{\theta^2}{2})} \right)
\]

where

\[
(1 - \frac{\alpha^2}{2}) = \frac{\theta}{2} \text{ and } \theta = \frac{1}{\alpha^2}
\]
Figure 1: Shape of copula densities when $\tau = 0.16$. Value of $\theta$ is provided for each copula.
(a) FGM - $\theta = 0.72$

(b) Cubic section - $\theta = 0.24$

(c) Clayton - $\theta = 0.38$

Figure 2: Table 1.
Figure 3: Shape of copula densities when $\tau = 0.70$. Value of $\theta$ is provided for each copula.
(a) Clayton - $\theta = 4.66$

(b) A12 - $\theta = 2.22$

(c) A14 - $\theta = 2.83$

Figure 4: Table 3.