Electromagnetic force on a metallic particle in the presence of a dielectric surface

P. C. Chaumet and M. Nieto-Vesperinas

Instituto de Ciencia de Materiales de Madrid, Consejo Superior de Investigaciones Científicas, Campus de Cantoblanco, Madrid 28049, Spain

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By using a method, previously established, to calculate electromagnetic fields, we compute the force of light upon a metallic particle. This procedure is based on both Maxwell's stress tensor and the couple dipole method. With these tools, we study the force when the particle is over a flat dielectric surface. The multiple interaction of light between the particle and the surface is fully taken into account. The wave illuminating the particle is either evanescent or propagating depending an whether or not total internal reflection takes place. We analyze the behavior of this force on either a small or a large particle in terms of the wavelength. A remarkable result obtained for evanescent field illumination is that the force on a small silver particle can be either attractive or repulsive depending on the wavelength. This behavior also varies as the particle becomes larger.

I. INTRODUCTION

Since the first demonstration of particle manipulation by the action of optical forces,^{1,2} optical tweezers³ and other configurations of light beams have been established to hold suspended particles like molecules, ⁴ or more recently, dielectric spheres.⁵⁻⁷ Also, the possibilities of creating microstructures by optical binding and resonance effects have been discussed⁸⁻¹² as well as the control of particles by evanescent waves.^{13,14} Only a few works exist on the interpretation, prediction, and control of the optical force acting on a small particle on a plane surface. To our knowledge, the only theoretical works dealing with this subject are those of Refs. 15-17. In Ref. 15 no multiple interaction of the light between the particle and the dielectric surface is considered. On the other hand, Ref. 16 deals with a two-dimensional (2D) situation. Only recently in Ref. 17 the full 3D case with multiple scattering was addressed for dielectric particles.

This paper, extends the study of Ref. 17 to metallic particles and, as such, this is the first theoretical study of light action on a metallic particle. We shall therefore present a rigorous procedure to evaluate the electromagnetic force in three dimensions. Further, we shall analyze how this force depends on the wavelength, distance between the particle and the surface, angle of incidence (whether the excitation is a plane propagating or an evanescent wave), and on the excitation of plasmons on the sphere. We shall make use of the couple dipole method previously employed, whose validity was analyzed in detail in Ref. 17.

In Sec. II we introduce a brief outline on the method used to compute the optical force on a particle. We also write its expression from the dipole approximation for a metallic sphere in the presence of a surface. Then, in Sec. III A we present the results and discussion obtained in the limit of a small sphere, and in Sec. III B we analyze the case of larger spheres compared to the wavelength.

II. COMPUTATION OF THE OPTICAL FORCES

The coupled dipole method (CDM) was introduced by Purcell and Pennypacker in 1973.¹⁸ In this paper we use this

procedure together with Maxwell's stress tensor¹⁹ in order to compute the optical forces on a metallic object in the presence of a surface. Since we developed this method in a previous paper,¹⁷ we shall now outline only its main features. It should be remarked that all calculations next will be written in cgs units for an object in vacuum.

The system under study is a sphere, represented by a cubic array of N polarizable subunits, above a dielectric flat surface. The field at each subunit can be written:

$$\mathbf{E}(\mathbf{r}_{i},\omega) = \mathbf{E}_{0}(\mathbf{r}_{i},\omega) + \sum_{j=1}^{N} \left[\mathbf{S}(\mathbf{r}_{i},\mathbf{r}_{j},\omega) + \mathbf{T}(\mathbf{r}_{i},\mathbf{r}_{j},\omega) \right] \alpha_{j}(\omega) \mathbf{E}(\mathbf{r}_{j},\omega)$$
(1)

where $\mathbf{E}_0(\mathbf{r}_i, \omega)$ is the field at the position \mathbf{r}_i in the absence of the scattering object, and \mathbf{T} and \mathbf{S} are the field susceptibilities associated to the free space²⁰ and the surface,^{21,22} respectively. $\alpha_i(\omega)$ is the polarizability of the *i*th subunit. Like in Ref. 17 we use the polarizability of the Clausius-Mossotti relation with the radiative reaction term given by Draine:²³

$$\alpha = \frac{\alpha_0}{1 - (2/3)ik_0^3 \alpha_0},$$
 (2)

where α_0 holds the usual Clausius-Mossotti relation $\alpha_0 = a^3(\varepsilon - 1)/(\varepsilon + 2)$.^{17,24} In a recent paper,²⁵ we have shown the importance to compute the optical forces taking into account the radiative reaction term in the equation for the polarizability of a sphere. For a metallic sphere, the polarizability is written as $\alpha = \alpha_0 [1 + (2/3)ik_0^3 \alpha_0^*]/D$ with $D = 1 + (4/3)k_0^3 \operatorname{Im}(\alpha_0) + (4/9)k_0^6 |\alpha_0|^2$, where the asterisk stands for the complex conjugate and $\Im m$ denotes the imaginary part.

The force²⁶ at each subunit is²⁵

$$F_{k}(\mathbf{r}_{i}) = (1/2) \operatorname{Re} \left[\alpha_{i} E_{il}(\mathbf{r}_{i}, \boldsymbol{\omega}) \left(\frac{\partial}{\partial k} E^{l}(\mathbf{r}, \boldsymbol{\omega}) \right)_{\mathbf{r}=\mathbf{r}_{i}}^{*} \right], \quad (3)$$



FIG. 1. Geometry of the configuration considered. Sphere of radius *a* on a dielectric flat surface (ϵ =2.25). The incident wave vector **k** is in the *XZ* plane.

where k and l stand for the components along either x, y or z, and Re denotes the real part. The object is a set of N small dipoles so that it is possible to compute the force on each one from Eq. (3). Hence, to obtain the total force on the particle, it suffices to sum the contributions $\mathbf{F}(\mathbf{r}_i)$ on each dipole.

Being the object under study a small sphere located at $\mathbf{r}_0 = (0,0,z_0)$ (see Fig. 1), we can employ the dipole approximation, and hence use directly Eq. (3) with N=1. Within the static approximation for the field susceptibility associated to the surface (SAFSAS) (that is to say $k_0=0$), we have found an analytical expression for $\mathbf{E}(\mathbf{r}_0,\omega)$ that yields the force components:¹⁷

$$F_{x} = \frac{\text{Re}}{2} \left[4 \alpha z_{0}^{3} (ik_{x})^{*} \left(\frac{2|E_{0x}|^{2}}{8z_{0}^{3} + \alpha \Delta} + \frac{|E_{0z}|^{2}}{4z_{0}^{3} + \alpha \Delta} \right) \right], \quad (4)$$

$$F_{z} = |E_{0x}|^{2} \frac{\operatorname{Re}}{2} \left(\frac{8z_{0}^{3}\alpha(ik_{z})^{*}}{8z_{0}^{3} + \alpha\Delta} + \frac{12z_{0}^{2}|\alpha|^{2}\Delta}{|8z_{0}^{3} + \alpha\Delta|^{2}} \right) + |E_{0z}|^{2} \frac{\operatorname{Re}}{2} \left(\frac{4z_{0}^{3}\alpha(ik_{z})^{*}}{4z_{0}^{3} + \alpha\Delta} + \frac{6z_{0}^{2}|\alpha|^{2}\Delta}{|4z_{0}^{3} + \alpha\Delta|^{2}} \right).$$
(5)

for *p*-polarization, and

$$F_{x} = |E_{0y}|^{2} \frac{\text{Re}}{2} \left[\frac{8z_{0}^{3}\alpha(ik_{x})^{*}}{8z_{0}^{3} + \alpha\Delta} \right],$$
(6)

$$F_{z} = |E_{0y}|^{2} \frac{\text{Re}}{2} \left(\frac{8z_{0}^{3}\alpha(ik_{z})^{*}}{8z_{0}^{3} + \alpha\Delta} + \frac{12z_{0}^{2}|\alpha|^{2}\Delta}{|8z_{0}^{3} + \alpha\Delta|^{2}} \right).$$
(7)

for s-polarization, with $\Delta = (1 - \epsilon)/(1 + \epsilon)$ being the Fresnel coefficient of the surface. We have assumed a dielectric surface, hence Δ is real.

From Fig. 1 is easy to see that k_x is always real whatever the angle θ , hence we can write Eqs. (4) and (6) for metallic particles as

$$F_{x} = \left(\frac{|E_{0x}|^{2} 64z_{0}^{6}}{|8z_{0}^{3} + \alpha\Delta|^{2}} + \frac{|E_{0z}|^{2} 16z_{0}^{6}}{|4z_{0}^{3} + \alpha\Delta|^{2}}\right) [k_{x} \operatorname{Im}(\alpha_{0})/(2D) + k_{x}k_{0}^{3}|\alpha_{0}|^{2}/(3D)]$$

$$(8)$$

for *p* polarization, and

$$F_{x} = \frac{|E_{0y}|^{2} 64z_{0}^{6}}{|8z_{0}^{3} + \alpha\Delta|^{2}} [k_{x} \operatorname{Im}(\alpha_{0})/(2D) + k_{x}k_{0}^{3}|\alpha_{0}|^{2}/(3D)]$$
(9)

for *s* polarization. In Eq. (8) the factor in front of $[k_x \operatorname{Im}(\alpha_0)/(2D) + k_x k_0^3 |\alpha_0|^2/(3D)]$ for the two polarizations constitutes the field intensity at z_0 . The first term within these square brackets corresponds to the absorbing force whereas the second represents the scattering force on the sphere. We see from Eqs. (8) and (9) that F_x always has the sign of k_x . Notice that it is not possible to write a general equation for the force along the Z direction, as k_z will be either real or imaginary, according to the angle of incidence.

III. RESULTS AND DISCUSSION

All forces calculated in this section are in cgs units with the modulus of the incident field normalized to unity.

A. Small particles

We first address a small isolated silver particle with radius a = 10 nm. In this case we can use the dipole approximation, hence we consider Eqs. (4)–(7) with $\Delta = 0$. Figure 2(a) presents the polarizability modulus $(|\alpha_0|)$ of the sphere. The maximum of the curve corresponds to the plasmon resonance, i.e., when the dielectric constant is equal to -2 in Drude's model. Notice that in this model the dielectric constant is real, on using experimental values,²⁷ the dielectric constant is complex and the resonance is not exactly at $\operatorname{Re}(\varepsilon) = -2$ but slightly shifted. In Fig. 2(b) we plot the real part of the polarizability [Re(α_0)], and Fig. 2(c) shows its imaginary part [Im(α_0)]. Figure 2(d) represents the force in free space computed from an exact Mie calculation (full line) and by the dipole approximation from Eqs. (4)–(7) with Δ =0 (dashed line) and Eq. (2) for α . In this case, the dipole approximation slightly departs from Mie's calculation between 350 nm and 375 nm. We can compute the polarizability α from the first Mie coefficient a_1 given by Dungey and Bohren (DB).²⁸ Therefore, the electric-dipole polarizability is $\alpha = 3ia_1/(2k_0^3)$.²⁹ The symbol + in Fig. 2(d) corresponds to the DB polarizability and it is exactly coincident with the Mie calculation. When the optical constant of the metallic sphere is close to the plasmon resonance, the calculation from the Clausius-Mossotti relation with the radiative reaction term, departs from the exact calculation, even for a small radius. We shall next use the polarizability of DB. Analytical calculations will always be done with Eq. (2) to get simple expressions, and thus a better understanding of the physics involved. The curve of the force obtained by



FIG. 2. From top to bottom: the first three curves represent the modulus, the real part, and the imaginary part of the polarizability of a silver sphere with radius a = 10 nm versus the wavelength. The fourth curve is the force on this particle in free space. Plain line: Mie calculation, dashed line: polarizability of Clausius-Mossotti relation with the radiative reaction term, symbol +: DB polarizability.

Mie's calculation has exactly the same shape as the imaginary part of the polarizability, this is due to the fact that for a small metallic sphere the absorbing force is larger than the scattering force.

Next, we consider the small sphere on a dielectric plane surface as shown by Fig. 1. Illumination takes place from the dielectric side with $\theta = 0^{\circ}$, hence in vacuum $k_z = k_0$ and $k_x = 0$. Figure 3 represents the force in the *Z* direction from Eq. (7) versus *z* for different wavelengths. Far from the surface the force tends to the Mie limit. Near the surface the force decreases, and, depending on the wavelength, it can become negative. For a better understanding of this force we write Eq. (7) as



FIG. 3. Force along the Z direction on a silver sphere with a = 10 nm versus distance z in the dipole approximation. The angle of incidence is $\theta = 0^{\circ}$. With the static approximation (no symbol) and in an exact calculation (+) for **k**. Dashed line $\lambda = 360$ nm, plain line $\lambda = 270$ nm, and dotted line $\lambda = 500$ nm. The inset shows details of the zero force.

$$F_{z} = \frac{64z_{0}^{6}|E_{0}|^{2}}{|8z_{0}^{3} + \alpha\Delta|^{2}} \left[\frac{k_{0}}{2} \operatorname{Im}(\alpha_{0}) + \frac{k_{0}^{4}}{3} |\alpha_{0}|^{2} + \frac{3|\alpha_{0}|^{2}\Delta}{32z_{0}^{4}} \right],$$
(10)

having made the approximation $|\alpha| \approx |\alpha_0|$, and $D \approx 1$ since we have a small sphere compared to the wavelength ($k_0 a$ $\ll 1$). The factor in front of the bracket corresponds to the intensity of the field at z_0 . The first and the second terms within brackets represent the interaction between the dipole moment associated to the sphere and the incident field: the first term is the absorbing force whereas the second one corresponds to the scattering force, hence these forces are always positive. The third term is due to the interaction between the dipole and the field radiated by the dipole and reflected by the surface. We can consider this term as a gradient force exerted on the sphere due to itself via the surface. Hence, this force is always negative whatever the relative permittivity ε . Since this term is proportional to $1/z_0^4$, it becomes more dominant as the sphere approaches the surface, hence the force decreases. To derive the point z_0 at which the force vanishes, if such a point exists, let us assume the scattering force smaller than the absorbing force, then from Eq. (10) the zero force is

$$z_0^4 = \frac{3|\alpha_0|^2}{16k_0 \operatorname{Im}(\alpha_0)} \frac{\epsilon - 1}{\epsilon + 2}.$$
 (11)

This equation always has a solution. We find z_0 for the three wavelengths used to be: $\lambda = 270 \text{ nm}, z_0 = 7.9 \text{ nm}, \lambda$ = 360 nm, $z_0 = 12.8$ nm, $\lambda = 500$ nm, and $z_0 = 21.6$ nm. Now, z_0 (the location of the center of the sphere) must be larger than the radius a, or else, the sphere would be buried in the surface. Therefore the first of those values of z_0 is not possible. Hence, the force is always positive. Thus, the distance between the sphere and the surface is z = 2.8 nm and 11.6 nm for $\lambda = 360$ nm and 500 nm, respectively. These values are very close to those shown in the inset of Fig. 2. Notice that near the plasmon resonance for the sphere (λ ≈ 360 nm) both $|\alpha_0|$ and Im(α_0) are maxima, hence the force is very large when the sphere is far from the surface and, due to the third term of Eq. (10), which depends of $|\alpha_0|^2$, the decay of this force is very fast. We have used the SAFSAS at large distance. We plot in Fig. 2 with crosses the force obtained from an exact calculation for S with the dipole approximation. As we see, these crosses coincide with those curves obtained with the SAFSAS whatever the distance z. Yet, we obtain for dielectric spheres the following: near the surface the SAFSAS is valid, whereas far from the surface the sphere does not feel its presence, and, thus, whether using the tensor susceptibility associated to the surface in its exact form, or within the static approximation, has no influence.

We next consider the surface illuminated at angle of incidence θ larger than the critical angle: $\theta = 50^{\circ} > 41.8^{\circ} = \theta_c$. Now the transmitted electromagnetic wave above the surface is evanescent. We plot in Fig. 4 from Eqs. (4)–(7) the force on the sphere for the two polarizations in the Z direction versus the wavelength, and in the X direction in Fig. 5, when the sphere is located at $z_0 = 30$ nm. In Fig. 4 we see that the force in the Z direction is also either positive or negative. In a previous work,¹⁷ we have observed that the force on a



FIG. 4. Force along the Z direction on a silver sphere with a = 10 nm versus the wavelength λ in the dipole approximation. The angle of incidence is $\theta = 50^{\circ}$. Plain line: p polarization and dashed line: s polarization.

small dielectric sphere is always attractive when the sphere is located in an evanescent wave. This is no longer the case for a metallic sphere. To understand this difference, and as the two polarizations have the same behavior, we take the analytical solution for F_z with $k_z = i\gamma$ ($\gamma > 0$) for *s* polarization. Then Eq. (7) can be written:

$$F_{z} = \frac{|E_{0y}|^{2}}{|8z_{0}^{3} + \alpha\Delta|^{2}} \frac{\text{Re}}{2} [-\gamma 8z_{0}^{3}\alpha(8z_{0}^{3} + \alpha^{*}\Delta) + 12z_{0}^{2}|\alpha|^{2}\Delta]$$
(12)

On using the approximation $D \approx 1$ and $|\alpha| \approx |\alpha_0|$ we obtain

$$F_{z} = \frac{64z_{0}^{6}|E_{0y}|^{2}}{|8z_{0}^{3} + \alpha\Delta|^{2}} \left(-\frac{\gamma \operatorname{Re}(\alpha_{0})}{2} - \frac{\gamma |\alpha_{0}|^{2}\Delta}{16z_{0}^{3}} + \frac{3|\alpha_{0}|^{2}\Delta}{32z_{0}^{4}}\right)$$
(13)



FIG. 5. Force along the *X* direction on a silver sphere with a = 10 nm versus the wavelenght λ in the dipole approximation. The angle of incidence is $\theta = 50^{\circ}$. Plain line: *p* polarization and dashed line: *s* polarization.

As shown by Fig. 3, when the sphere is located at z_0 =30 nm, the influence of the surface becomes negligible, thus we can use Eq. (13) with the hypothesis that z_0 is large. Hence, $F_z \approx -|E_{0y}|^2 \gamma \operatorname{Re}(\alpha_0)/2$. This is the gradient force due to the incident field, and therefore due to the interaction between the dipole associated to the sphere and the applied field. This force exactly follows the behavior of $\operatorname{Re}(\alpha_0)$ (cf. Fig. 2). When $\operatorname{Re}(\alpha_0)$ is negative, the dipole moment of the sphere oscillates in opposition to the applied field and so the sphere is attracted towards the weaker field. Notice that the same phenomenon is used to build an atomic mirror: for frequencies of oscillation higher than the atomic frequency of resonance, the induced dipole oscillates in phase opposition with respect to the field. The atom then undergoes a force directed towards the region of weaker field.³⁰ For ppolarization, the force can be written $F_z \approx -(|E_{0x}|^2)$ $+|E_{0z}|^2)\gamma \operatorname{Re}(\alpha_0)/2$. As the modulus of the field becomes more predominant in p polarization, the magnitude of the force becomes more important. We now search more carefully the change of sign in the force. Writing $\varepsilon = \varepsilon' + i\varepsilon''$ for the relative permittivity, we get

$$\operatorname{Re}(\alpha_0) = a^3 \frac{(\varepsilon'-1)(\varepsilon'+2) + \varepsilon''^2}{(\varepsilon'+2)^2 + \varepsilon''^2}, \quad (14)$$

$$\operatorname{Im}(\alpha_0) = a^3 \frac{3\varepsilon''}{(\varepsilon'+2)^2 + \varepsilon''^2} \tag{15}$$

If the damping is weak, then the change of sign of F_{z} happens both for $\varepsilon' \approx 1$ and at the plasmon resonance for the sphere, i.e., $\varepsilon' \approx -2$. Between these two values, the gradient force is positive. In fact, the limiting values of the positive gradient force are always strictly in the interval [-2,1] due to damping. For example, the force vanishes at $\lambda = 352$ nm with $\varepsilon = -1.91 + 0.6i$ and $\lambda = 317$ nm with $\varepsilon = 0.66 + 0.95i$. We notice that the change of sign happens steeply at the plasmon resonance since then the denominator of the real part of the polarizability becomes very weak [see Eq. (14)], hence the zero force is surrounded by the two maxima of the force (one positive and the other negative). At $\lambda = 317$ nm the change of sign is smoother, as in that case, the denominator is far from zero. The third case, $\lambda = 259$ nm, lies between those two previous cases as the damping of the relative permittivity is important: $\varepsilon = -1.65 + 1.12i$. We have also investigated the cases of gold and copper spheres, where a plasmon easily takes place, but we found a change of sign not possible for F_{τ} for these two materials as the damping is then too important: if $\varepsilon'' > 3/2$, then Re(α_0) is always positive whatever ε' . However, if the particle is embedded in a liquid with a relative permittivity 2, then it is possible to get $\operatorname{Re}(\alpha) \leq 0$ for gold. Notice that if θ is close to θ_c , then γ ≈ 0 and so we only have the third term of Eq. (13), then the force is always negative whatever the wavelength. In Fig. 5 we see, as previously, that the force in the X direction has the sign of k_x and, as the absorbing force is the most predominant one, the curve has the same shape as the imaginary part of the polarizability (cf. Fig. 2). In that case, the maximum of the force F_x is at the plasmon resonance [see Eq. (15)] namely, at $\lambda = 354$ nm.



FIG. 6. Force along the Z direction on a silver sphere with a = 10 nm versus distance z in the dipole approximation. The angle of incidence is $\theta = 50^{\circ}$. Without symbol: p polarization, with cross (+): s polarization. Plain line: $\lambda = 340$ nm, dashed line: $\lambda = 260$ nm, and dotted line: $\lambda = 317.5$ nm.

Figure 6 shows the force in the Z direction versus z for both s (symbol +) and p polarization (without any symbol) at three different wavelengths ($\lambda = 265,340,317.5$ nm) for $\theta = 50^{\circ}$. The behavior is the same for both polarizations, only appearing as a difference of magnitude, this is due to the component of the field perpendicular to the surface in ppolarization. All curves manifest that near the surface the force is attractive, this is due, as seen before, with a propagating wave, to the term of Δ/z_0^4 in Eqs. (10) and (13) which is always negative, irrespective of the kind of wave above the surface. At $\lambda = 317.5$ nm (dotted line), we have Re(α) =0 which is why the force very quickly goes to zero when z grows. The two other cases correspond to $\operatorname{Re}(\alpha) > 0$ (λ = 260 nm) and Re(α) < 0 (λ = 340 nm) and far from the surface the force tends to zero. As the force is proportional to $|\mathbf{E}_0|^2$, we have $F_z \propto e^{(-2\gamma z)}$.

Notice, that when the sphere is close to the surface, the Casimir-Polder force³¹ may be not negligible. In fact, as the light force depends on the intensity of the incident beam, in practice, a comparison of the two forces must be done for each specific configuration under study. In the case of a small sphere, either dielectric or metallic, in front of a dielectric plane surface, one can look at the discussion of Ref. 32.

B. Large particles

It is difficult to obtain convergence of the CDM calculations when the relative permittivity of the medium to be discretized is large. This imposes a very fine sampling. In this section, we use the range 250-355 nm for the wavelength, the real part of the relative permittivity being small. In that case, the difference between the force upon a sphere of radius a = 100 nm, in free space, calculated from the CDM and that obtained from the exact calculation³³ is less than 7% at the plasmon resonance, and outside this range it is less than 4%. In this range of wavelengths, we have a good convergence of the CDM, and in addition, this is the most interesting case as $Re(\alpha)$ crosses three times the axis $Re(\alpha)=0$ in this interval of wavelengths. We do not take



FIG. 7. Force along the Z direction on a silver sphere versus distance z with $\theta = 0$ and a = 100 nm for the following wavelengths: Plain line: $\lambda = 255$ nm, dashed line: $\lambda = 300$ nm, and dotted line: $\lambda = 340$ nm. Dots correspond to computed points.

into account the Casimir-Polder force yet, but it can be computed in first approximation from Ref. 34.

In Fig. 7 we plot the force in the Z direction for an incident propagating wave ($\theta = 0^{\circ}$) versus the distance between the sphere and the surface at three different wavelengths: λ =255 nm, 300 nm, and 340 nm. The calculations are done without any approximation. The curves have a similar magnitude and behavior at the three wavelengths. The forces present oscillations due to the multiple reflection of the radiative waves between the sphere and the surface, hence the period of these oscillations is $\lambda/2$. The magnitude of these oscillations depends on the reflectivity of the sphere, so the higher the Fresnel coefficient is, the longer these oscillations are. As expected, they are less remarkable when the sphere goes far from the surface. We notice that the decay of the force when the metallic sphere gets close to the surface is not comparable to that on a dielectric sphere (see Ref. 17). This is due to strong absorbing and scattering forces on the metallic sphere in comparison to the gradient force induced by the presence of the dielectric plane.

In Fig. 8 we plot for $\theta = 50^{\circ}$ the normalized force in the Z direction, i.e., $F_z/|\mathbf{E}_0|^2$, \mathbf{E}_0 being the field at z_0 in the absence of the sphere. We relate two important facts at this angle of incidence. First, the decay of the force when the sphere is near the surface is more important in *p* polarization. Notice that with the CDM it is not possible to numerically split the scattering, absorbing, and gradient forces. Therefore, when the sphere is large we shall argue on the set of dipoles forming it. In p polarization, due to the z component of the incident field, the dipoles also have a component perpendicular to the surface, and this is larger than in s polarization. Hence, in agreement with Fig. 6, due to this z component, the attraction of the sphere towards the surface is larger for p polarization. Second, all forces are positive when the sphere is far from the surface except for $\lambda = 300$ nm in p polarization. At this wavelength, for a small sphere, the force is negative for both s and p polarization, hence we can assume an effect due to the size of the sphere. Just to see this effect, we present in Fig. 9, the force in the Z direction versus the radius a, on a sphere located at $z_0 = 100$ nm for an angle of incidence $\theta = 50^{\circ}$ but without taking into account



FIG. 8. Force along the Z direction on a silver sphere with a = 100 nm versus distance z with $\theta = 50^{\circ}$ for the following wavelengths: Plain line: $\lambda = 255$ nm, dashed line: $\lambda = 300$ nm, and dotted line: $\lambda = 340$ nm. symbol +: *s*-polarization and without symbol: *p*-polarization.

the multiple interaction with the surface (i.e., S=0). We take the previous wavelength of Fig. 8 ($\lambda = 255$ nm, 300 nm, and 340 nm) more the wavelength at the plasmon resonance, λ = 351.5 nm, where Re(α_0) = 0. For small radius, we observe the same behavior as in the previous section for an incident evanescent wave. These curves show a dependence proportional to the cube of the radius as $\operatorname{Re}(\alpha_0) \propto a^3$. At the plasmon resonance, the force is slightly positive as there is no gradient force, but only weak absorbing and scattering forces. But as shown by Fig. 9, when the radius grows, in s polarization at $\lambda = 300$ nm, the force sign changes and it becomes positive around 82 nm as p polarization keep the same behavior. This confirms the fact that the positive force obtained in Fig. 8 for $\lambda = 300$ nm is only a size effect. For the cases $\lambda = 255$ nm, and 340 nm, the gradient force is positive in the Z direction, like the scattering and absorbing forces. Hence, the force, is always positive whatever the radius. Nevertheless, for $\lambda = 300$ nm, there is a negative gradient force, the two other forces being positive. As previously said, it is not possible to know the relative contribution of the different forces, but since the dipoles are mainly oriented along the direction of the incident electric field, namely, parallel to the surface for s polarization, and in the plane of incidence for p polarization, we can assume that due to the field radiative part in the normal direction, which is larger in s polarization, the absorbing and scattering forces acting on each subunit in the sphere, become relevant when its radius increases, thus counterbalancing the negative gradient force. At the plasmon resonance $\lambda = 351.5$ nm, when the radius grows, the absorbing and scattering forces become larger, but then as no gradient force exists, the force is lower than those obtained at $\lambda = 255$ nm and 340 nm. In fact, the curve shown at $\lambda = 340$ nm is the one showing the largest force contribution, due to the onset of the plasmon resonance, thus $|\alpha_0|$ and Im(α_0) are near their maximum, and Re(α_0) is close to its minimum. In this case, the gradient force is maximum and positive.

IV. CONCLUSIONS

We have presented a theoretical study of the optical forces acting upon a metallic particle on a dielectric plane



FIG. 9. Force along the Z direction on a silver sphere located at $z_0 = 100$ nm, with $\theta = 50^{\circ}$, versus the radius *a* for plain line: $\lambda = 255$ nm, dashed line: $\lambda = 300$ nm, dotted line: $\lambda = 340$ nm, and thick line: $\lambda = 351.5$ nm. Symbol +: *s*-polarization and without symbol: *p*-polarization. The interaction between the sphere and the surface is not taken into account.

surface either illuminated at normal incidence or under total internal reflection. This paper is done both with the coupled dipole method and Maxwell's stress tensor. We observe that when the incident wave is propagating, the difference between the force acting on a dielectric sphere and that on a metallic sphere stems from the absorbing force. Due to this contribution, the force upon a small silver sphere close to the dielectric surface can be positive in spite of the gradient force. The opposite happens with a dielectric sphere. The main difference between the two cases (dielectric and metallic) arises however on illumination under total internal reflection. In that case, the effect on a small silver sphere is completely different to that observed on a dielectric sphere. Depending on the wavelength, the gradient force due to the incident field can be either repulsive or attractive. The change of sign happens both at the plasmon resonance and when ε becomes close to one. In the interval between these two values the gradient force is positive. The explanation is very similar to that on the effect used to build an atomic mirror. At a wavelength where the gradient force on a small sphere is negative, we see that when the sphere radius grows, the force along the Z direction stays negative for p polarization, and it becomes positive for s polarization due to the size effect. Nevertheless, at any arbitrary wavelength and angle of incidence, as the sphere approaches close to the surface, the attraction of the surface on the sphere increases, repulsive forces diminish and even can change sign, eventually becoming attractive at certain wavelengths. Attractive forces, on the other hand, increase their magnitude.

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